ACTIVITY of STUDY/RESEARCH in the whole course of doctorate — XXIX cycle
Curriculum in Mathematics for Engineering

Pierluigi Vellucci

Courses and doctoral schools

During the whole course of doctorate, I have attended the following PhD courses:

• Complementi di Analisi Funzionale. Prof. Antonio Avantaggiati (2013/2014).

• Equazioni alle Derivate Parziali. Prof. D.Andreucci (2013/2014).

• Control Theory of Partial Differential Equations. Prof. V. Komornik (2013/2014).

• Homogenization techniques and applications to biological tissues. Prof. M. Amar (2014/2015).

• Modeling and simulation of emerging collective behavior. Prof. E. Cristiani, L. Pareschi and A. Tosin (2014/2015).

• Onde Nonlineari. Prof. P. D'Ancona (2014/2015).

Moreover, from 20.10.2014 to 24.10.2014 I have attended the doctoral school course "Computational Harmonic Analysis — with Applications to Signal and Image Processing " at CIRM, Jean Morlet Chair (Marseille). The organizing committee was composed by: Hans Georg Feichtinger (Aix-Marseille and Vienna); Bruno Torrésani (Aix-Marseille); Sandrine Anthoine (Aix-Marseille); Caroline Chaux (Aix-Marseille); Clothilde Mélot (Aix-Marseille). This winter school brought together PhD-students and young Postdocs (as well as a few experts) in the field of computational harmonic analysis, in order to explain the background and the efficiency as well as the range of applications of a number of numerical algorithms which are based on the Fourier-, the wavelet and the Short-Time Fourier Transform (Time-Frequency and Gabor Analysis), as well as other atomic decomposition techniques, in particular in higher dimensions (shearlets, curvelets, etc.).

Conferences

During the whole course of doctorate I have attended, as speaker, the following conferences:

• Computationally Assisted Mathematical Discovery and Experimental Mathematics 12-15 May 2016, London, Ontario, Canada.

• XIII Congresso Nazionale SIMAI — Bi-annual congress of the Italian Society of Industrial and Applied Mathematics 13-16 September 2016, Milano.

• XIII International Conference on Informatics in Control, Automation and Robotics, ICINCO, 29-31 July 2016, Lisbon, Portugal.

• XII International Conference on Informatics in Control, Automation and Robotics, ICINCO, 21-23 July 2015, Colmar, Alsace, France

• XII Congresso Nazionale SIMAI — Bi-annual congress of the Italian Society of Industrial and Applied Mathematics 7 -10 July 2014, Taormina (ME).

Research activities

My research interests, developed during the whole course of doctorate, are: orthogonal polynomials, non-integer bases, Riesz bases and frames. These researches were finalised to some areas of mathematical modeling, pure and applied mathematics: models of intracellular enzymatic interactions; signal reconstruction; Robotics (hyper-redundant manipulators); infinite nested square roots; π-formulas.

Basis Expansions. The topics of basis expansions in Mathematics are the main directions of my research activity, pursued mainly with Prof. A. M. Bersani and Prof. P. Loreti.

Exponential and Sinc Bases. It is well known that the system $\{e^{int}\}_{n\in\mathbb{Z}}$ is an orthonormal and a Riesz basis for $L^2(-\pi,\pi)$. The question on the *stability* of the exponential system consists in asking whether expansion property of the system is still valid if we replace n with a *perturbation* λ_n. Kadec's theorem is a classical and famous result which gives a criterion for the nodes $\{\lambda_n \in \mathbb{R} : n \in \mathbb{Z}\}$ so that $\{e^{i\lambda_n t}\}_{n\in\mathbb{Z}}$ forms a Riesz basis for $L^2(-\pi,\pi)$. The paper [**8**] took its origin from the cycle of lessons given by Prof. A. Avantaggiati (Complementi di Analisi Funzionale - 2013/2014) and contains a simple point of view to generalize Kadec's theorem to complex field.

