Quantum optics and metamaterials

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Motivation

- Quantum optics a well-developed field for studying interaction of light with atoms
- Analogies of metamaterials systems to atomic and molecular scatterers
- Analogous phenomena to quantum coherence and semi-classical effects in atomic gases
- Nanostructured resonators interacting strongly with EM fields
- Less emphasis on microscopic approach
- Cooperative response in large system

Start with a two-level system interacting with EM fields

Quantum coherence phenomena, slow light, nanaofabricated analogs

Collective effects in large systems

Standard quantum optics:
Light propagation in polarization medium

Weak interactions between scatterers in the medium

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = 0 \quad \text{(no free charges)}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad \text{(no free currents)}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}.$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} - \mathbf{M}$$

Assume no magnetization

$$\boldsymbol{\nabla} \cdot \mathbf{E} \simeq 0$$

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{E} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \simeq -\nabla^2 \mathbf{E}$$

$$-\nabla^2 \mathbf{E} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{1}{\epsilon_0 c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Plane wave propagation

$$\mathbf{E}(\mathbf{r},t) = \tfrac{1}{2}e^{i(kz-\omega t)}\mathcal{E}(z,t)\,\hat{\mathbf{e}} + \text{c.c.},$$
$$\mathbf{P}(\mathbf{r},t) = \tfrac{1}{2}e^{i(kz-\omega t)}\mathcal{P}(z,t)\,\hat{\mathbf{e}} + \text{c.c.}$$

$$k = \omega/c$$

Slowly varying envelope approximation

$$\left|\frac{\partial \mathcal{E}}{\partial t}\right| \ll \omega|\mathcal{E}|, \quad \left|\frac{\partial \mathcal{E}}{\partial z}\right| \ll k|\mathcal{E}|\,,$$

$$\begin{aligned}
-\nabla^2\mathbf{E} &= -\tfrac{1}{2}\hat{\mathbf{e}}\frac{\partial^2}{\partial z^2}\left[\mathcal{E}e^{i(kz-\omega t)}\right] + \text{c.c.} \\
&= -\tfrac{1}{2}\hat{\mathbf{e}}\left[\mathcal{E}'' + 2ik\mathcal{E}' - k^2\mathcal{E}\right]e^{i(kz-\omega t)} + \text{c.c.} \\
&\simeq -\tfrac{1}{2}\hat{\mathbf{e}}\left[2ik\mathcal{E}' - k^2\mathcal{E}\right]e^{i(kz-\omega t)} + \text{c.c.}, \\
\frac{\partial^2\mathbf{E}}{\partial t^2} &\simeq \tfrac{1}{2}\hat{\mathbf{e}}\left[-2i\omega\dot{\mathcal{E}} - \omega^2\mathcal{E}\right]e^{i(kz-\omega t)} + \text{c.c.}, \\
\frac{\partial^2\mathbf{P}}{\partial t^2} &\simeq \tfrac{1}{2}\hat{\mathbf{e}}\left[-\omega^2\mathcal{P}\right]e^{i(kz-\omega t)} + \text{c.c.}\,.
\end{aligned}$$

slowly varying envelope approximation

$$\frac{\partial \mathcal{E}}{\partial z} + \frac{1}{c}\frac{\partial \mathcal{E}}{\partial t} = i\frac{k}{2\epsilon_0}\mathcal{P}\,.$$

steady-state solution for electric field in linear medium

$$\frac{\partial \mathcal{E}}{\partial t} = 0, \quad \mathcal{P} = \epsilon_0(\chi' + i\chi'')\mathcal{E}\,.$$

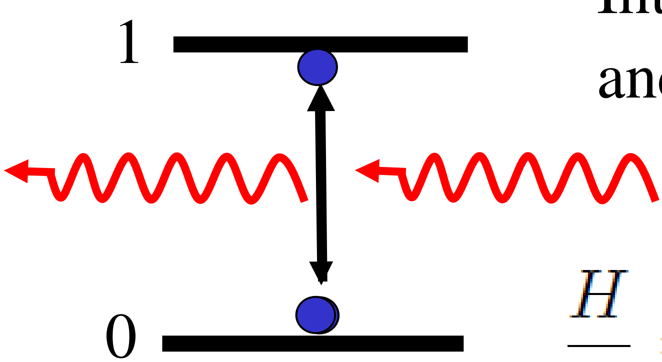
real and imaginary parts of electric susceptibility

$$\mathcal{E}(z) = \mathcal{E}(0)\, e^{\frac{1}{2}k(i\chi' - \chi'')z}$$

$$I(z) \propto |\mathcal{E}(z)|^2 \propto e^{-k\chi'' z}$$

absorption $\alpha = k\chi''$

Two-level medium



Interaction between dipole and electric field $H' = -\hat{\mathbf{d}} \cdot \mathbf{E}(t)$

$$\frac{H}{\hbar} = \omega_0 |1\rangle\langle 1| - \frac{\mathbf{E}(t)}{\hbar} \cdot \left(\mathbf{d}^* |0\rangle\langle 1| + \mathbf{d}|1\rangle\langle 0|\right)$$

$$\mathbf{d} = \langle 1|\hat{\mathbf{d}}|0\rangle$$

quantum state $|\psi(t)\rangle = c_0(t)|0\rangle + c_1(t)|1\rangle$

$$\dot{c}_0 = i\frac{\mathbf{d}^* \cdot \mathbf{E}}{\hbar} c_1, \quad \dot{c}_1 = -i\omega_0\, c_1 + i\frac{\mathbf{d} \cdot \mathbf{E}}{\hbar}\, c_0$$

see e.g. Meystre, Sargent, Elements of quantum optics

Introduce new slowly varying amplitude

$$c_0(t) = C_0(t), \quad c_1(t) = C_1(t) e^{-i\omega t}.$$

$$\dot{C}_0 = \frac{i}{2}\Omega^* C_1, \quad \dot{C}_1 = -i\Delta C_1 + \frac{i}{2}\Omega C_0.$$

$\Omega = \dfrac{\mathbf{d}\cdot\boldsymbol{\mathcal{E}}}{\hbar}$ Rabi frequency; $\Delta = \omega_0 - \omega$ detuning

here $\mathbf{E}(\mathbf{r},t) = \frac{1}{2}\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t} + \frac{1}{2}\boldsymbol{\mathcal{E}}^*(\mathbf{r})e^{i\omega t}$

$$\frac{H}{\hbar} = \Delta\,|1\rangle\langle 1| - \tfrac{1}{2}(\Omega|1\rangle\langle 0| + \Omega^*|0\rangle\langle 1|)\,,$$

$$\frac{H}{\hbar} = \begin{bmatrix} 0 & -\frac{1}{2}\Omega^* \\ -\frac{1}{2}\Omega & \Delta \end{bmatrix}$$

Relaxation terms

$$|\psi(t)\rangle = c_a(t)|a\rangle + c_b(t)|b\rangle$$

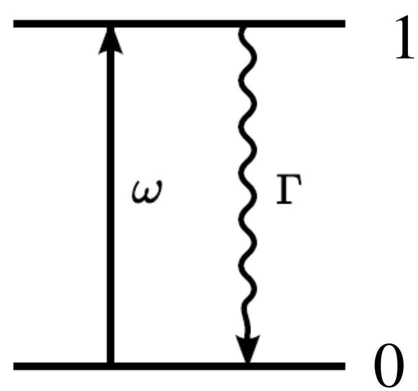
density matrix

$$\rho = \begin{pmatrix} c_a c_a^* & c_a c_b^* \\ c_b c_a^* & c_b c_b^* \end{pmatrix} = \begin{pmatrix} \rho_{aa} & \rho_{ab} \\ \rho_{ba} & \rho_{bb} \end{pmatrix}$$

$$\rho_{aa} = c_a c_a^*$$
$$\rho_{ab} = c_a c_b^*$$
$$\rho_{ba} = c_b c_a^*$$
$$\rho_{bb} = c_b c_b^*$$

$$\left.\frac{d}{dt}\right|_{\mathrm{R}} \rho_{11} = -\Gamma\rho_{11}, \quad \left.\frac{d}{dt}\right|_{\mathrm{R}} \rho_{00} = \Gamma\rho_{11}$$

$$\left.\frac{d}{dt}\right|_{\mathrm{R}} \rho_{01} = -\gamma\rho_{01}, \quad \left.\frac{d}{dt}\right|_{\mathrm{R}} \rho_{10} = -\gamma\rho_{10}$$



choose $\gamma = \frac{\Gamma}{2}$

closed two-state system

$$
\begin{aligned}
\dot{\rho}_{00} &= \Gamma\rho_{11} + \tfrac{1}{2}i(\Omega^*\rho_{10} - \Omega\rho_{01}), \\
\dot{\rho}_{11} &= -\Gamma\rho_{11} - \tfrac{1}{2}i(\Omega^*\rho_{10} - \Omega\rho_{01}), \\
\dot{\rho}_{01} &= (i\Delta - \gamma)\rho_{01} + \tfrac{1}{2}i\Omega^*(\rho_{11} - \rho_{00}), \\
\dot{\rho}_{10} &= (-i\Delta - \gamma)\rho_{10} - \tfrac{1}{2}i\Omega(\rho_{11} - \rho_{00}) = (\dot{\rho}_{01})^* .
\end{aligned}
$$

longitudinal and transverse relaxation times

$T_1 = 1/\Gamma$ and $T_2 = 1/\gamma$.

Steady-state solutions, conserved population $\rho_{00} + \rho_{11} = 1$

$$
\rho_{11} = 1 - \rho_{00} = \frac{|\Omega|^2/4}{\Delta^2 + \gamma^2 + |\Omega|^2/2}, \quad \rho_{10} = (\rho_{01})^* = \frac{\frac{1}{2}\Omega(i\gamma + \Delta)}{\Delta^2 + \gamma^2 + |\Omega|^2/2} .
$$

Optical response

$$\langle\hat{\mathbf{d}}\rangle = (d\rho_{01} + d^*\rho_{10})\hat{\mathbf{e}}$$ electric dipole excitation

$$\boldsymbol{\mathcal{E}} = e^{ikz}\mathcal{E}(z)\,\hat{\mathbf{e}} \qquad \Omega = \Omega(z) = de^{ikz}\mathcal{E}(z)$$

$$\langle\hat{\mathbf{d}}\rangle(z,t) = \frac{|d|^2(i\gamma + \Delta)/2\hbar}{\Delta^2 + \gamma^2 + |\Omega|^2/2}\,\mathcal{E}(z)e^{i(kz-\omega t)}\hat{\mathbf{e}} + \text{c.c.}\,.$$

$$\mathbf{P}(z,t) = N\langle\hat{\mathbf{d}}\rangle(z,t) = \tfrac{1}{2}\mathcal{P}(z)e^{i(kz-\omega t)}\hat{\mathbf{e}} + \text{c.c.}\,,$$

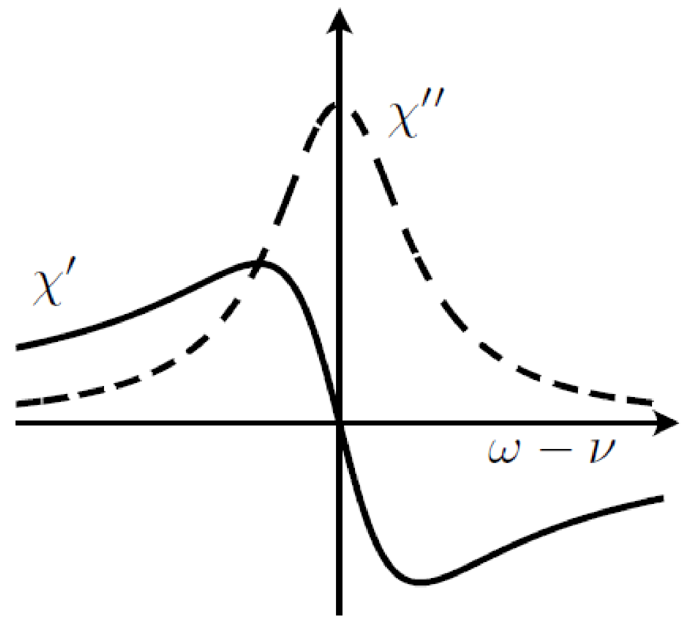
$$\mathcal{P}(z) = \frac{|d|^2 N(i\gamma + \Delta)/\hbar}{\Delta^2 + \gamma^2 + |\Omega|^2/2}\,\mathcal{E}(z)$$

$$\chi = \frac{\mathcal{P}}{\epsilon_0\mathcal{E}} = \frac{|d|^2 N}{\hbar\epsilon_0}\,\frac{i\gamma + \Delta}{\Delta^2 + \gamma^2 + |\Omega|^2/2}$$ electric susceptibility

Absorption

$$\alpha = k\Im(\chi) = \alpha_0 \frac{\gamma^2}{\Delta^2 + \gamma^2 + |\Omega|^2/2} \, .$$

$$\alpha_0 = \frac{|d|^2 k N}{\hbar \gamma \epsilon_0}$$



Laser

Laser

Coherence may be expressed in terms of population difference

$$\rho_{10} = \frac{\frac{1}{2}\Omega(\rho_{00} - \rho_{11})}{\Delta - i\gamma} \qquad \alpha \propto \Im(\rho_{10}) \propto \rho_{00} - \rho_{11}$$

$\rho_{00} - \rho_{11}$ usually positive and propagating light is absorbed

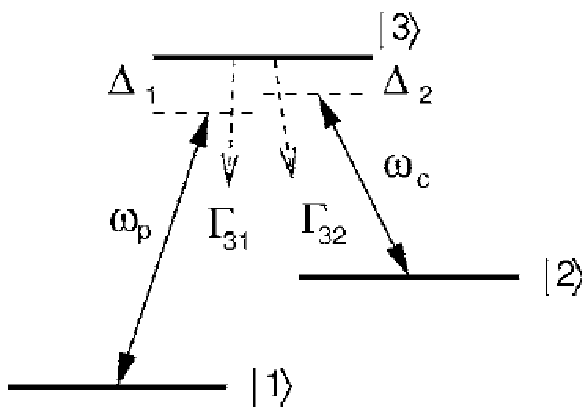
One can also arrange (usually in multi-level system) so that atoms are transferred to $|1\rangle$ and depleted from $|0\rangle$

Inversion $\rho_{11} > \rho_{00}$

Light coupling to transition $|0\rangle \rightarrow |1\rangle$ is amplified

Electromagnetically induced transparency & ultra slow light

Three-level system



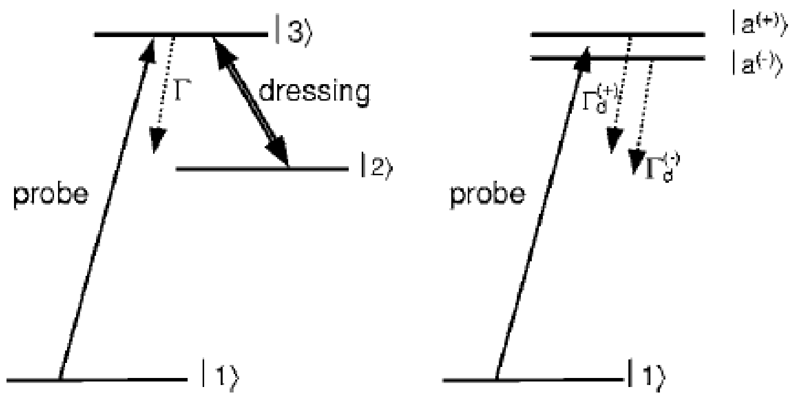
$$H_{\text{int}} = -\frac{\hbar}{2}\begin{bmatrix} 0 & 0 & \Omega_p \\ 0 & -2(\Delta_1 - \Delta_2) & \Omega_c \\ \Omega_p & \Omega_c & -2\Delta_1 \end{bmatrix}$$

see e.g.

