Existence and multiplicity of bound-ground states for a system of NLS-KdV equations

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Abstract.

We will show existence of solutions for systems of coupled NLS-KdV equations,

\[
\begin{aligned}
if_t + f_{xx} + \beta fg + |f|^2f &= 0, \quad x \in \mathbb{R}, \quad t > 0 \\
g_t + g_{xxx} + gg_x + \frac{1}{2}\beta(|f|^2)_x &= 0, \quad x \in \mathbb{R}, \quad t > 0,
\end{aligned}
\]  

(1)

where \(f(x,t) \in \mathbb{C}, g(x,t) \in \mathbb{R}, \beta \in \mathbb{R}\) and \(|f|, |g| \to 0\) as \(|x| \to \infty\).

Precisely, we will show existence of positive bound and ground states for the corresponding stationary system (2) when one looks for solitary-traveling wave solutions of the form

\[
(f(x,t), g(x,t)) = (e^{i\lambda_1 t}e^{i\lambda_2 x}u(x-ct), v(x-ct))
\]

with \(\lambda_1 = \omega + \frac{c^2}{4}, \lambda_2 = c\), and \(u, v\) are real functions which correspond to solutions of the following stationary system

\[
\begin{aligned}
-u'' + \lambda_1 u &= u^3 + \beta uv \\
-v'' + \lambda_2 v &= \frac{1}{2}v^2 + \frac{1}{2}\beta u^2.
\end{aligned}
\]  

(2)

References
