Book of Abstracts

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Contents

A time slicing approach to parabolic equations on non-cylindrical domains

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Abstract.

Parabolic equations on domains or surfaces that change over time are now receiving a renewed interest, motivated by many different applications (see for instance the recent survey paper [7]). There are a number of ways to address these problems in the mathematical literature, including semigroup methods [1, 8], adding a time viscosity [4], mapping the spacetime domain to a cylindrical domain [2] or using De Giorgi's minimizing movements [6, 3]. We will introduce here a time slicing approach in order to study this type of problems [5].

References

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Some extensions of the div-curl lemma and applications

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Abstract.

Abstract. The classical div-curl lemma by F. Murat and L. Tartar asserts that the limit of the scalar product of two vectorial sequences converging weakly in L^p and L^q respectively (p and q conjugated exponents) converges weakly to the product of the limits if we also know that the divergence of one of the sequences and the curl of the other one are compact in a suitable topology. This result was extended in a work in collaboration with M. Briane and F. Murat to the case where p and q satisfy $1/p+1/q \leq 1+1/N$ (N the dimension of the space). Recently, in a work with M. Briane we have extended the result to $1/p+1/q < 1+1/(N-1)$ or even $1/p+1/q \leq 1+1/(N-1)$ with some equiintegrability condition. The result applies to the convergence of the Jacobian of a sequence converging weakly in a Sobolev space and to the homogenization of partial differential systems with unbounded coefficients.

Existence and multiplicity of bound-ground states for a system of NLS-KdV equations

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Abstract.

We will show existence of solutions for systems of coupled NLS-KdV equations,

$$\left\{\begin{array}{rcl} \mathrm{i}f_t + f_{xx} + \beta fg + |f|^2 f &=& 0, \quad x \in \mathbb{R}, \quad t > 0 \\ g_t + g_{xxx} + gg_x + \frac{1}{2}\beta(|f|^2)_x &=& 0, \quad x \in \mathbb{R}, \quad t > 0, \end{array}\right. \tag{1}$$

where $f(x,t) \in \mathbb{C}$, $g(x,t) \in R$, $\beta \in \mathbb{R}$ and $|f|$, $|g| \to 0$ as $|x| \to \infty$.

Precisely, we will show existence of positive bound and ground states for the corresponding stationary system (2) when one looks for solitary-traveling wave solutions of the form

$$(f(x,t), g(x,t)) = \big(e^{i\omega t} e^{i\frac{c}{2}x} u(x-ct), v(x-ct)\big)$$

with $\lambda_1 = \omega + \frac{c^2}{4}$, $\lambda_2 = c$, and u, v are real functions which correspond to solutions of the following stationary system

$$\left\{\begin{array}{rcl} -u'' + \lambda_1 u &=& u^3 + \beta uv \\ -v'' + \lambda_2 v &=& \frac{1}{2}v^2 + \frac{1}{2}\beta u^2. \end{array}\right. \tag{2}$$

References

[1] E. Colorado, *Existence of bound and ground states for a system of coupled nonlinear Schrödinger-KdV equations.* C. R. Acad. Sci. Paris Ser. I (2015), http://dx.doi.org/10.1016/j.crma.2015.03.011

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Pulsating solutions of a kinetic equation

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Abstract.

Abstract: I will briefly introduce a family of kinetic equations that appear in wave turbulence theory. Then I will present an existence result of a very particuler type of global solutions that exhibit an interesting type of asymptotic behaviour.The existence of such solutions was unsuspected until now.

Improved time decay for magnetic Schrodinger evolutions

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Abstract.

We will show some recent results about the relation between Laplace-Beltrami operators of the form

$$L_{A,a} = (i\nabla_{\mathbb{S}^{d-1}} + A(\theta))^2 + a(\theta), \qquad A : \mathbb{S}^{d-1} \to \mathbb{R}^d, \quad a : \mathbb{S}^{d-1} \to \mathbb{R}$$

and dispersive properties of electromagnetic Schrödinger flows e^{itH}, with

$$H = H_{A,a} = (i\nabla + |x|^{-1}A(\theta))^2 + |x|^{-2}a(\theta).$$

This family includes relevant examples, as the Aharonov-Bohm potential and the point-dipole. Notice that H is a critical perturbation of $H_{0,0}$ with respect to the scale of H^1, by the Hardy inequality. We will show a mechanism connecting spectral properties of $L_{A,a}$ to time-dispersive properties of e^{itH}.

In addition, we will prove some quantitative diamagnetic improvement of the free decay, in suitable topologies.