Fleischhauer, M., Imamoglu, A. & Marangos, J. P. Electromagnetically induced transparency: Optics in coherent media. *Rev. Mod. Phys.* **77,** 633–673 (2005).

Probe and coupling fields, nonlinear response

dressed level picture (eigenstates)



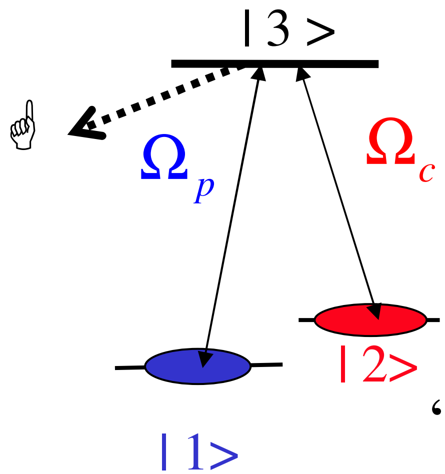
Autler-Townes splitting

Quantum interference of different excitation paths

|1⟩-|3⟩ pathway

|1⟩-|3⟩-|2⟩-|3⟩ pathway

Electromagnetically induced transparency

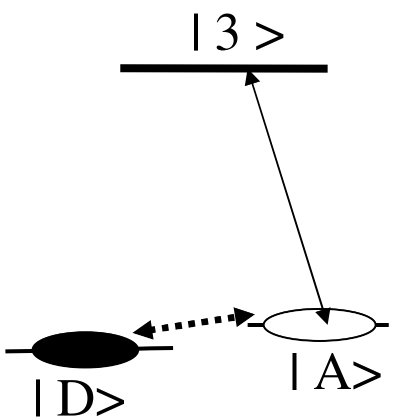


Three-level system

Two ground states coupled to excited state $\Omega_p \Omega_c$

‘Dark state’: $|D\rangle \propto \left(\Omega_c |1\rangle - \Omega_p |2\rangle\right)$

‘Absorbing state’: $|A\rangle \propto \left(\Omega_p |1\rangle + \Omega_c |2\rangle\right)$



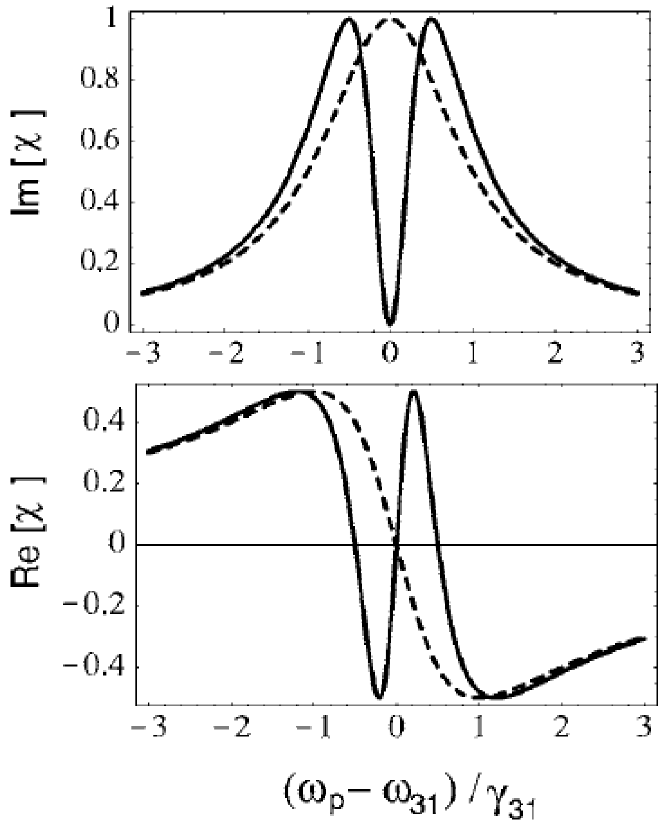
Dark state decoupled from light (quantum interference)

Atoms driven to dark state via spontaneous emission

Electromagnetically induced transparency (EIT)

Otherwise opaque medium made transparent

EIT response



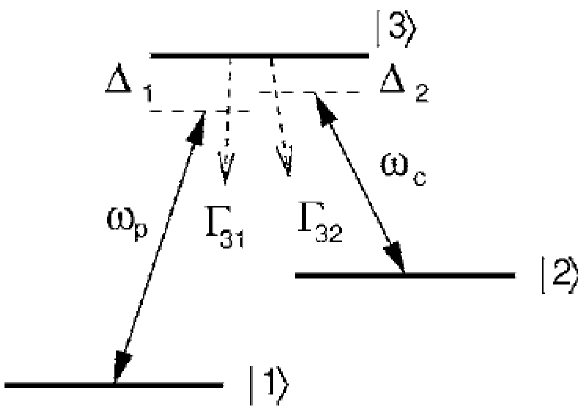
Narrow resonance at which opaque medium transparent

Steep anomalous dispersion has changed to normal dispersion with controllable steepness (by coupling laser)

Steep and linear dispersion where absorption small

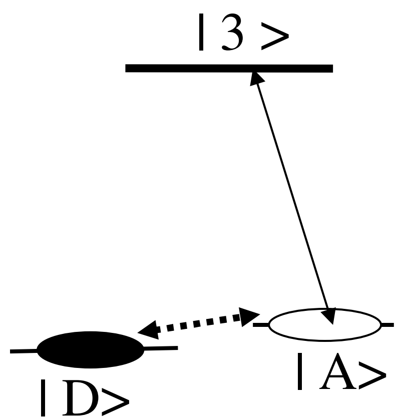
Constructive interference of the nonlinear processes $\chi^{(3)}$

Dark and bright states

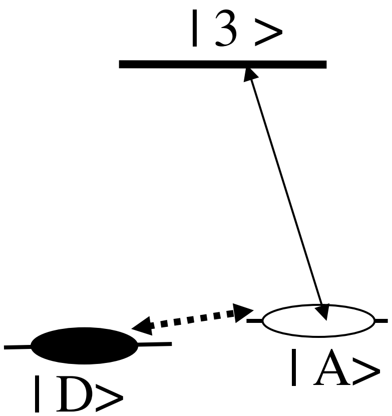


$$|D\rangle = \frac{\Omega_2|1\rangle - \Omega_1|2\rangle}{\sqrt{|\Omega_1|^2 + |\Omega_2|^2}}$$

$$|B\rangle = \frac{\Omega_1^*|1\rangle + \Omega_2^*|2\rangle}{\sqrt{|\Omega_1|^2 + |\Omega_2|^2}}$$



Ultra-slow light propagation



- Atoms adiabatically follow dark state
- Small perturbations from |D> lead to coherent driving between ground states with no absorption
- Reversible process
- Coherent driving and the associated exchange of photons between two light beams result in slow group velocity for light

Hau, Harris, Dutton, Behroozi, Nature **397**, 594 (1999)
Liu, Dutton, Behroozi, Hau, Nature **409**, 490 (2001)

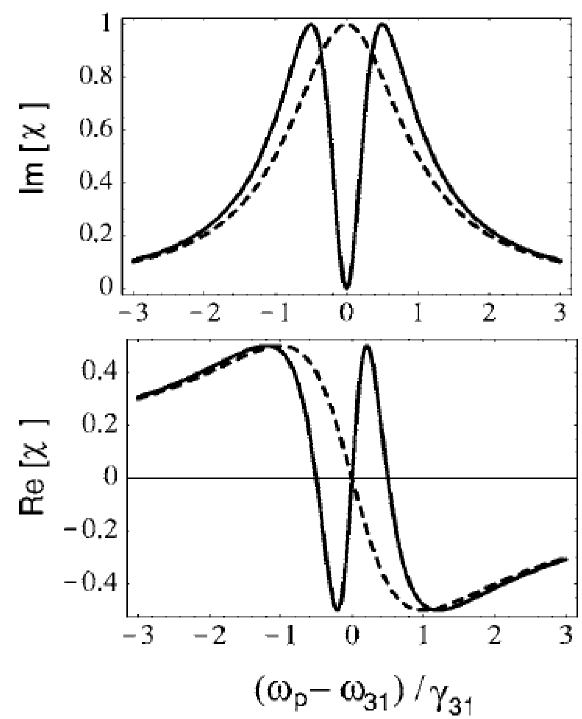
Ultra-slow and stopped light

Linear dependence of susceptibility close to the absorption dip

$dn/d\omega_p$ $\qquad n=\sqrt{1+\mathrm{Re}[\chi]}$

extreme pulse compression

$$v_{\mathrm{gr}} \equiv \frac{d\omega_p}{dk_p} = \frac{c}{n+\omega_p(dn/d\omega_p)},$$

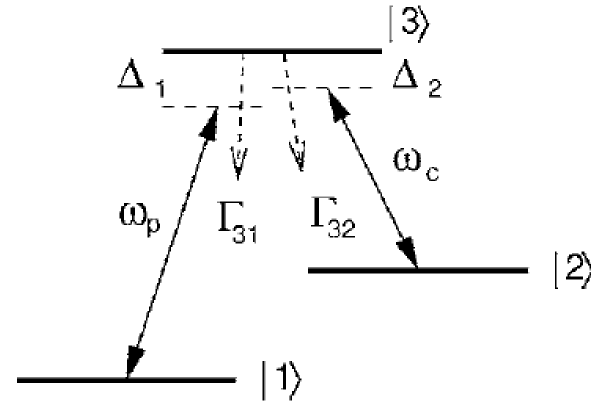


light pulses

Dutton, Hau, Phys. Rev. A 70, 053831 (2004)
Dutton, Ruostekoski, Phys. Rev. Lett 93, 193602 (2004)

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p=-\frac{N_c f_{13}\sigma_0}{2A}(\Omega_p|\psi_1|^2+\Omega_c\psi_1{}^*\psi_2),$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_c=-\frac{N_c f_{23}\sigma_0}{2A}(\Omega_c|\psi_2|^2+\Omega_p\psi_1\psi_2{}^*),$$



$v_g \propto \Omega_c^{(\mathrm{in})2}/|\psi_1|^2$

population transfer from 1 to 2

dark state $\psi_2 = -\psi_1^{(\mathrm{G})}(\Omega_p/\Omega_c^{(\mathrm{in})})$

switch coupling off

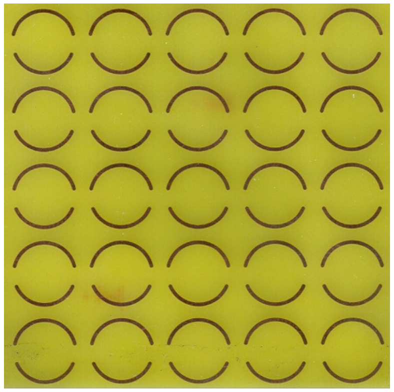
Also probe pulse goes to zero

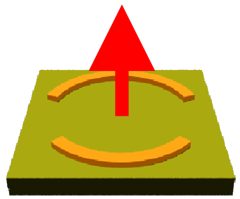
Revive the probe pulse later on

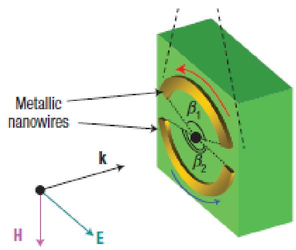
by switching the coupling field on

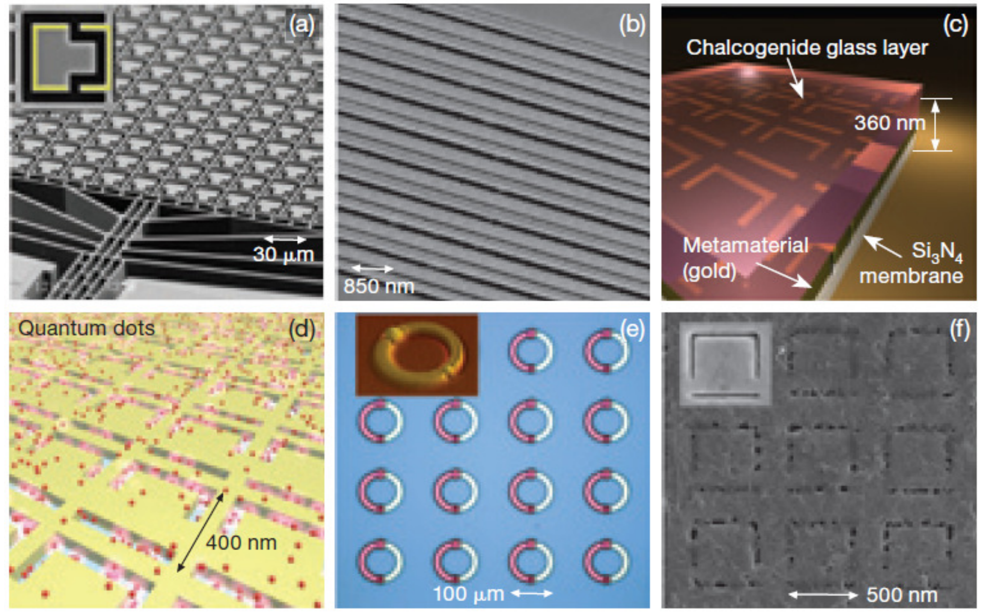
$$\Omega_p = -\Omega_c^{(\mathrm{in})}(\psi_2/\psi_1^{(\mathrm{G})})$$