The stability of exponential Riesz bases is related to *sampling theorem* and is also related to stability of another, important system — the *sinc basis* $\{\mathrm{sinc}(t-n)\}_{n\in\mathbb{Z}}$. If f represents the signal, assuming that $f \in L^2(\mathbb{R})$ (the energy of the signal is finite), then f is said band-limited to $[-\pi,\pi]$ if $\hat{f}$ vanishes outside the set $[-\pi,\pi]$, where $\hat{f}$ denotes the Fourier transform. The space of functions which are band-limited on $[-\pi,\pi]$ is the Paley-Wiener space, usually denoted by PW_π. The space PW_π plays a significant role in signal processing applications. As well known, any function $f \in PW_\pi$ can be expanded in terms of the orthonormal basis $\{e^{int}\}_{n\in\mathbb{Z}}$ (for $\hat{f}$) and $\{\mathrm{sinc}(t-n)\}_{n\in\mathbb{Z}}$ (for f). This is the Shannon's sampling theorem.

In [**1**] we perform a preliminary study on the perturbation of the sinc bases $\{\mathrm{sinc}(t-\lambda_n)\}_{n\in\mathbb{Z}}$, and in a paper already submitted [**12**], we give a more complete result. In [**5**] we introduce a mathematical model for the energy of the signal at the output of an ideal DAC, expressed as sinc expansion

$$f(t) = \sum_{n\in\mathbb{Z}} a_n \,\mathrm{sinc}(t-n)\,,$$

in presence of sampling clock jitter. The energy of the signal $f(t)$ is defined by

$$E_f := \int_{-\infty}^{\infty} |f(t)|^2 dt\,,$$

and we also denote jitter as ϵ_n. Then the signal at the output of an ideal DAC assumes the form

$$f(t) = \sum_{n\in\mathbb{Z}} a_n \,\mathrm{sinc}(t-\lambda_n),$$

where $\lambda_n = n + \epsilon_n$. Hence, when sampling clock jitter occurs, the energy of the signal at the output of an ideal DAC does not satisfy a Parseval identity. Nevertheless, an estimation of the signal energy is shown in [**5**] by a direct method involving sinc functions.

The paper presented at Proceedings of the 13-th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2016) [**5**] has been selected to be included in the series published by Springer [**17**] and its extended version is now in preparation.

Non-integer bases and Robotics. Non-integer bases — whose study was started by Rényi and Parry in the 1950s, and subsequently deepened by a group of Hungarian mathematicians led by Paul Erdös — were applied, in two previous papers ([**2**] and [**3**]), to mathematical models in Robotics.

In [**2**] the model of a robot finger is introduced. A configuration of a finger is the sequence $(\mathbf{x}_k)_{k=0}^{K} \subset \mathbb{R}^3$ of its junctions. The configurations of every finger are ruled by two phalanx-at-phalanx motions: extension and rotation. In particular, the length of k-th phalanx of the finger is either 0 or $\dfrac{f_k}{\rho^k}$. Parameter $\rho > 1$ is a fixed ratio: this choice is ruled by a binary control we denote by using the symbol u_k, so that the length l_k of the k-th phalanx is

$$l_k := \| \mathbf{x}_k - \mathbf{x}_{k-1} \| = \frac{u_k f_k}{\rho^k}.$$

Doing so we introduced a robot hand model composed by an arbitrarily large number of hyper-redundant binary planar manipulators, where the length of each link scales according to the Fibonacci sequence. In [**3**] we introduced the model of a planar hyper-redundant manipulator that is analogous in morphology to robotic snakes and tentacles, based on a discrete linear dynamical system involving the Fibonacci sequence. The hyper-redundant manipulator is controlled by a sequence of couples of discrete actuators on the junctions, ruling both the length and the orientation of every link.

In a recently submitted paper, [**4**], we introduce a control model for an octopus robot tentacle. The model extends to a continuous setting a class of hyper-redundant planar manipulators characterized by a self-similar structure. The proposed model encompasses manipulators with a links scaling according to Fibonacci sequence.