The results are obtained in collaboration with V. Felli (Milano-Bicocca), M. Fontelos (Madrid - ICMAT), G. Grillo (Milano - Politecnico), H. Kovařík (Brescia - Università), A. Primo (Madrid - UAM).

Blow-up for non-local p-laplacian equation

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Abstract.

We study the blow-up phenomenon for the non-local equation

$$u_t(x,t) = \int_{R^N} J(x-y)|u(y,t)-u(x,t)|^{p-2}(u(y,t)-u(x,t))\,dy + u^q(x,t),$$

for $x \in \Omega$, $t \in [0,T]$, with Dirichlet conditions, $u \equiv 0$ in $R^N \setminus \Omega$.

We determine the global existence exponents, the blow-up rate and the blow-up set. We also compare our result with the analogous local problem

$$u_t = \Delta_p u + u^q.$$

An isoperimetric problem with a non local repulsive term

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Abstract.

I will discuss local and global minimality properties of balls with respect to the volume-constrained minimization of a free energy consisting of a nonlocal s-perimeter plus a non-local repulsive interaction term. In the particular case $s = 1$ the s-perimeter coincides with the classical perimeter, and our results improve the ones of [3, 1] concerning minimality of balls of small volume in isoperimetric problems with a competition between perimeter and a nonlocal potential term. More precisely, their result is extended to its maximal range of validity concerning the type of nonlocal potentials considered, and is also generalized to the case where local perimeters are replaced by their nonlocal counterparts. The results presented in the talk are contained in the recent paper [2].

References

[1] M. Bonacini & R. Cristoferi, *Local and global minimality results for a nonlocal isoperimetric problem on R^N.* SIAM J. Math. Anal. bf 46 (2014), 2310-2349.

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Finite time blow up for the incompressible Navier-Stokes equation with a free boundary

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Abstract.

In this talk we consider weak solutions of the inhomogeneous Navier-Stokes equations with variable viscosity. Both, the density and the viscosity functions, are given by steps functions taking constant values in complementary connected open sets. It models the evolution of incompressible fluids with different constant densities and viscosities. In this talk we will study the one fluid case, where the density and viscosity in one set are equal to zero. We will show finite time blow-up for this system. The singularity is a splash-type: a smooth fluid boundary collapses due to two different particles evolve to collide at a single point. This is the first example of a singularity for a Navier-Stokes system.

References

[1] Splash singularities for the free boundary Navier-Stokes equations. A. Castro, D. Crdoba, C. Fefferman, F. Gancedo and J. Gmez-Serrano. Submitted (2015).

Multiplicity of solutions for some semilinear elliptic equations*

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Abstract.

We consider the elliptic problem

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where f is a nonnegative, locally Lipschitz nonlinearity, Ω a smooth bounded domain and $\lambda > 0$ a parameter. Assuming f has an isolated zero α such that

$$\frac{f(t)}{(t-\alpha)^{\frac{N+2}{N-2}}} \text{ is decreasing in } (\alpha, \alpha+\delta),$$

for some $\delta > 0$, we show that for sufficiently large λ there exist at least two solutions $u_\lambda < v_\lambda$, verifying $\|u_\lambda\|_\infty < \alpha < \|v_\lambda\|_\infty$ and $u_\lambda, v_\lambda \to \alpha$ uniformly on compact sets of Ω as $\lambda \to +\infty$. The existence of these solutions do not depend on the behavior of f near zero or infinity. We also show that our condition for existence is somehow sharp.

* Joint work with B. Barrios and L. Iturriaga ([1])

References

[1] B. Barrios, J. García Melián, L. Iturriaga, *Semilinear elliptic equations and nonlinearities with zeros*, Submitted.

Evolution of support for limited flux porous medium equations

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Abstract.

Limited flux porous medium equations (LFPMEs) are nonlinear parabolic equations whose flux both degenerates as the unknown vanishes and remains bounded as the unknown's gradient diverges. Two prototypes within this class are

$$u_t = \operatorname{div}\left(\frac{u^m \nabla u}{\sqrt{u^2 + |\nabla u|^2}}\right), \quad u_t = \operatorname{div}\left(\frac{u \nabla u^{M-1}}{\sqrt{1 + |\nabla u^{M-1}|^2}}\right), \quad m, M > 1.$$

In this talk I will present recent results concerning the evolution of the solutions' support [1]. Generalizing an earlier work of Andreu, Caselles, Mazon, and Moll [2], I will argue that entropy solutions to the Cauchy problem for LFPMEs satisfy the finite speed of propagation property, with upper bounds that I expect to be optimal in terms of time-scaling. For the two aforementioned prototypes, I will also give a growth condition on the initial datum that guarantees the occurrence of a waiting-time phenomenon and I will present a heuristic argument in favor of its optimality. Finally, I will discuss possible strategies for two open questions: (i) generalizing the waiting-time results from the two prototypes to the general class of LFPMEs; (ii) showing optimality of the aforementioned growth condition.