Nanostructures





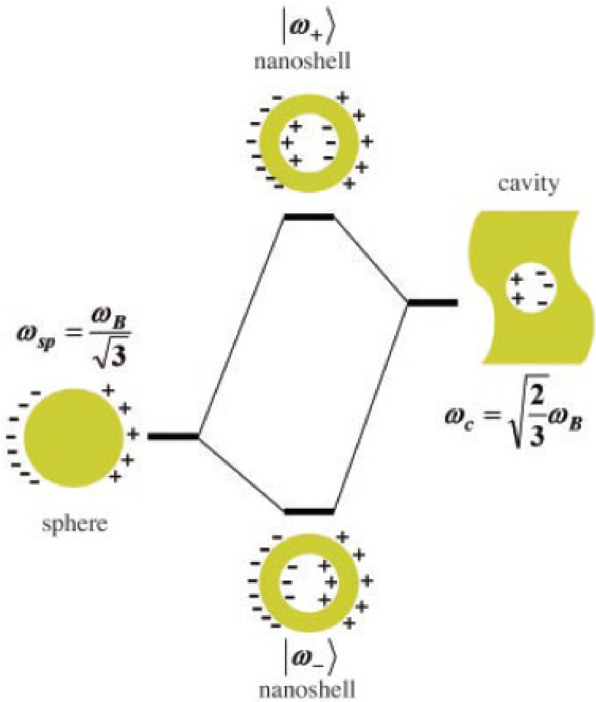




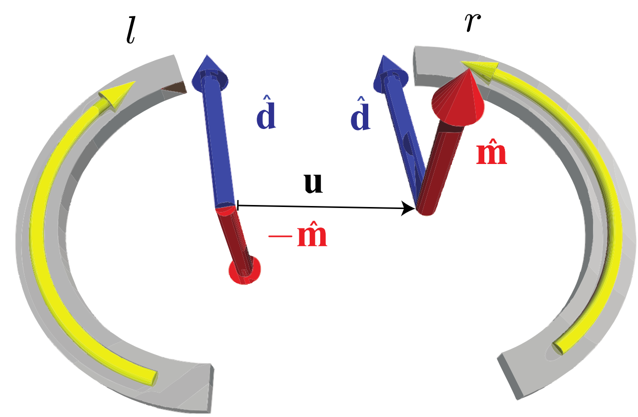
Hybridization model of plasmons supported by nanostructures of elementary geometries

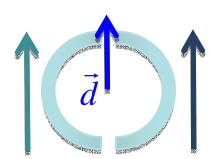
Analog of molecular orbital theory [Prodan et al., Science 302, 419 (2003)]

Hybridisation of metal nanoshell
from interaction of sphere and cavity plasmons

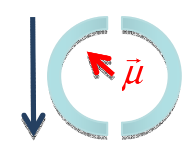


Split ring resonator





electric



magnetic

Quantum optics and metamaterials

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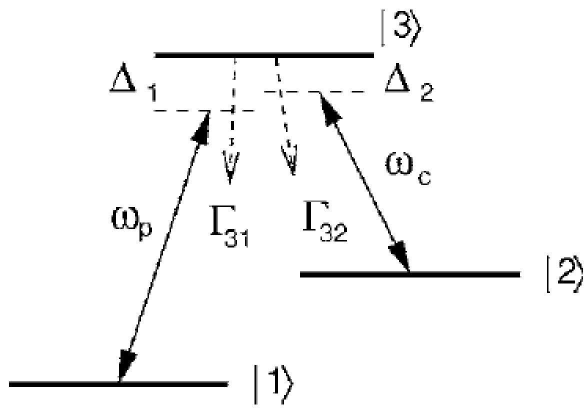
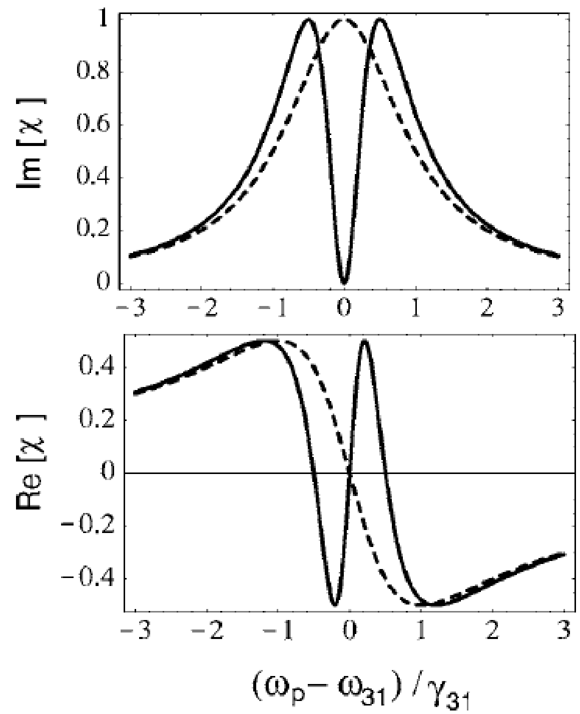
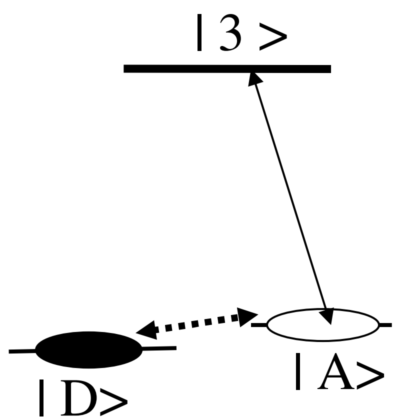
Electromagnetically induced transparency and slow light



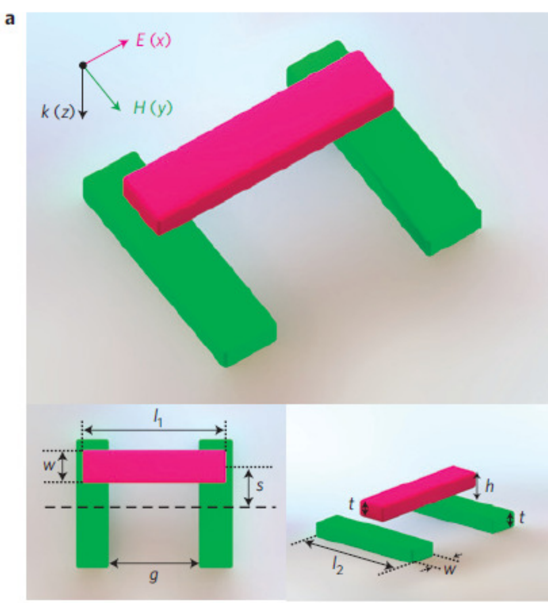


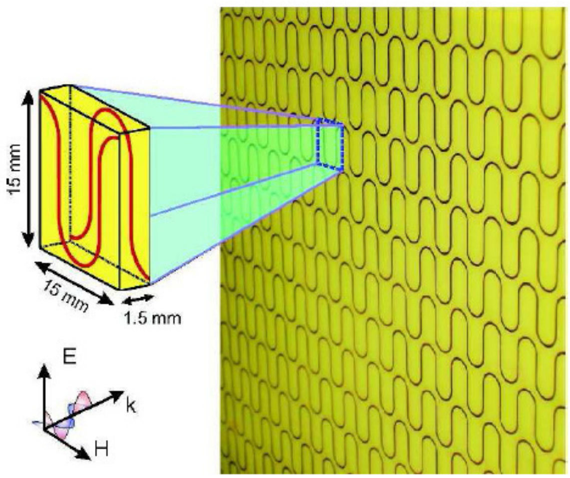


EIT in metamaterials

Instead of coupling atomic transitions to obtain quantum interference of transition amplitudes, create the interference using plasmonic structures

Plasmonic structures allow large field strengths in small volumes





Metamaterial Analog of Electromagnetically Induced Transparency

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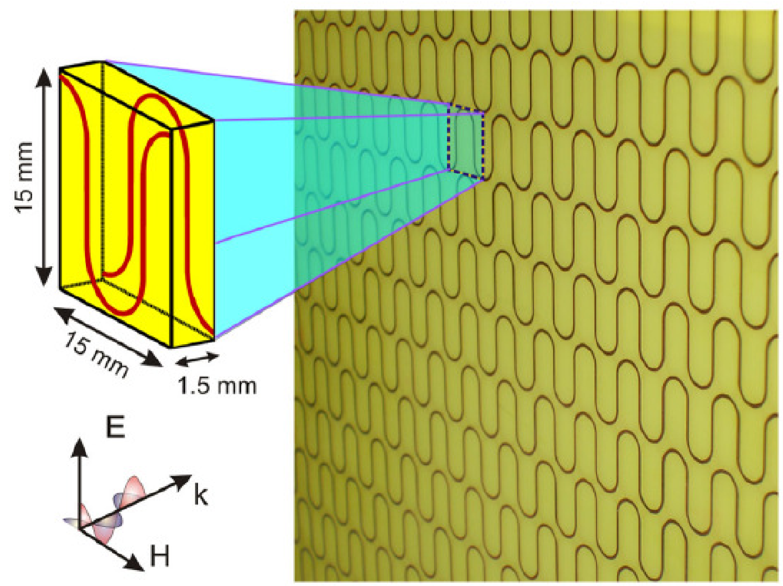
We demonstrate a classical analog of electromagnetically induced transparency in a planar metamaterial. We show that pulses propagating through such metamaterials experience considerable delay. The thickness of the structure along the direction of wave propagation is much smaller than the wavelength, which allows successive stacking of multiple metamaterial slabs leading to increased transmission and bandwidth.