Orthogonal polynomials: the class of Lucas-Lehmer polynomials. In [**10**] we introduced a sequence of polynomials, which follow the same recursive rule of the well-known Lucas-Lehmer integer sequence: $L_n(x) = L_{n-1}(x)^2 - 2$, where $L_0(x) = x$. Lucas-Lehmer polynomials are related to the Chebyshev polynomials of the first and second kind. As we know, the Chebyshev polynomials of first and second kind ($T_n(x)$ and $U_n(x)$) satisfy the recurrence relations

$$\begin{cases} T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) \qquad n \geq 2 \\ T_0(x) = 1, \quad T_1(x) = x \end{cases}$$

and

$$\begin{cases} U_n(x) = 2xU_{n-1}(x) - U_{n-2}(x) \quad n \geq 2 \\ U_0(x) = 1, \quad U_1(x) = 2x \,, \end{cases}$$

respectively. In [**10**] we proved, among other properties, that

$$L_n(x) = 2\ T_{2^{n-1}}\left(\frac{x^2}{2} - 1\right) \quad \text{and} \quad \prod_{i=1}^{n} L_i(x) = U_{2^n-1}\left(\frac{x^2}{2} - 1\right).$$

In [**9**] we reinvestigated the structure of the solution of a well-known Love's problem, related to the electrostatic field generated by two circular co-axial conducting disks, in terms of orthogonal polynomial expansions, enlightening the role of the Lucas-Lehmer polynomials.

In [**11**] we discussed some relations between zeros of Lucas-Lehmer polynomials and Gray code. Gray code is a particular binary code which is widely used in Informatics. Given a binary code, we say that its **order** is the number of bits with which the code is built, while its **length** is the number of strings that compose it. The Gray code is a binary code of order n and length 2^n. If the code for $n - 1$ bits is formed by binary strings

$$\begin{array}{l} g_{n-1,1} \\ \cdots \\ g_{n-1,2^{n-1}-1} \\ g_{n-1,2^{n-1}} \end{array}$$

the Gray code for n bits is built from the previous one in the following way:

$$\begin{array}{l} 0g_{n-1,1} \\ \ldots \\ 0g_{n-1,2^{n-1}-1} \\ 0g_{n-1,2^{n-1}} \\ 1g_{n-1,2^{n-1}} \\ 1g_{n-1,2^{n-1}-1} \\ \ldots \\ 1g_{n-1,1} \end{array}$$

Just as an example, we have: for $n = 1$: $g_{1,1} = 0$; $g_{1,2} = 1$; for $n = 2$: $g_{2,1} = 00$; $g_{2,2} = 01$; $g_{2,3} = 11$; $g_{2,4} = 10$; for $n = 3$: $g_{3,1} = 000$; $g_{3,2} = 001$; $g_{3,3} = 011$; $g_{3,4} = 010$; $g_{3,5} = 110$; $g_{3,6} = 111$; $g_{3,7} = 101$; $g_{3,8} = 100$; and so on. We apply this binary law to the study of nested square roots of 2 expressed by

$$\sqrt{2 \pm \sqrt{2 \pm \sqrt{2 \pm \sqrt{2 \pm \ldots \pm \sqrt{2}}}}},$$

associating bits 0 and 1 to $\oplus$ and $\ominus$ signs in the nested form. This gives the possibility to obtain an ordering for the zeros of Lucas-Lehmer polynomials, which assume the form of nested square roots of 2 above. This paper was exposed by myself at a conference on the so-called "experimental mathematics" [**23**].

In a paper recently submitted [**13**], we obtain π as the limit of a sequence related to the zeros of the class of polynomials $L_n(x)$. The results obtained here are based on the placement of the zeros of the polynomials $L_n(x)$. Since zeros have a structure of nested radicals, in this way we can build infinite sequences of nested radicals converging to π.

Finally, the investigation of other properties of the Lucas-Lehmer polynomials — characterizing them as a class of classical orthogonal polynomials — is the subject of a paper in preparation [**18**].

Other projects. Many papers belong to different research areas. Below I will give a brief description of them.

Collective decision-making. Starting from the cycle of lessons for the PhD course, given by Prof. E. Cristiani, L. Pareschi and A. Tosin, we published two works [**6**], [**14**].

In [**6**] we introduce and discuss kinetic models describing the influence of the competence in the evolution of decisions in a multi-agent system. The original exchange mechanism, which is based on the human tendency to compromise and change opinion through self-thinking, is here modified to include the role of the agents' competence. In particular, we take into account the agents' tendency to behave in the same way as if they were as good, or as bad, as their partner: the so-called *equality bias.* This occurs, for example, in a situation where a wide gap separates the competence of the group members. We discuss the main properties of the kinetic models and numerically investigate some examples of collective decision under the influence of the equality bias. The results confirm that the equality bias leads the group to suboptimal decisions.