References

[1] L. Giacomelli. Finite speed of propagation and waiting-time phenomena for degenerate parabolic equations with linear growth Lagrangian. SIAM J. Math. Anal., to appear.

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Asymptotic analysis and sign-changing bubble towers for Lane-Emden problems

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Abstract.

We analyze the asymptotic behavior for $p \to +\infty$ of the solutions of the semilinear Lane-Emden problem

$$\begin{array}{ll} -\Delta u = |u|^{p-1}u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{array} \tag{3}$$

where $\Omega \subset \mathbb{R}^2$ is a smooth bounded domain.
It turns out that any family of finite energy solutions concentrates at a finite number of points, moreover in the case of positive solutions we can give a complete description of this concentration phenomenon. For sign-changing solutions the analysis is much more more delicate. In particular we show that, under suitable symmetry assumptions, the positive and negative parts of families of symmetric solutions concentrate at the same point, and that the limit profile looks like a tower of two different bubbles given by the superposition of a regular and a singular solution of the Liouville problem in the plane.
The results are obtained in collaboration with F. De Marchis and F. Pacella (Roma Sapienza).

Variational aspects of Singular Liouville Equations

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Abstract.

We consider Liouville equations arising from curvature prescription problems and from models in Electroweak and Chern-Simons theory. We show how improved versions of the Moser-Trudinger inequality may reduce these PDEs to the study of finite-dimensional objects. With the aid of compactness results proved via blow-up analysis, one can then derive existence of solutions via min-max or Morse theory.

Nonlinear diffusion in transparent media.

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Abstract.

In this talk, we will discuss about some recent results concerning nonlinear diffusion in transparent media. The focus will be the study of the Dirichlet problems associated to some elliptic and parabolic PDEs whose diffusion term takes the form

$$\operatorname{div}\left(u^m \frac{Du}{|Du|}\right),$$

with $m \in]-\infty, +\infty[$. We will present existence and uniqueness results of entropy solutions and some qualitative properties of them. This is a work in progress in collaboration with L. Giacomelli and F. Petitta (SBAI, Università di Roma I, "La Sapienza").

Some Quasilinear Dirichlet Problems with natural growth

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Abstract.

We are going to show a brief summary of a kind of quasilinear Dirichlet problems with quadratic growth respect to the gradient. These problems can be seen, at least formally, as a generalization of the Euler-Lagrange equation of the funcional

$$J(v) = \frac{1}{2}\int_\Omega (a(x) + |v|^r)|\nabla v|^2 - \int_\Omega f\, v\,,$$

defined on a suitable subset of $W_0^{1,2}(\Omega)$. In particular, we will consider problems whose basic model is the following one:

$$\begin{gathered} -\mathrm{div}\left[(a(x) + |u|^r)\,\nabla u(x)\right] + \tfrac{r}{2}\, u\,|u|^{r-2}\,|\nabla u|^2 = f(x) \,\mathrm{in}\,\Omega \\ u(x) = 0 \,\mathrm{on}\,\partial\Omega\,, \end{gathered} \tag{4}$$

where Ω is an open bounded subset of $\mathbb{R}^N$, $a(x)$ is a strictly positive bounded function, $r > 0$ and f belongs to $L^m(\Omega)$ with $m \geq 1$.

The study of problems with these features was developed in several recent papers ([1], [2], [3] and [4]). We show a complete study (depending on the parameters) of the existence of solution of non variational problems whose basic model is the problem (4).

We remark that there is a difference between the case $r > 1$ and $0 < r < 1$. Indeed, if $0 < r < 1$ a new difficulty appears: *"the lower order term of the problem (4) becomes singular where the solution is zero"* since it depends on a negative power of the solution.

References

[1] L. Boccardo, Dirichlet problems with singular and quadratic gradient lower order terms. ESAIM Control Optim. Calc. Var. **14** (2008), 411–426.

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Quasi-Linear corrections of the Schrödinger equations in bounded domains

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Abstract.