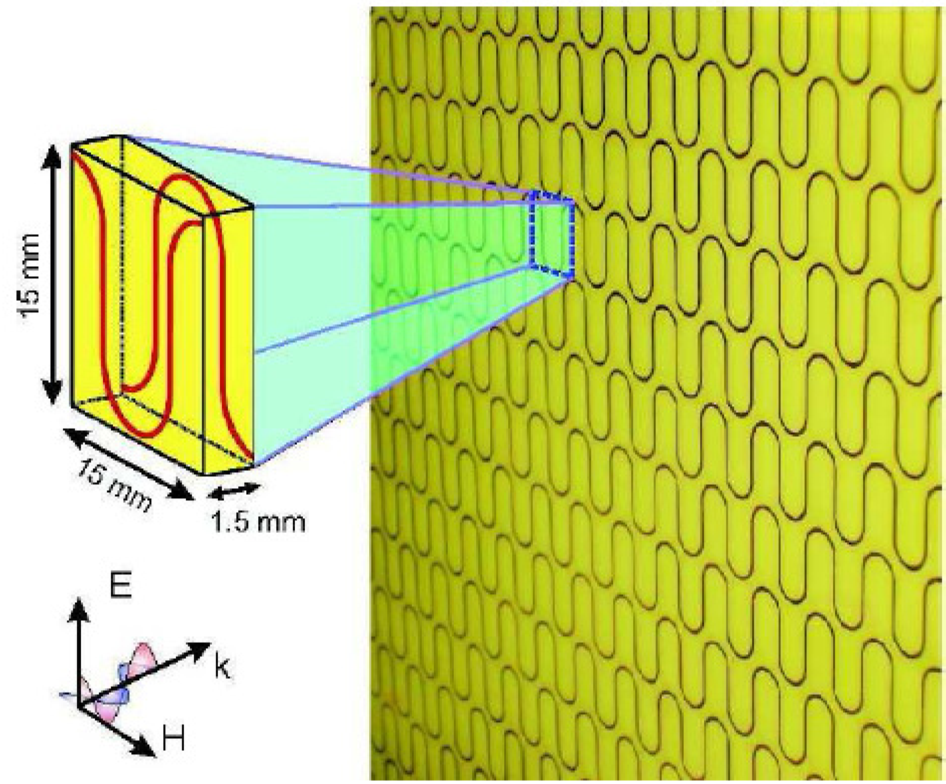


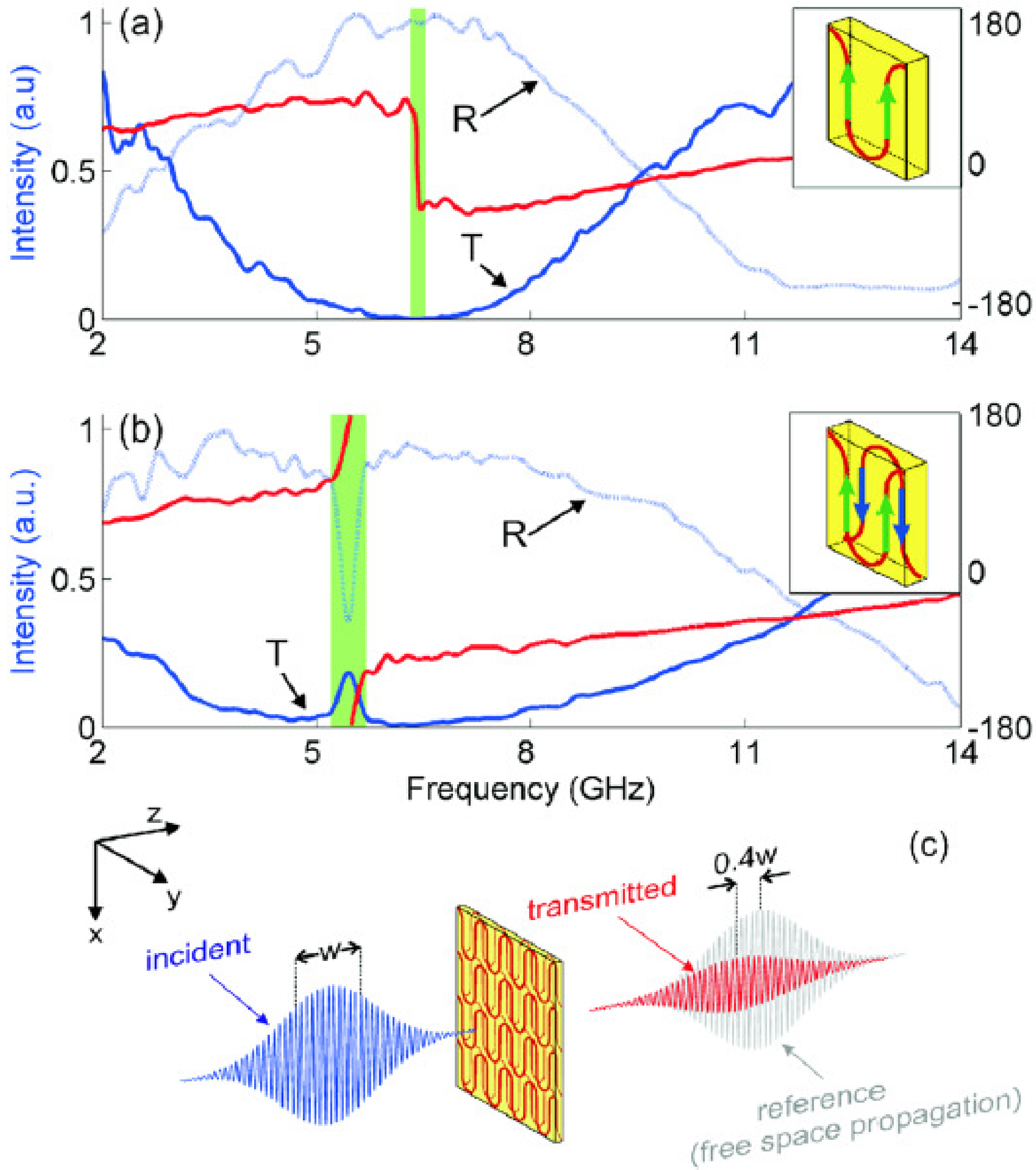
Interference of layered copper patterns

Shifted double fish-scale copper pattern

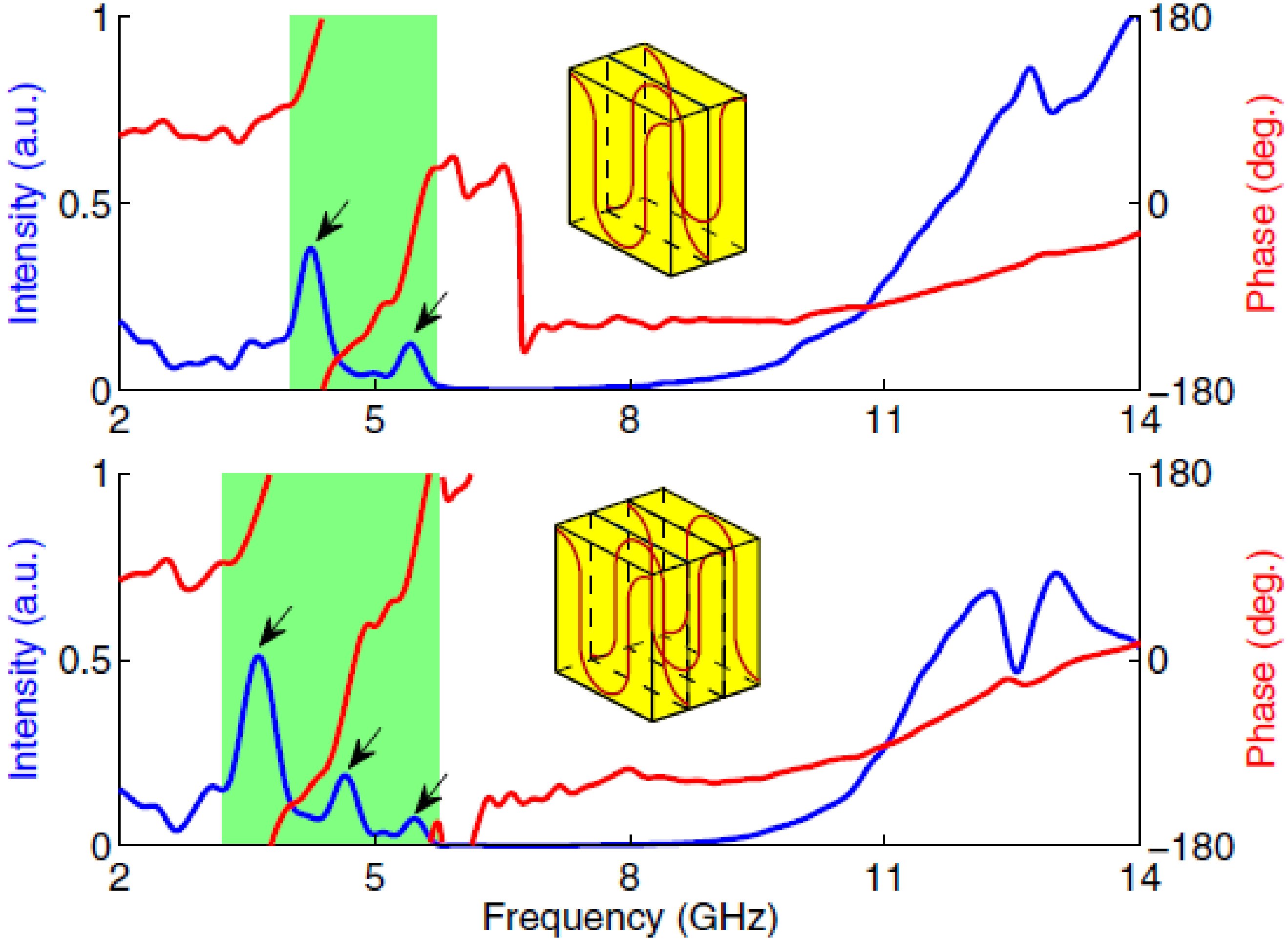
currents oscillate pi out of phase

Papasimakis et al., PRL 101, 253903 (2008)





(a)
Intensity (a.u)
Phase (deg.)
R
T
1
0.5
0
180
0
-180
2
5
8
11
14
(b)
Intensity (a.u.)
Phase (deg.)
R
T
Frequency (GHz)
(c)
z
y
x
incident
w
transmitted
0.4w
reference
(free space propagation)

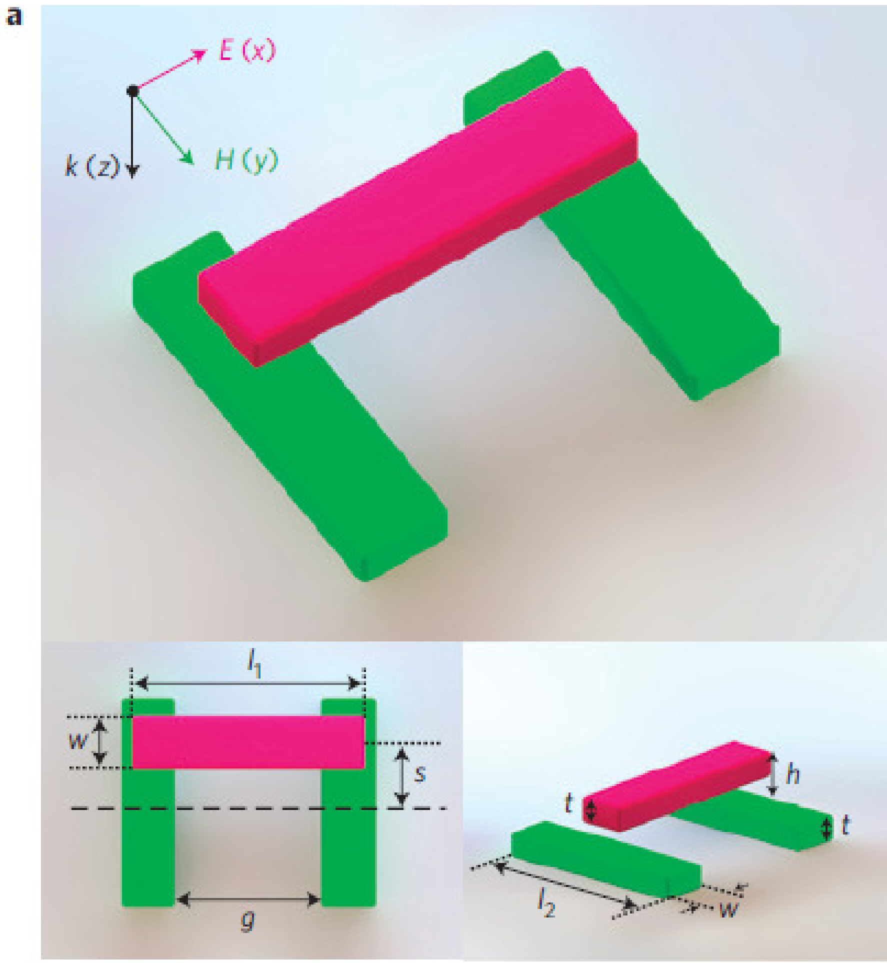


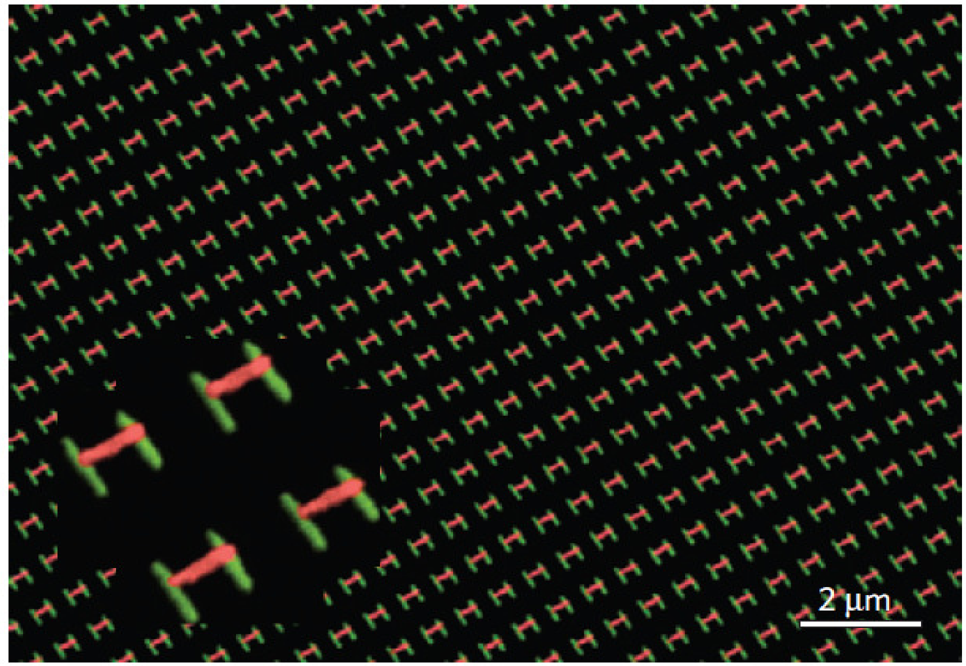
Intensity (a.u.)
Phase (deg.)
0
0.5
1
180
0
−180
2
5
8
11
14
Intensity (a.u.)
Phase (deg.)
0
0.5
1
180
0
−180
2
5
8
11
14
Frequency (GHz)

Coupling of a rod to two wires

Coupled dipole and quadrupole antennas

Liu et al., Nature Materials 8, 758 (2009)





Two layers: gold rod on top
two gold wires below
h=70nm; g=220nm; w=80nm

Broad and narrow resonances

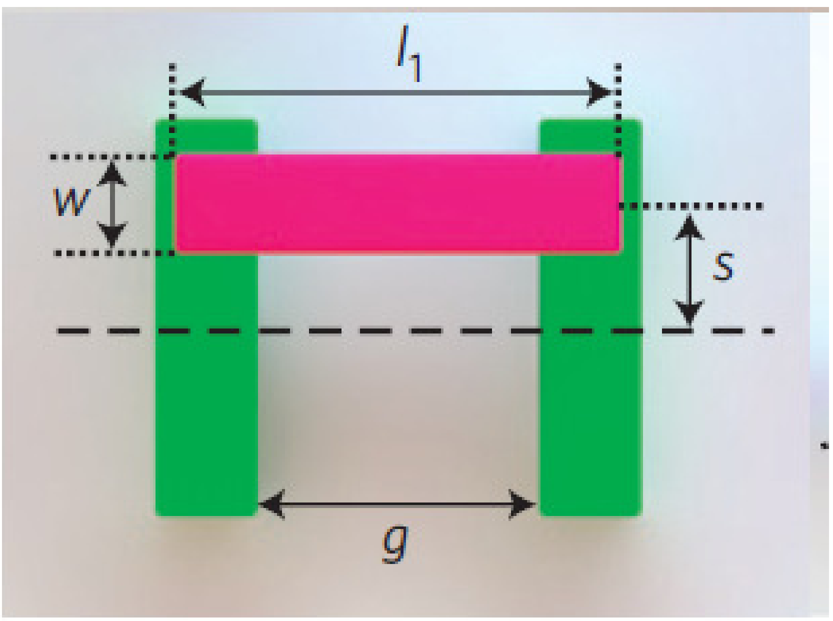
dipole with large radiative damping (rod on top)

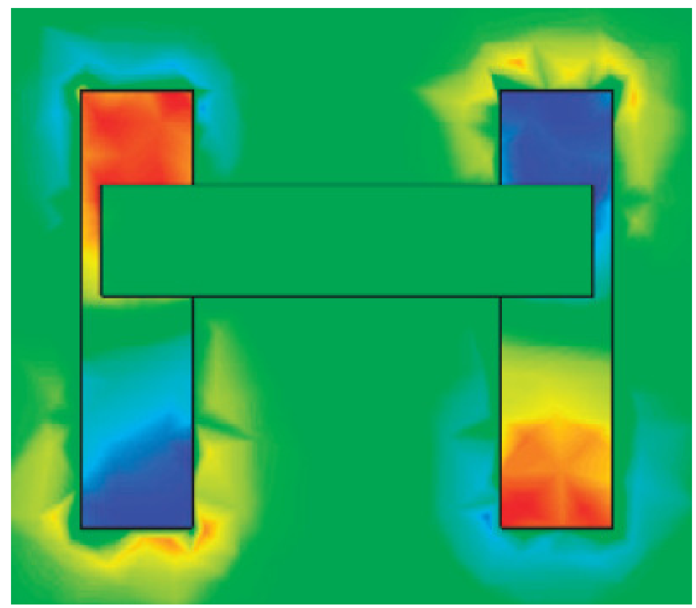
quadrupole almost non-radiative (two wires below)

Coupling between the two layers

no coupling for s=0 due to symmetry

for non-zero s the coupling induced

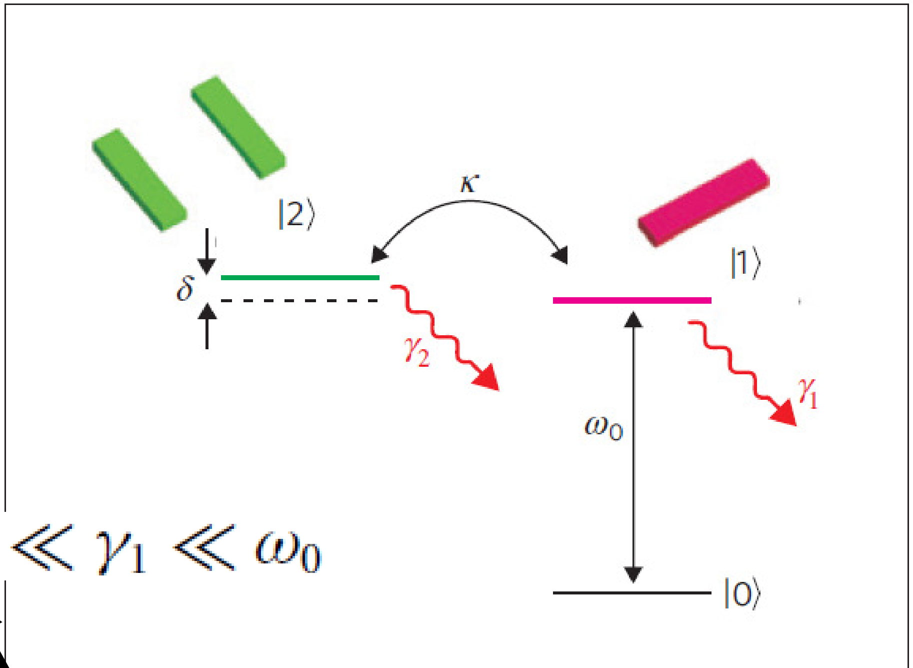




Level structure

$|1\rangle$ dipole excitation in the top rod

$|2\rangle$ quadrupole excitation in the bottom wires can be excited due to structural asymmetry