In a paper recently submitted [**14**], we discuss a novel microscopic model for collective decision-making interacting multi-agent systems. In particular we are interested in modeling the equality bias phenomena. We analyze the introduced problem showing the suboptimality of the collective decision-making in the presence of equality bias.

Elliptic integrals. In [**7**], power series in the complementary modulus for the first and second complete elliptic integrals are deduced in terms of binomial series, by exploiting a suitable decomposition of the integration domain. Despite the procedure is simple, it needs some results about binomial series proved in the appendix of the paper.

Mathematical models of biological systems. I am continuing the study of the topics covered in my graduate thesis, i.e. the mathematical models of enzyme kinetics, with particular attention to reactions where a substrate S binds reversibly to an enzyme E to form a complex C. The complex can decay irreversibly to a product P and the enzyme, which is then free to bind another substrate molecule. This is summarized in the scheme

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} C \xrightarrow{k_2} E + P\,,$$

where k_1, k_{-1}, and k_2 are kinetic parameters (supposed constant) associated with the reaction rates.

These reactions are usually treated by means of some approximations, in order to capture their biochemical properties. The most common ones are the standard quasi-steady-state approximation (sQSSA) and the more recent total QSSA (tQSSA). In our studies we are defining the correct mathematical foundations of these approximations, which are related to the presence of a sufficiently small parameter ϵ.

Let x be a scalar and $\mathbf{y}$ be a two-dimensional vector, i.e. $\mathbf{y} = (y_1, y_2)^t$. All variables are real, and ϵ is positive. We know that the sQSSA is related to Tihonov's theorem, which ensures that, under certain conditions, the solution $x(t,\epsilon)$, $\mathbf{y}(t,\epsilon)$ of the full initial value problem

$$\begin{aligned} \frac{dx}{dt} &= f(x,\mathbf{y}) \\ \epsilon\frac{d\mathbf{y}}{dt} &= \mathbf{g}(x,\mathbf{y}), \end{aligned}$$

$$x = \alpha, \quad \mathbf{y} = \boldsymbol{\beta}, \quad \text{for } t = 0$$

is connected with the solution $x_0(t)$, $\mathbf{y}_0(t) = \boldsymbol{\phi}(x_0(t))$ of the reduced problem

$$\begin{aligned} \frac{dx}{dt} &= f(x, \boldsymbol{\phi}(x)) \\ \mathbf{y} &= \boldsymbol{\phi}(x), \\ x = \alpha, &\quad \text{for } t = 0 \end{aligned}$$

by the limit relations

$$\begin{aligned} &\lim_{\epsilon\to 0} x(t,\epsilon) = x_0(t), && 0 \le t \le T_0 \\ &\lim_{\epsilon\to 0} \mathbf{y}(t,\epsilon) = \mathbf{y}_0(t) = \boldsymbol{\phi}(x_0(t)) && 0 < t \le T_0 \end{aligned}$$

for some number T_0. For the tQSSA, Tihonov's theorem is related to the Theory of Center Manifold in a way enlightened in a paper in preparation [**15**]. This paper was exposed in [**20**] by Prof. A. M. Bersani while its possible generalization to n-state biological systems (as, for example, the well-known Goldbeter-Koshland cycle and the double phosphorylation mechanism in a tQSSA framework) was exposed at SIMAI conference [**21**] by myself. Moreover, a preliminary study was exposed in a previous SIMAI conference [**19**] by myself.

An extended work of the talk [**21**] is now in preparation for a special volume dedicated to SIMAI2016 conference [**16**].

A recent collaboration with a research team of Policlinico is part of this line of research. In this case we focused on microRNAs (miRNAs), which are small non coding RNA molecules, composed of about twenty nucleotides, targeting specific portions of RNA messenger (mRNA) molecules, reducing their stability and enhancing translation repression. This study employs the experimental tests performed by the research team of Policlinico and it was exposed in [**22**] by Prof. A. M. Bersani.

Accepted papers

[1] A. Avantaggiati, P. Loreti, P. Vellucci, *An explicit bound for stability of Sinc Bases*, Proceedings of 12-th International Conference on Informatics in Control, Automation and Robotics (2015), 473–480.

[2] A.C. Lai, P. Loreti, P. Vellucci,*A model for robotic hand based on Fibonacci sequence*, Proceedings of 11-th International Conference on Informatics in Control, Automation and Robotics (2014), 577–584.