We will prove the existence of a Mountain Pass solution of a quasi-linear Schrödinger equation in a bounded domain via variational methods. In order to do this we will have to study non-differentiable, and possibly unbounded functionals.

Existence of solutions to supercritical problems on manifolds

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Abstract.

I will talk about the problem

$$-\Delta_g u + h(x)u = u^p,\ u > 0,\ in\ (M, g)$$

where (M, g) is a$n-$dimensional Riemannian manifold without boundary and $p > 1$. In particular, I will present some recent results about existence of solutions concentrating along $k-$dimensional minimal submanifolds of M for any integer k between 0 and $n-1$, provided the exponent p is close the $(k+1)-$st critical exponent.

Natural growth and beyond

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Abstract.

We discuss second order equations with first order terms having super linear growth. We discuss the threshold often called natural growth in terms of gradient estimates, barriers and maximal stationary solutions, as well as the role played by nonlinear additive eigenvalues for the existence of solutions and the long time behavior of evolution problems.

References

[1] I. Capuzzo Dolcetta, F. Leoni, A. Porretta, *Hölder estimates for degenerate elliptic equations with coercive Hamiltonians*, Trans. Amer. Math. Soc. **362** (2010), 4511–4536.

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[4] A. Dall'Aglio, A. Porretta, *Local and global regularity of weak solutions of elliptic equations with superquadratic Hamiltonian*, Trans. Amer. Math. Soc. 367 (2015), 3017-3039.

[5] T. Leonori, A. Porretta, *Large solutions and gradient bounds for quasilinear elliptic equations*, in preparation.

On the regularization phenomena and time behavior of the solutions to some parabolic PDE

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Abstract.

It is well known that many parabolic PDE (for example the heat equation or the porous medium equation) exhibit a very strong regularization phenomenon: the solutions become "immediately bounded" also in presence of only summable initial data. Besides, the L^∞-norms of these solutions decay for large values of time. In this talk we will briefly describe this phenomenon. Moreover we will show that if this "strong regularization" does not appear many different behaviors are possible. Finally, we present a new method to investigate these different situations and to describe the time behavior of the solutions to many various parabolic PDE.

On the mass transport problem with relativistic cost

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Abstract.

In this talk, we will describe the mass transport problem with a relavistic cost function; these are cost functions which are convex and bounded in a closed, convex subset of $\mathbb{R}^N$, and $+\infty$ outside of it; the prototype is the "relativistic heat cost", which is

$$c_{\mathbf{heat}}(x,y) := \begin{cases} 1-\sqrt{1-|y-x|^2} & \text{if } |y-x|\le 1,\\ +\infty & \text{otherwise.}\end{cases}$$

We will list and discuss the main properties and open questions regarding this specific kind of cost. Joint work with J. Bertrand and M. Puel.

References

[1] J. Bertrand, A. Pratelli & M. Puel, Kantorovich potentials and continuity of total cost for relativistic cost functions, preprint (2015).

Basic estimates for solutions of nonlocal elliptic and parabolic equations

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Abstract.

We study the following problems

$$\begin{cases} \mathcal{L}\,u &= f \text{ in } \Omega,\\ u &= 0 \text{ in } \mathbb{R}^N\setminus\Omega,\end{cases} \qquad \text{and} \qquad \begin{cases} u_t+\mathcal{L}\,u &= f \text{ in } \Omega\times(0,T),\\ u(x,t) &= 0 \text{ in } \big(\mathbb{R}^N\setminus\Omega\big)\times[0,T),\\ u(x,0) &= 0 \text{ in } \Omega,\end{cases}$$

where $\mathcal{L}$ is a nonlocal integral operator related to the Fractional Laplacian and Ω is a smooth domain in $\mathbb{R}^N$.

We study existence, uniqueness and summability of the solution u with respect to the summability of the datum f. In the process, we develop some useful techniques, inequalities and some applications.

References

[1] T. Leonori, F. Soria, I. Peral & A. Primo, Basic estimates for solutions of a class of nonlocal elliptic and parabolic equations, Discrete and Continuous Dynamical System. **35** (2015)

A rigidity result for overdetermined elliptic problems in the plane

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Abstract.

A widely open problem is to classify the set of domains $\Omega \subset \mathbb{R}^n$ where there exists a bounded solution u to the overdetermined elliptic problem

$$\begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \\ \frac{\partial u}{\partial \vec{\nu}} = 1 & \text{on } \partial\Omega \end{cases} \tag{5}$$

for some Lipschitz function f. The case of a bounded domain was solved by J. Serrin in 1971: the ball is the unique such domain. Instead, the case of unbounded domains is far from being completely understood.