$|0\rangle - |1\rangle$

dipole-allowed transition

$|0\rangle - |2\rangle$

dipole-forbidden

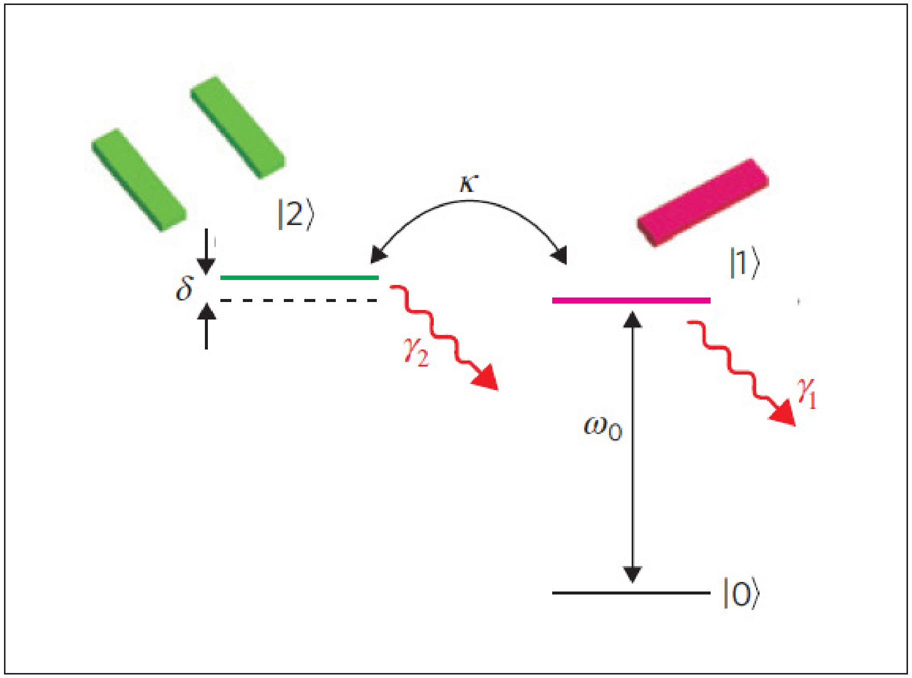
$\delta \ll \gamma_1$

$\gamma_2 \ll \gamma_1 \ll \omega_0$

non-radiative due to ohmic losses

κ transition rate between dipole and quadrupole excitations

Resonances



At s=0, there is a single resonance at $\omega_0 = 170\,\mathrm{THz}$
No coupling between layers

At s=10nm resonance properties change
Transmittance peak at 173THz

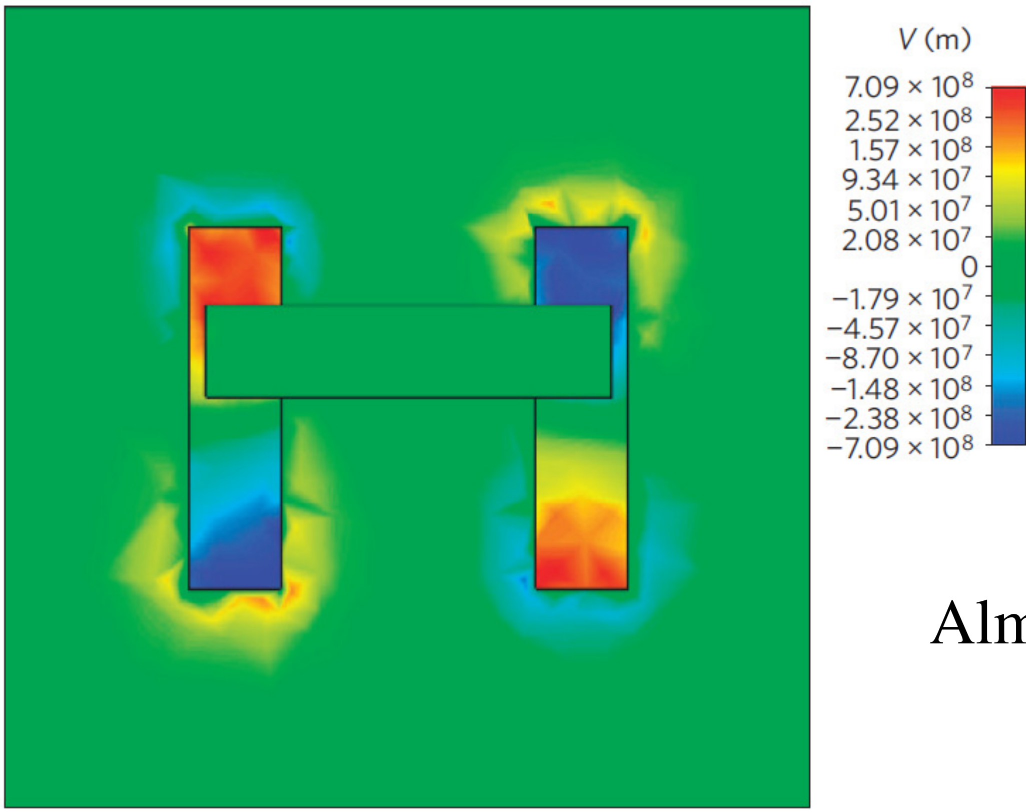
δ is approximately $3\,\mathrm{THz}$
frequency difference between dipole and quadrupole resonances

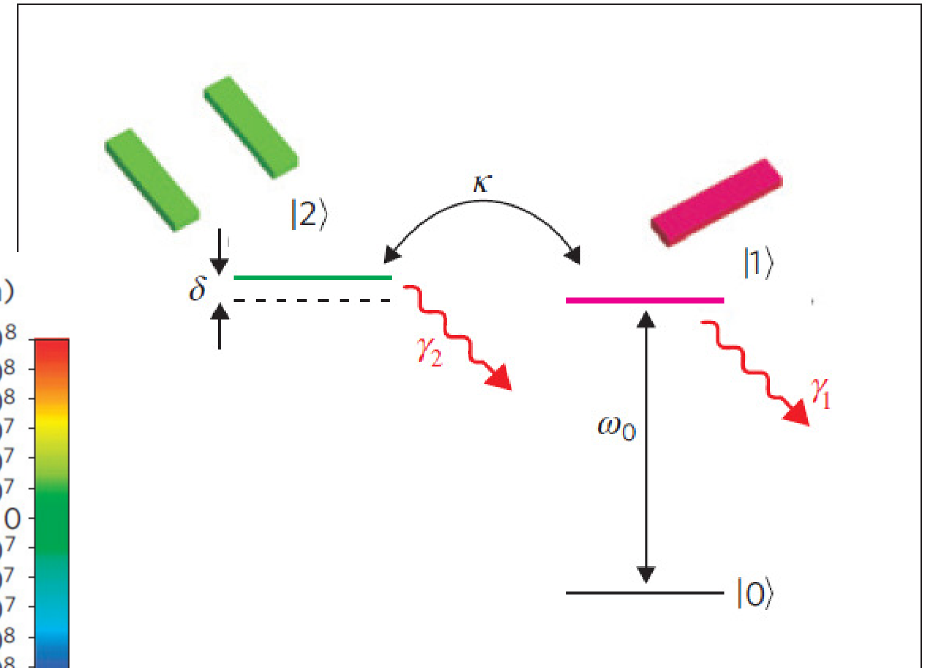
Interference of transitions

destructive interference between excitation paths

$$|0\rangle - |1\rangle$$

$$|0\rangle - |1\rangle - |2\rangle - |1\rangle$$





Almost no excitation in dipole rod

Analytic model

$$\ddot{q}_1(t)+\gamma_1\dot{q}_1(t)+\omega_0^2 q_1(t)+\kappa\dot{q}_2=E(t)$$

$$\ddot{q}_2(t)+\gamma_2\dot{q}_2(t)+(\omega_0+\delta)^2 q_2(t)-\kappa\dot{q}_1=0$$

$$P(\omega)=\frac{i}{2}\frac{(\omega-\omega_0-\delta)+i\frac{\gamma_2}{2}}{\left(\omega-\omega_0+i\frac{\gamma_1}{2}\right)\left(\omega-\omega_0-\delta+i\frac{\gamma_2}{2}\right)-\frac{\kappa^2}{4}}$$

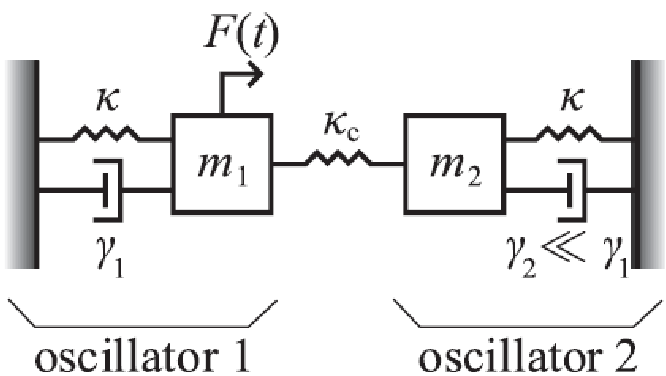
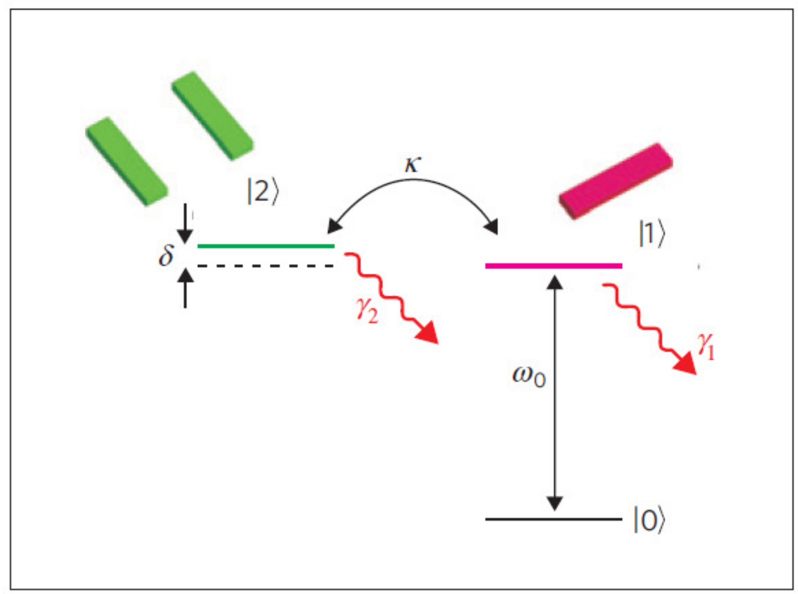
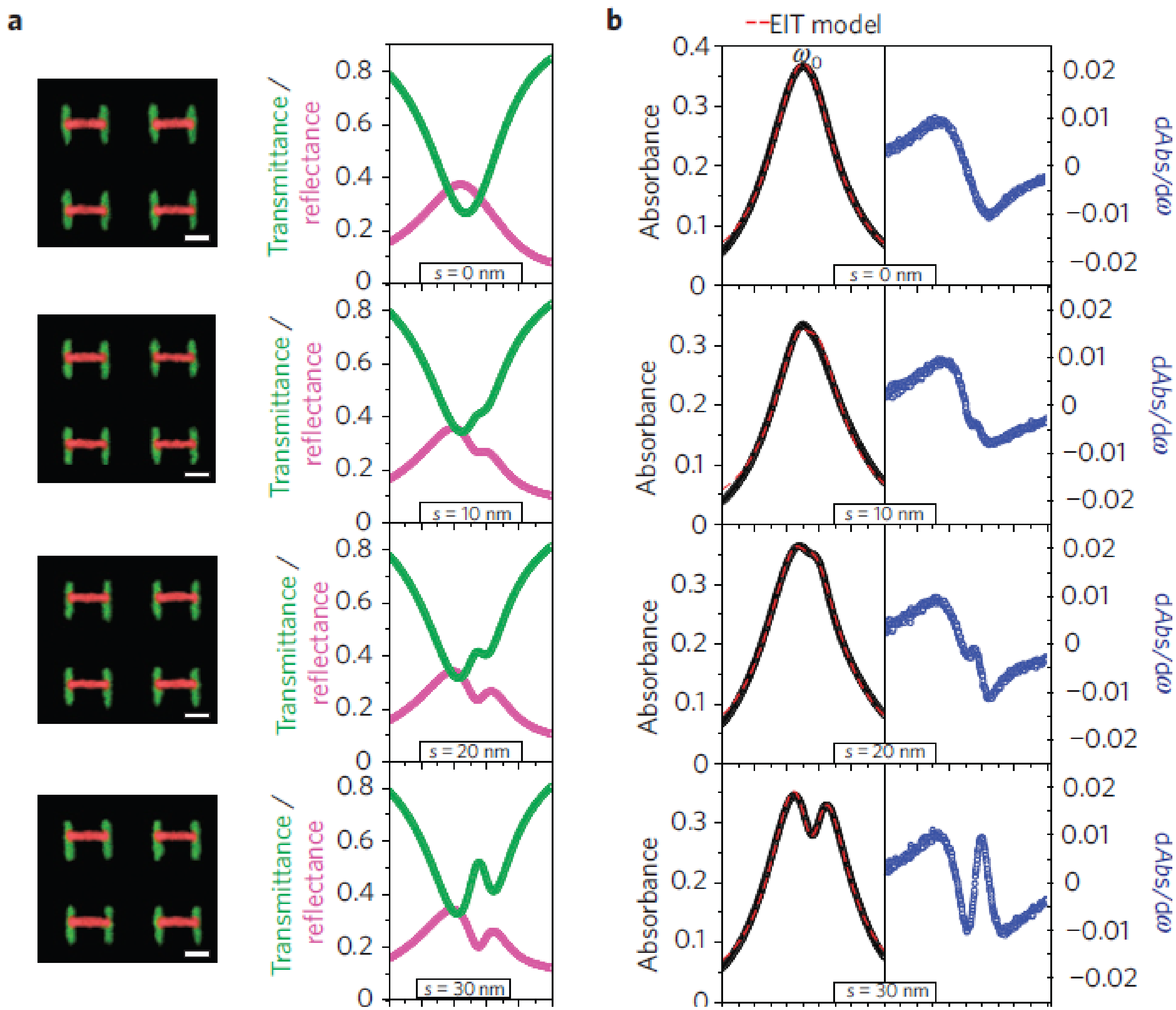
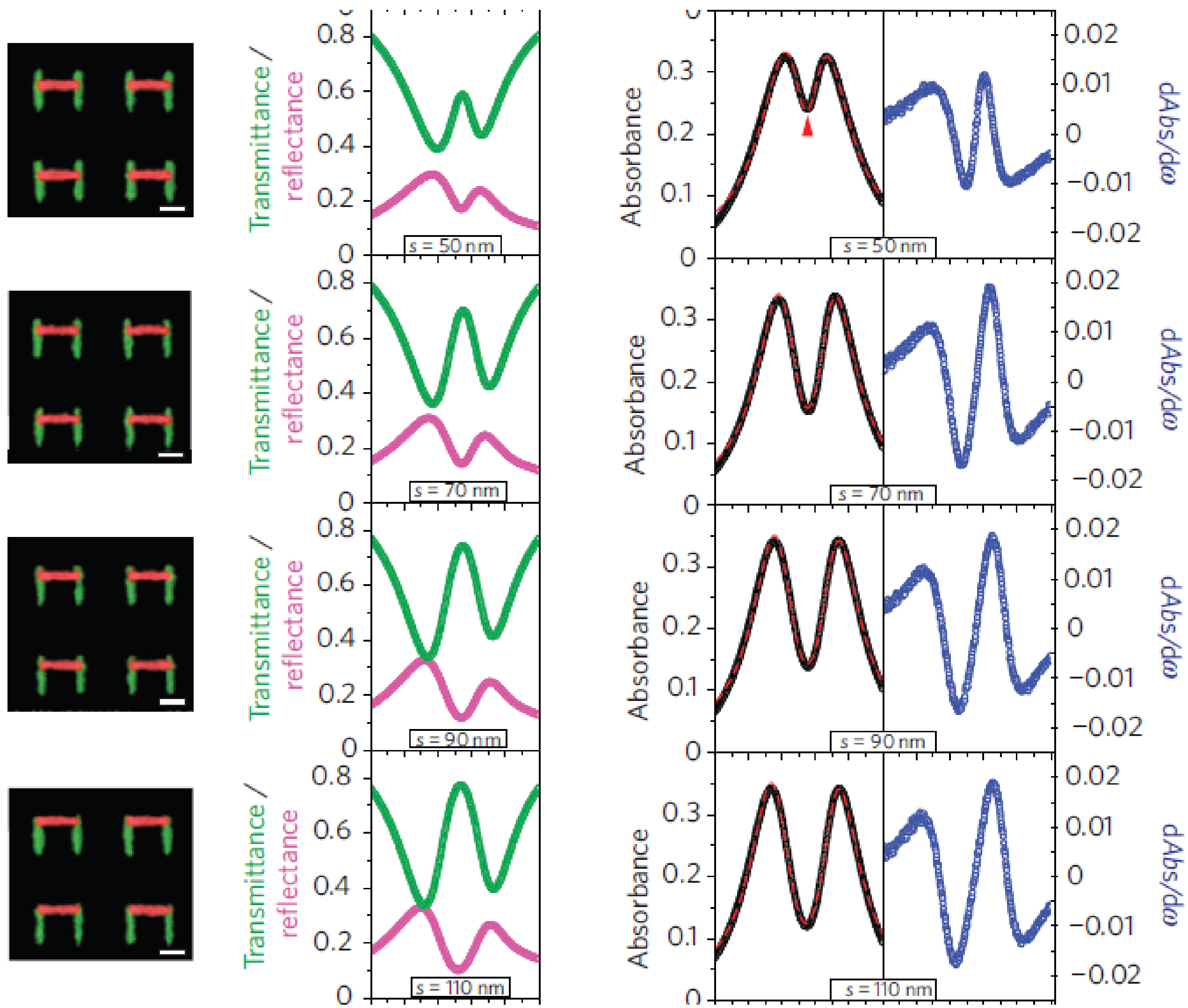