[3] A.C. Lai, P. Loreti, P. Vellucci, *A Fibonacci control system with application to hyper-redundant manipulators*, Math. Control Signal **28** (2) (2016), 1–32.

[4] A.C. Lai, P. Loreti, P. Vellucci, *A continuous Fibonacci model for robotic octopus arm*, European Modelling Symposium on Mathematical Modelling and Computer Simulation 2016 (in press).

[5] P. Loreti, P. Vellucci, *A Mathematical Model for Signal's Energy at the Output of an Ideal DAC*, Proceedings of 13-th International Conference on Informatics in Control, Automation and Robotics, (2016) 347–352.

[6] L. Pareschi, P. Vellucci, M. Zanella *Kinetic models of collective decision-making in the presence of equality bias*, Physica A, 467 (2017), 201 – 217.

[7] G. Riccardi, P. Vellucci, E. De Bernardis, *Asymptotic expansions of the complete elliptic integrals about unitary modulus*, CAIM. **5** (2015), 1–12.

[8] P. Vellucci, *A simple pointview for Kadec-1/4 theorem in the complex case*, Ric. Mat. **64** (1) (2014), 1–6.

[9] P. Vellucci, A. M. Bersani, *Orthogonal polynomials and Riesz bases applied to the solution of Love's equation*, Math. Mech. Complex Syst. **4** (1) (2016) 55–66.

[10] P. Vellucci, A. Bersani, *The class of Lucas-Lehmer polynomials*, Rend. Mat. Appl. **37** (7) (2016), 1–20 (in press).

[11] P. Vellucci, A. M. Bersani, *Ordering of nested square roots of 2 according to Gray code*, Ramanujan J. doi:10.1007/s11139-016-9862-5 (in press).

Submitted papers

[12] A. Avantaggiati, P. Loreti, P. Vellucci, *Kadec's* 1/4-*Theorem for Sinc Bases*, submitted to: Journal of Mathematical Analysis and Applications.

[13] P. Vellucci, A. M. Bersani, *New formulas for π involving infinite nested square roots and Gray code*, submitted to: Milan J. Math.

[14] P. Vellucci, M. Zanella, *Microscopic modeling and analysis of collective decision making: equality bias leads non optimal solutions*, submitted to: Annali dell'Università di Ferrara.

Papers in preparation

[15] A. M. Bersani, E. Bersani, A. Borri, P. Vellucci, *Dynamical aspects of the Total QSSA in Enzyme Kinematics*, to submit to: Nonlinear Dynamics.

[16] A. M. Bersani, A. Borri, A. Milanesi, P. Vellucci, *Tihonov approach for multidimensional systems in bio-informatics*, to submit to: Special volume of CAIM for SIMAI 2016.

[17] P. Loreti, S. Sarv Ahravi, P. Vellucci, *A Mathematical Model for Signal's Energy at the Output of an Ideal DAC*, to submit to: Lecture Notes in Electrical Engineering (LNEE), Springer.

[18] P. Vellucci, A.M. Bersani, *Lucas-Lehmer polynomials as a class of classical orthogonal polynomials.*

Talks

[19] A.M. Bersani, P. Vellucci, *Time Scale Separation, Normal Modes and Quasi-Steady State Approximations in Enzyme Kinetics*, XII congresso SIMAI, Hotel Villa Diodoro, Taormina (ME), 2014.

[20] A. M. Bersani, E. Bersani, A. Borri, P. Vellucci, *Theoretical foundations of the total quasi-steady state approximation in enzyme kinetics*, Abstracts Bringing Maths to Life (BMTL) 2015, Napoli.

[21] A. M. Bersani, A. Borri, A. Milanesi, P. Vellucci, *Tihonov approach for multidimensional systems in bio-informatics*, XIII congresso SIMAI, Politecnico di Milano, 2016.

[22] A. M. Bersani, E. Bersani, P. Vellucci, *Modelling miRNA intracellular regulation activity*, XIII congresso SIMAI, Politecnico di Milano, 2016.

[23] P. Vellucci, A.M. Bersani, *Nested square roots of 2 and Gray code*, Computationally Assisted Mathematical Discovery and Experimental Mathematics, London, Ontario, Canada, 2016.