In this talk we show that if $n = 2$ and $\partial\Omega$ is unbounded and connected, then Ω is a halfplane. This is joint work with Antonio Ros (U. Granada) and Pieralberto Sicbaldi (U. Aix Marseille).

Antisymmetry of solutions for some weighted elliptic problems

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Abstract.

The main purpose of this talk is to study antisymmetry, uniqueness, and monotonicity of solutions for some elliptic functionals involving weights and a double well potential. In the one-dimensional case, we introduce the continuous odd rearrangement of an increasing function and we show that it decreases the energy functional when the weights satisfy a certain convexity-type hypothesis. This leads to the antisymmetry or oddness of increasing solutions (and not only of minimizers). We also prove a uniqueness result (which leads to antisymmetry) where a previously known convexity-type condition on the weights is improved to a monotonicity condition. In addition, we provide with a large class of problems where antisymmetry does not hold. Finally, some rather partial extensions in higher dimensions are also given.

This is a joint work with Xavier Cabré, Marcello Lucia and Salvador Villegas.

Qualitative properties of positive solutions to nonlocal critical problems involving the Hardy-Leray potential

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Abstract.

We consider positive solutions to the problem:

$$(-\Delta)^s u = \vartheta \frac{u}{|x|^{2s}} + u^{2_s^*-1} \quad \text{in } \mathbb{R}^N, \tag{6}$$

with $N > 2s$, $0 < \vartheta < \Lambda_{N,s}$ ($\Lambda_{N,s}$ is the sharp constant of the Hardy-Sobolev inequality) and $2_s^* = \frac{2N}{N-2s}$. The existence of the solutions follows by a constrained minimization problem and we study the qualitative properties of the solutions. In particular we show that any solution is radial and radially decreasing about the origin. Furthermore we derive the behaviour of the solutions at the origin (solutions are singular near the origin) and the behaviour of the solutions at infinity.

The results have been obtained in collaboration with SERENA DIPIERRO, LUIGI MONTORO and IRENEO PERAL.

Restriction, multiplier and waves

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Abstract.

In the seventies, C. Fefferman showed the failure of the spherical convergence of Fourier integrals in L^p for $p \neq 2$. For other summation methods, such as the Bochner-Riesz means, the problem is still open in high dimensions.

In a joint work with Sanghyuk Lee [2], we considered a similar question for the cone. This has some implications for the wave equation. In particular, it is connected to the "local smoothing" of the solutions after averages on time. We proved the sharp result in $L^3(R^2)$.

In our analysis, a key role is played by the multilinear restriction theorem by Bennett–Carbery–Tao and some recent developments due to Bourgain–Guth. More recently, Bourgain and Demeter [1] have proved some estimated for spheres related to our result. They also pointed out the connections with some problems for dispersive equations in tori.

References

[1] Bourgain, Jean; Demeter, Ciprian. The proof of the l^2 Decoupling Conjecture. Arxiv:1403.5335.

[2] Lee, Sanghyuk; Vargas, Ana. On the cone multiplier in R3. J. Funct. Anal. 263 (2012), no. 4, 925-940.

Numerical hypocoercivity for the Kolmogorov equation

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Abstract.

In this talk present a recent joint work with Alessio Porretta.

We prove that a finite-difference centered approximation for the Kolmogorov equation in the whole space preserves the decay properties of continuous solutions as $t \to \infty$, independently of the mesh-size parameters.

This is a manifestation of the property of numerical hypo-coercivity and it holds both for semi-discrete and fully discrete approximations.

The method of proof is based on the energy methods developed by Herau and Villani, employing well-balanced Lyapunov functionals mixing different energies, suitably weighted and equilibrated by multiplicative powers in time.

The decreasing character of this Lyapunov functional leads to the optimal decay of the L^2-norms of solutions and partial derivatives, which are of different order because of the anisotropy of the model.

References

[1] A. Porretta, E. Zuazua. Numerical hypocoercivity for the Kolmogorov equation. Preprint, 2014. [http://www.bcamath.org/documentos_public/archivos/publicaciones/1-01-2015.pdf]

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10:45–11:05	**COFFEE BREAK**	**COFFEE BREAK**	**COFFEE BREAK**
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15:20–15:50	Dino Sciunzi	Raúl Ferreira	
15:55–16:15	**COFFEE BREAK**	**COFFEE BREAK**	
16:20–16:50	Jorge García–Melián	Juan Casado	
16:55–17:25	Alessio Porretta	Luca Fanelli	