image: Tassin etc.

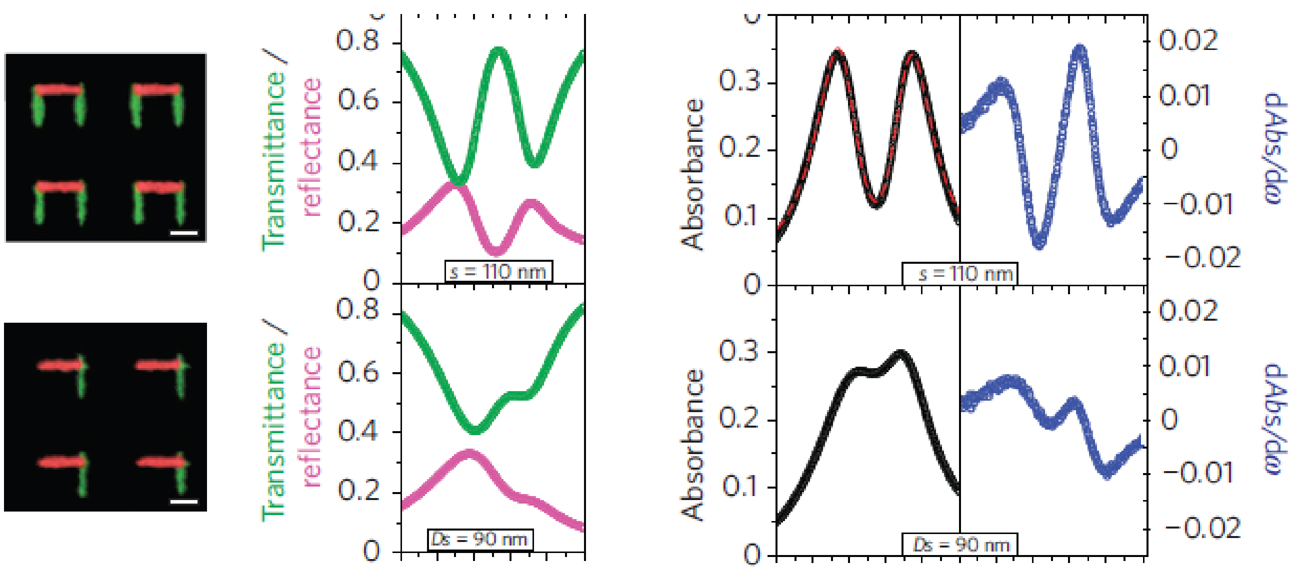



a
b
--EIT model
ω_0
Transmittance / reflectance
Absorbance
dAbs/dω
s = 0 nm
s = 10 nm
s = 20 nm
s = 30 nm
0
0.2
0.4
0.6
0.8
0.1
0.3
0.02
0.01
−0.01
−0.02


Transmittance / reflectance
Absorbance
dAbs/dω
s = 50 nm
s = 70 nm
s = 90 nm
s = 110 nm
0
0.2
0.4
0.6
0.8
0.1
0.3
0.01
0.02
−0.01
−0.02

Dipole-quadrupole vs dipole-dipole

Two single rods on top of each other

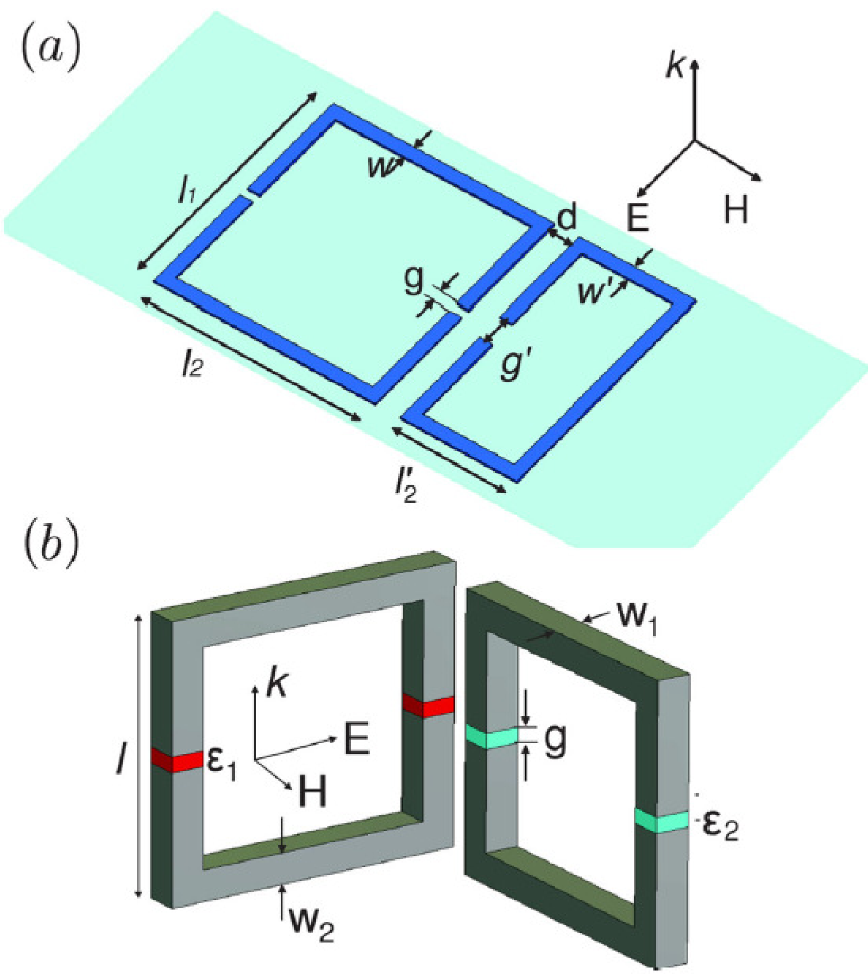


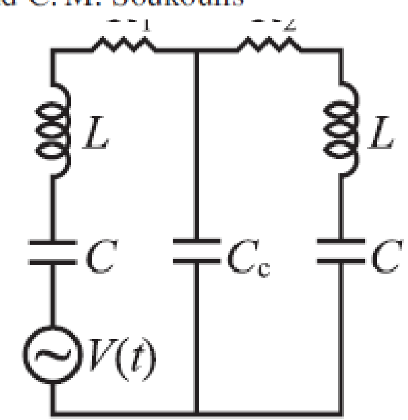
Circuit analog

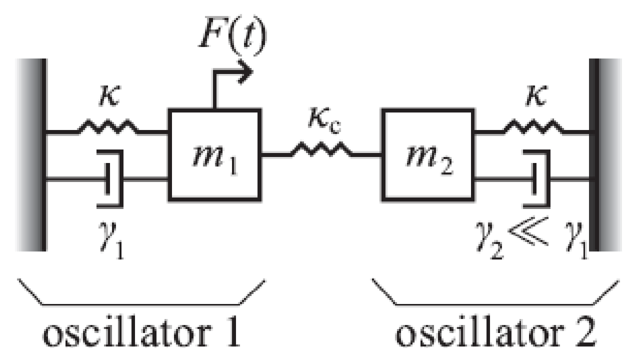
PRL **102,** 053901 (2009) PHYSICAL REVIEW LETTERS week ending 6 FEBRUARY 2009

Low-Loss Metamaterials Based on Classical Electromagnetically Induced Transparency

P. Tassin,[1] Lei Zhang,[2] Th. Koschny,[2,3] E. N. Economou,[3] and C. M. Soukoulis[2,3]

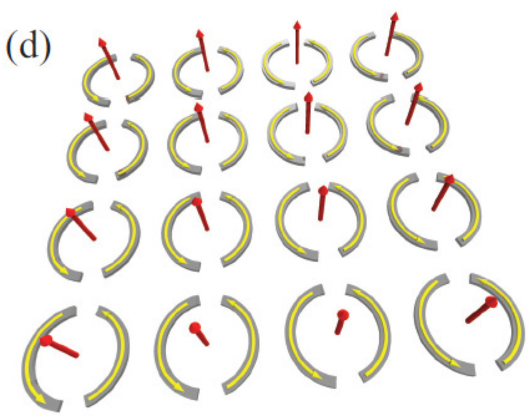


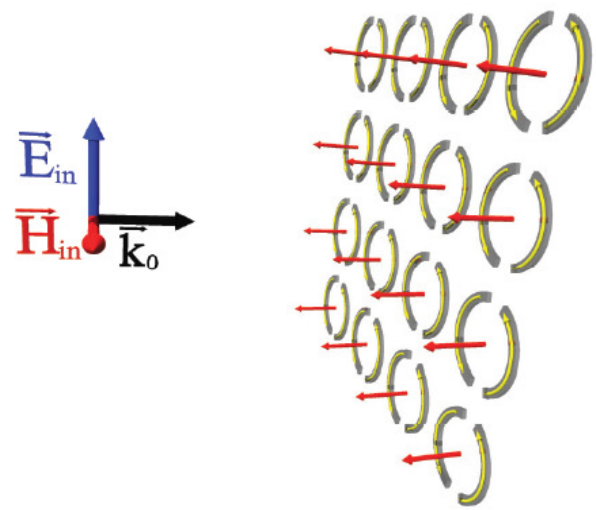




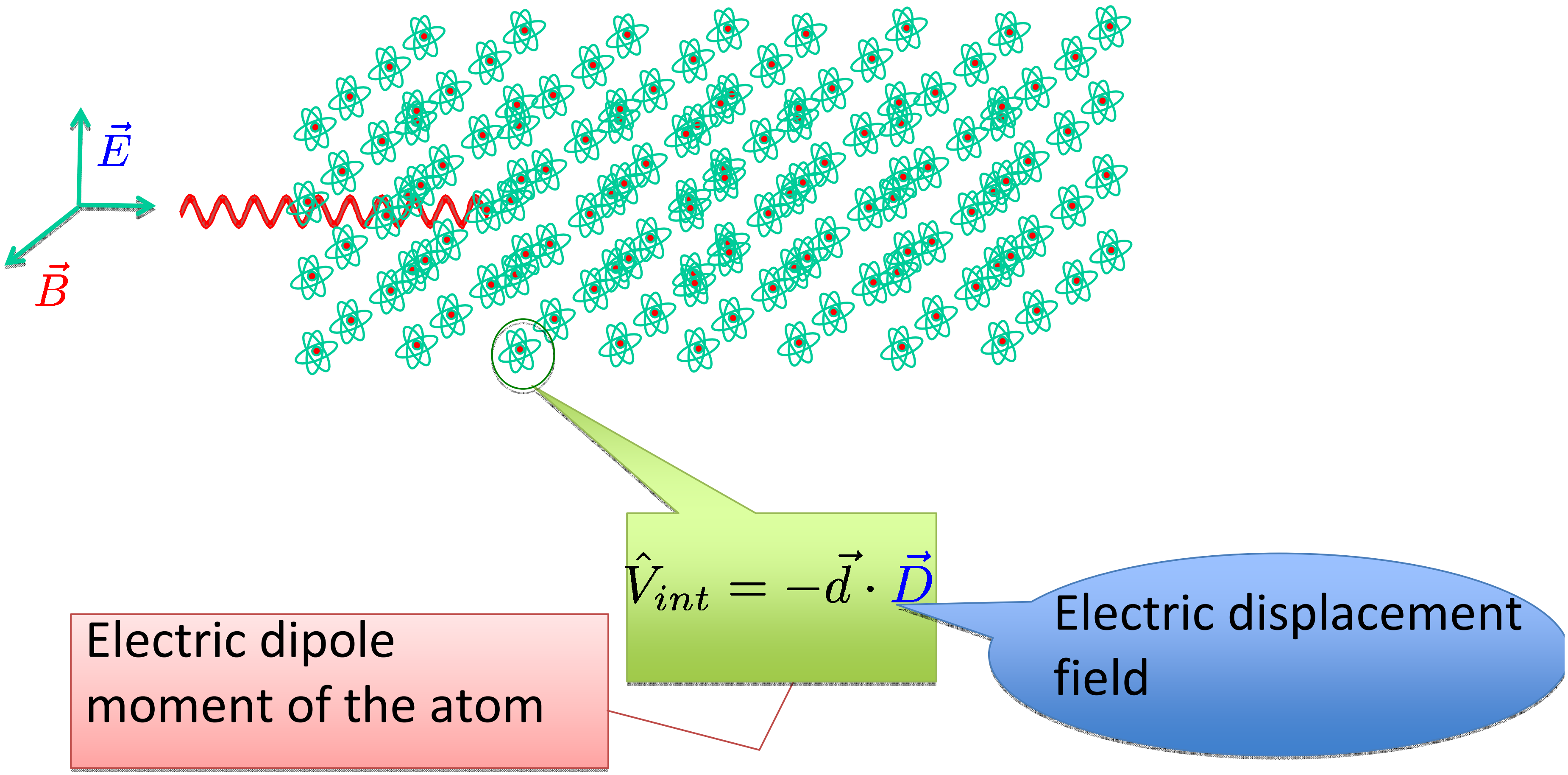
Strong interactions to EM fields – collective response

- Nanostructured resonators interacting strongly with EM fields
- Cooperative response in large system
- Analogies of metamaterials systems to molecular scatterers
- Analogous phenomena to quantum coherence effects in atomic gases
- Treatment of resonators as discrete scatterers

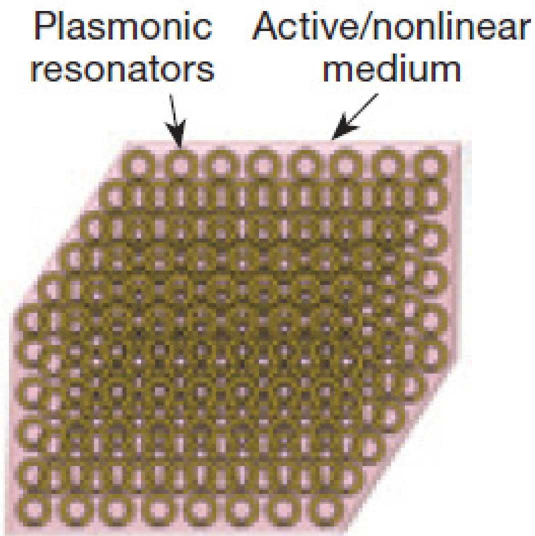




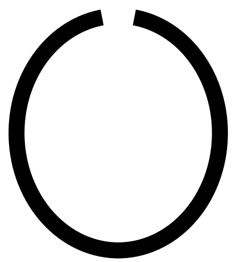
Natural medium and light propagation

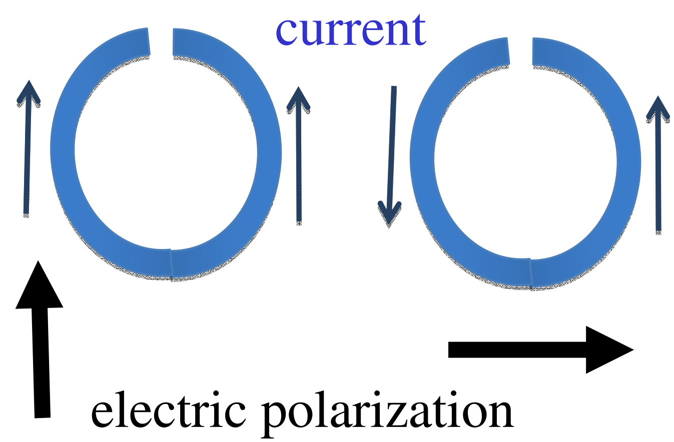


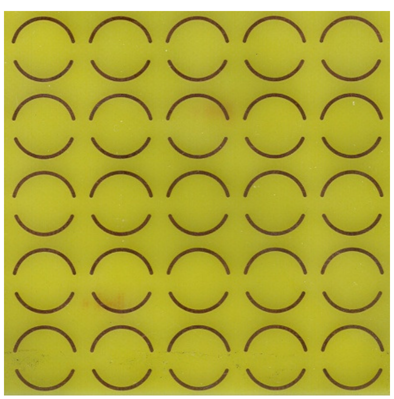
Nanofabricated resonators

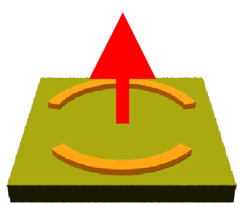


split-ring resonator (SSR)





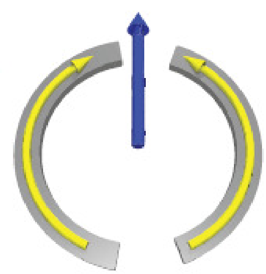




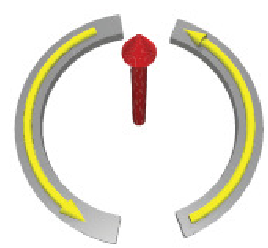
Net electric dipole

Net magnetic dipole

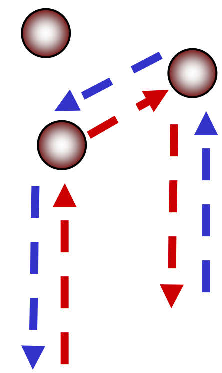
(b)



(c)



Co-operative light scattering



Coherent back-scattering

Simplest manifestation of coherence in multiple scattering

Enhanced scattering intensity in back-scattering direction

Co-operative response

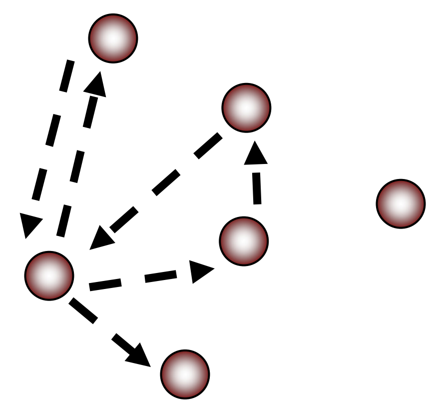
Strong scattering – Interference between different scattering paths

Localisation of light (observed in semiconductor GaAs powder)

Wiersma et al., Nature **390**, 671 (1997)

Analogous to Anderson localisation of electrons

Localisation achievable in atomic condensates



Simple model of oscillating current in meta-molecule constituents

single mode of current oscillation

Jenkins, Ruostekoski PRB

dynamic variable $Q_j(t)$

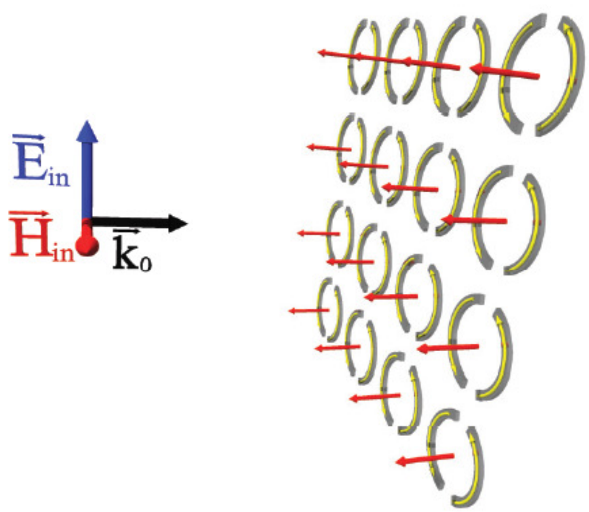
Polarization and Magnetization Densities

$$\vec{P}_j(\vec{r},t) = Q_j(t)\mathbf{p}_j(\mathbf{r})$$
$$\vec{M}_j(\vec{r},t) = \dot{Q}_j(t)\mathbf{w}_j(\mathbf{r})$$

Spatial mode functions

$$I_j(t) = dQ_j/dt$$

meta-atom ensemble



Charge and current density

$$\rho(\vec{r},t) = -\sum_j \nabla\cdot\vec{P}_j(\vec{r},t)$$

$$\vec{J}(\vec{r},t) = \sum_j \left(\frac{d\vec{P}_j}{dt} + \nabla\times\vec{M}_j(\vec{r},t) \right)$$

Lagrangian description

Coulomb gauge, Power-Zienau-Woolley transformation

(Cohen-Tannoudji, Dupont-Roc, Grynberg, Photons and atoms)

System Lagrangian

$$\mathcal{K} = \sum_j \frac{1}{2} l I_j^2(t)$$

Inertial inductance due, e.g. to masses of charge carriers

Lagrangian for free electromagnetic field

Coulomb interaction

$$\mathcal{L} = \mathcal{K} + \mathcal{L}_{EM} + V_{Coul} + \sum_j \left[Q_j(t)\mathcal{E}_j(t) + I_j(t)\Phi_j(t) \right]$$

Magnetic flux through circuit element

$$\mathcal{E}_j \equiv \int d^3r\, \vec{E}(\vec{r},t)\cdot \mathbf{p}_j(\mathbf{r})$$

Effective electromotive force induced by electric field

$$\Phi_j(t) \equiv \int d^3r \vec{B}(\vec{r},t)\cdot \mathbf{w}_j(\mathbf{r})$$

$\mathbf{\Pi}(\mathbf{r},t) = -\mathbf{D}(\mathbf{r},t),$ conjugate momentum for vector potential

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$\phi_j = l I_j + \Phi_j.$ conjugate momentum for charges

Hamiltonian

Introduce normal modes for the EM field, a_q

$$\mathbf{D}(\mathbf{r},t) = \sum_q \xi_q \hat{\mathbf{e}}_q a_q e^{i\mathbf{q}\cdot\mathbf{r}} + \text{C.c.} \qquad \xi_q \equiv i\sqrt{cq\epsilon_0/2V},$$

$$\mathbf{B}(\mathbf{r},t) = \sqrt{\frac{\mu_0}{\epsilon_0}} \sum_q \xi_q \hat{\mathbf{q}} \times \hat{\mathbf{e}}_q a_q e^{i\mathbf{q}\cdot\mathbf{r}} + \text{C.c.}$$

sum over transverse polarizations $\hat{\mathbf{e}}_{\lambda,\mathbf{q}}$
and wavevectors $\mathbf{q}$

$$H = H_{\text{EM}} + \sum_j \left[\frac{1}{2l}(\phi_j - \Phi_j)^2 + \frac{1}{2\epsilon_0} \int \mathbf{P}(\mathbf{r}) \cdot \mathbf{P}(\mathbf{r}) - \frac{1}{\epsilon_0} \int d^3r\, \mathbf{D}(\mathbf{r}) \cdot \mathbf{P}(\mathbf{r}) \right]$$

kinetic energy $-\mathbf{M}(\mathbf{r}) \cdot \mathbf{B}(\mathbf{r})$
+ quadratic term (diamagnetic energy)

$$H_{\text{EM}} = \sum_q cq a_q^* a_q \,.$$

Radiated fields

Total EM fields = incident field + scattered fields from all meta-atoms

$$\boldsymbol{E}_{\mathrm{S}}(\boldsymbol{r},t) = \sum_j \boldsymbol{E}_{\mathrm{S},j}(\boldsymbol{r},t)\,,$$

$$\boldsymbol{H}_{\mathrm{S}}(\boldsymbol{r},t) = \sum_j \boldsymbol{H}_{\mathrm{S},j}(\boldsymbol{r},t)\,,$$

$$\boldsymbol{E}^{+}_{\mathrm{S},j}(\boldsymbol{r},\Omega) = \frac{k^3}{4\pi\epsilon_0}\int \mathrm{d}^3r' \Big[\mathbf{G}(\boldsymbol{r}-\boldsymbol{r}',\Omega)\cdot\boldsymbol{P}^{+}_j(\boldsymbol{r}',\Omega) + \frac{1}{c}\mathbf{G}_{\times}(\boldsymbol{r}-\boldsymbol{r}',\Omega)\cdot\boldsymbol{M}^{+}_j(\boldsymbol{r}',\Omega)\Big],$$

$$\boldsymbol{H}^{+}_{\mathrm{S},j}(\boldsymbol{r},\Omega) = \frac{k^3}{4\pi}\int \mathrm{d}^3r' \Big[\mathbf{G}(\boldsymbol{r}-\boldsymbol{r}',\Omega)\cdot\boldsymbol{M}_j(\boldsymbol{r}',\Omega) - c\mathbf{G}_{\times}(\boldsymbol{r}-\boldsymbol{r}',\Omega)\cdot\boldsymbol{P}^{+}_j(\boldsymbol{r}',\Omega)\Big]\,,$$

Jackson, Classical electrodynamics

$$\mathbf{G}_{\times}(\boldsymbol{r},\Omega)\cdot\boldsymbol{v} = \frac{\mathrm{e}^{\mathrm{i}kr}}{kr}\left(1-\frac{1}{\mathrm{i}kr}\right)\hat{\boldsymbol{r}}\times\boldsymbol{v}\,.$$

magnetic (electric) field from oscillating electric (magnetic) dipole

$$\mathbf{G}(\boldsymbol{r},\Omega)\cdot\boldsymbol{v} = (\hat{\boldsymbol{r}}\times\boldsymbol{v})\times\hat{\boldsymbol{r}}\frac{\mathrm{e}^{\mathrm{i}kr}}{kr} + [3\hat{\boldsymbol{r}}(\hat{\boldsymbol{r}}\cdot\boldsymbol{v})-\boldsymbol{v}] \times\left[\frac{1}{(kr)^3}-\frac{\mathrm{i}}{(kr)^2}\right]\mathrm{e}^{\mathrm{i}kr} - \frac{4\pi}{3}\delta(k\boldsymbol{r})\boldsymbol{v}\,,$$

electric (magnetic) field from oscillating electric (magnetic) dipole

$$k \equiv \Omega/c$$

$$\boldsymbol{E}^{+}_{\mathrm{S},j}(\boldsymbol{r},\Omega) = \frac{k^3}{4\pi\epsilon_0}\int \mathrm{d}^3r' \Big[\mathbf{G}(\boldsymbol{r}-\boldsymbol{r'},\Omega)\cdot\boldsymbol{P}^{+}_{j}(\boldsymbol{r'},\Omega)$$
$$+\frac{1}{c}\mathbf{G}_{\times}(\boldsymbol{r}-\boldsymbol{r'},\Omega)\cdot\boldsymbol{M}^{+}_{j}(\boldsymbol{r'},\Omega)\Big],$$
$$\boldsymbol{H}^{+}_{\mathrm{S},j}(\boldsymbol{r},\Omega) = \frac{k^3}{4\pi}\int \mathrm{d}^3r' \Big[\mathbf{G}(\boldsymbol{r}-\boldsymbol{r'},\Omega)\cdot\boldsymbol{M}_{j}(\boldsymbol{r'},\Omega)$$
$$-c\mathbf{G}_{\times}(\boldsymbol{r}-\boldsymbol{r'},\Omega)\cdot\boldsymbol{P}^{+}_{j}(\boldsymbol{r'},\Omega)\Big],$$

Integral representation of Maxwell's wave equations in magnetodielectric medium (in freq space)

$$\begin{aligned}
(\nabla^2 + k^2)\mathbf{D}^{(\pm)} &= -\nabla\times(\nabla\times\mathbf{P}^{(\pm)}) \\
&\quad \mp i\frac{k}{c}\nabla\times\mathbf{M}^{(\pm)} \\
(\nabla^2 + k^2)\mathbf{B}^{(\pm)} &= -\mu_0\nabla\times(\nabla\times\mathbf{M}^{(\pm)}) \\
&\quad \pm i\mu_0 ck\nabla\times\mathbf{P}^{(\pm)}
\end{aligned}$$

Resonator dynamics

Charges driven by net magnetic flux
their conjugate momenta driven by net electromotive force

$$\dot{\phi}_j(t) \;=\; \mathcal{E}_j(t),$$

Interaction with self-radiated fields results in radiative damping
electric radiative emission depends on self-capacitance

$$\Gamma_{E,j}(k) \;=\; h_j^2 \omega_j C_j k^3/(6\pi\epsilon_0)$$

magnetic depends on self-inductance

$$\Gamma_{M,j}(k) = \mu_0 \omega_j A_j^2 k^3/(6\pi L_j)$$

resonance frequency $\omega_j \;\equiv\; 1/\sqrt{L_j C_j}$

⇒ Simple LC circuit include also ohmic losses Γ_O;

Coupled system for resonators

Strong coupling mediated by scattered fields

$$\dot{\mathrm{b}} = \mathcal{C}\mathrm{b} + \mathrm{f}_{\mathrm{in}} \ ,$$

$$b_j(t) \equiv \frac{\mathrm{e}^{\mathrm{i}\Omega_0 t}}{\sqrt{2}} \left(\frac{Q_j(t)}{\sqrt{\omega_j C_j}} + \mathrm{i}\frac{\phi_j(t)}{\sqrt{\omega_j L_j}} \right) .$$

$$\mathrm{b}(t) \equiv \begin{pmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_{nN}(t) \end{pmatrix} , \qquad \mathrm{f}_{\mathrm{in}}(t) \equiv \begin{pmatrix} f_{1,\mathrm{in}}(t) \\ f_{2,\mathrm{in}}(t) \\ \vdots \\ f_{nN,\mathrm{in}}(t) \end{pmatrix} .$$

$$\mathcal{C} = -\mathrm{i}\Delta - \frac{\Gamma}{2}\mathrm{I} + \frac{1}{2}\left(\mathrm{i}\mathcal{C}_{\mathrm{E}} + \mathrm{i}\mathcal{C}_{\mathrm{M}} + \mathcal{C}_{\times} + \mathcal{C}_{\times}^{T}\right) ,$$

$$\Gamma \equiv \Gamma_{\mathrm{E}} + \Gamma_{\mathrm{M}} + \Gamma_{\mathrm{O}}$$

$$[\mathcal{C}_{\mathrm{E}}]_{j,j'} = \frac{3}{2}\Gamma_{\mathrm{E}}\, \hat{\boldsymbol{d}}_j \cdot \mathbf{G}(\boldsymbol{r}_j - \boldsymbol{r}_{j'}, \Omega_0) \cdot \hat{\boldsymbol{d}}_{j'},$$

$$[\mathcal{C}_{\mathrm{M}}]_{j,j'} = \frac{3}{2}\Gamma_{\mathrm{M}}\, \hat{\boldsymbol{m}}_j \cdot \mathbf{G}(\boldsymbol{r}_j - \boldsymbol{r}_{j'}, \Omega_0) \cdot \hat{\boldsymbol{m}}_{j'},$$

$$[\mathcal{C}_{\times}]_{j,j'} = \frac{3}{2}\bar{\Gamma}\, \hat{\boldsymbol{d}}_j \cdot \mathbf{G}_{\times}(\boldsymbol{r}_j - \boldsymbol{r}_{j'}, \Omega_0) \cdot \hat{\boldsymbol{m}}_{j'} . \qquad \bar{\Gamma} \equiv \sqrt{\Gamma_{\mathrm{E}}\Gamma_{\mathrm{M}}}$$

Quantization

Photon plane-wave mode amplitudes, replace photon annihilation and creation operators $a_{\mathbf{q},\lambda} \rightarrow \sqrt{\hbar}\hat{a}_{\mathbf{q},\lambda}$

$$\left[\hat{a}_{\mathbf{q},\lambda}, \hat{a}_{\mathbf{q}',\lambda'}\right] = \left[\hat{a}^{\dagger}_{\mathbf{q},\lambda}, \hat{a}^{\dagger}_{\mathbf{q}',\lambda'}\right] = 0$$

$$\left[\hat{a}_{\mathbf{q},\lambda}, \hat{a}^{\dagger}_{\mathbf{q}',\lambda'}\right] = \delta_{\lambda,\lambda'}\delta(\mathbf{q}-\mathbf{q}')\,.$$

$$H = \sum_n \hbar\omega_n (a_n^{\dagger} a_n + \tfrac{1}{2})\,.$$

Resonators $b_j(t) \equiv \frac{e^{i\Omega_0 t}}{\sqrt{2}}\left(\frac{Q_j(t)}{\sqrt{\omega_j C_j}} + i\frac{\phi_j(t)}{\sqrt{\omega_j L_j}}\right)\,.$

format $\frac{1}{\sqrt{2}}(x_n + ip_n),$

$$[x_n, x_{n'}] = 0, \quad [p_n, p_{n'}] = 0, \quad [x_n, p_{n'}] = i\hbar\delta_{nn'}\,.$$

$$b_j \rightarrow \sqrt{\hbar}\hat{b}_j$$

$$\left[\hat{Q}_j, \hat{Q}_{j'}\right] = \left[\hat{\phi}_j, \hat{\phi}_{j'}\right] = 0$$
$$\left[\hat{Q}_j, \hat{\phi}_{j'}\right] = i\hbar\delta_{j,j'}$$

$$\left[\hat{b}_j, \hat{b}_{j'}\right] = \left[\hat{b}_j^\dagger, \hat{b}_{h'}^\dagger\right] = 0$$
$$\left[\hat{b}_j, \hat{b}_{j'}^\dagger\right] = \delta_{j,j'}$$

Quantum features – illuminate by quantum light

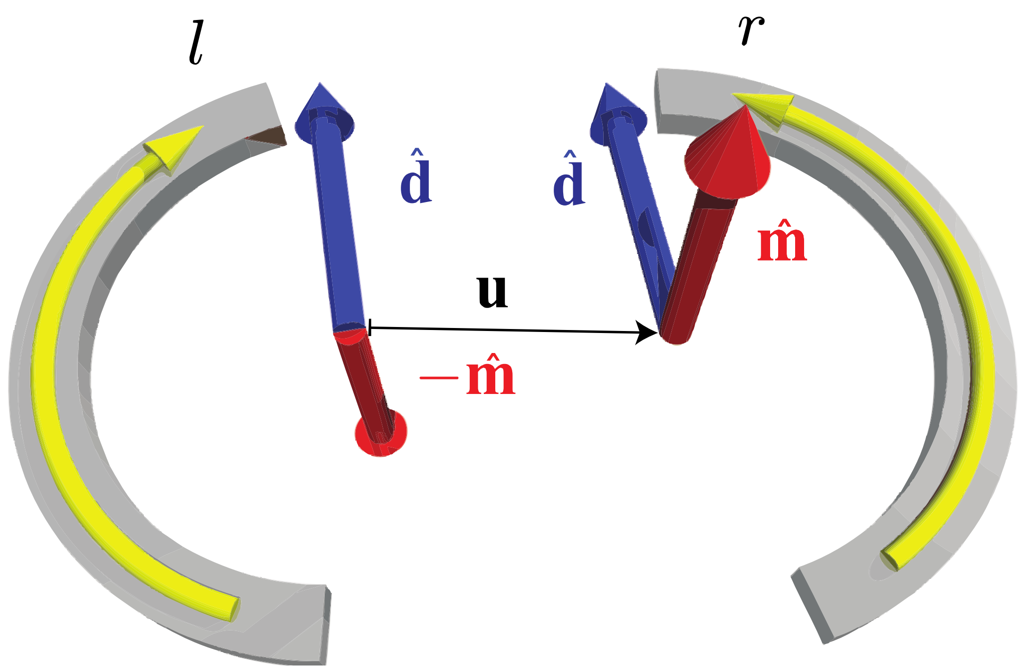
Nonlinear resonators (SQUID circuits)

Split ring resonator:
Simple model of oscillating current

single mode of current oscillation in each half of the unit-cell resonator

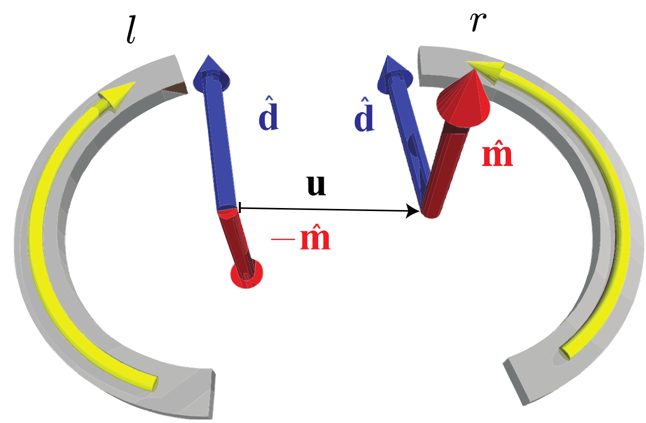
dynamics variable charge Q

polarization and magnetization densities



Single split ring resonator

$$\begin{pmatrix} \dot{b}_r \\ \dot{b}_l \end{pmatrix} = \mathcal{C}_{\mathrm{SRR}} \begin{pmatrix} b_r \\ b_l \end{pmatrix} + \begin{pmatrix} f_{r,\mathrm{in}} \\ f_{l,\mathrm{in}} \end{pmatrix}$$



$$\mathcal{C}_{\mathrm{SRR}} = \begin{pmatrix} -\Gamma/2 & i\mathrm{d}\Gamma G - \bar{\Gamma} S \\ i\mathrm{d}\Gamma G - \bar{\Gamma} S & -\Gamma/2 \end{pmatrix}$$

$$\Gamma \equiv \Gamma_{\mathrm{E}} + \Gamma_{\mathrm{M}} + \Gamma_{\mathrm{O}}$$

$$\bar{\Gamma} \equiv \sqrt{\Gamma_{\mathrm{E}}\Gamma_{\mathrm{M}}},$$

$$\mathrm{d}\Gamma \equiv \Gamma_{\mathrm{E}} - \Gamma_{\mathrm{M}}\,.$$

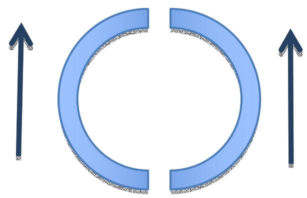
$$G \equiv \frac{3}{4}\hat{\mathbf{d}} \cdot \mathbf{G}(\mathbf{u}, \Omega_0) \cdot \hat{\mathbf{d}} = \frac{3}{4}\hat{\mathbf{m}} \cdot \mathbf{G}(\mathbf{u}, \Omega_0) \cdot \hat{\mathbf{m}}.$$

$$S \equiv \frac{3}{4}\hat{\mathbf{d}} \cdot \mathbf{G}_{\times}(\mathbf{u}, \Omega_0) \cdot \hat{\mathbf{m}}_r\,,$$

Unit-cell resonator current excitations (eigenmodes)

Symmetric oscillation

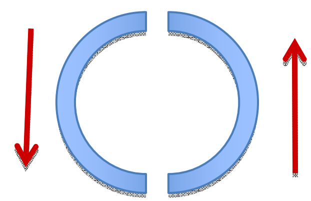
$$c(t) = \frac{1}{\sqrt{2}}\left[b_r(t) + b_l(t)\right]$$



Net electric dipole

Antisymmetric oscillation

$$d(t) = \frac{1}{\sqrt{2}}\left[b_r(t) - b_l(t)\right]$$

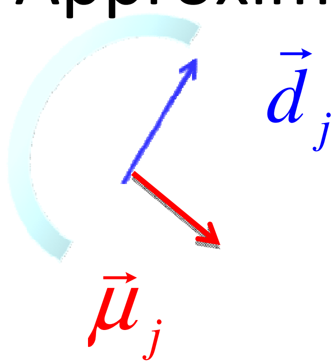


Net magnetic dipole

$$\gamma_+ \approx 2\Gamma_E + \Gamma_O \ ,$$
$$\gamma_- \approx 2\Gamma_M + \Gamma_O \ .$$

Dipole Approximation

$$\vec{P}_j \approx \vec{d}_j(t)\delta(\vec{r} - \vec{r}_j)$$
$$\vec{d}_j(t) = Q_j(t) h_j \hat{p}_j$$

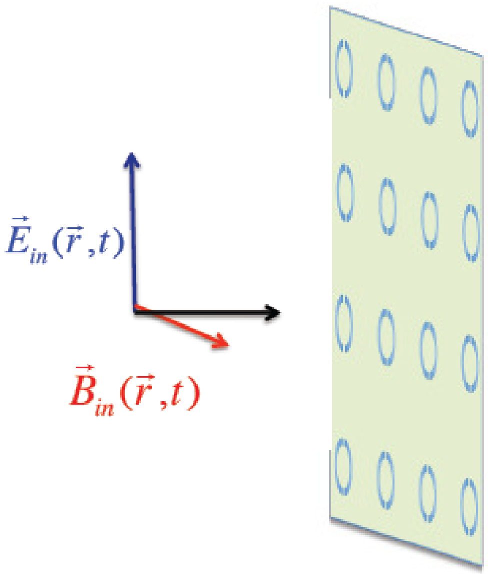


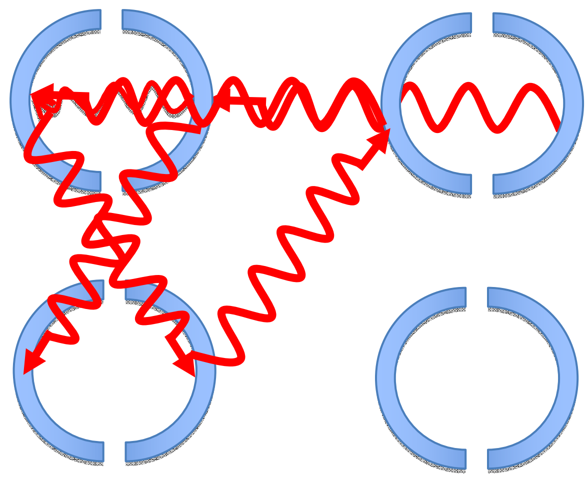
$$\vec{M}_j(\vec{r},t) \approx \vec{\mu}_j(t)\delta(\vec{r} - \vec{r}_j)$$
$$\vec{\mu}_j(t) = I_j(t) A_j \hat{q}_j$$

Metamaterial samples

Unit-cell resonators (and their sub-elements) interact strongly

Long-range coupling due to electric & magnetic radiation from current excitations





Recurrent scattering:

Wave is scattered more than once

by the same resonator

Metamaterial sample responds to EM fields cooperatively

Exhibits collective excitation eigenmodes, resonance linewidths, frequencies

Cooperative response

Uniform eigenmode model breaks down due to
boundary effects, dislocations, defects,
inhomogeneous sample, disorder

Advantages of the discrete model

Computationally efficient model for large systems
Physically tractable – providing insight and understanding
Can be build up to more complex systems

Quantum optics and metamaterials

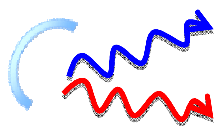
Janne Ruostekoski
Mathematics & Centre for Photonic Metamaterials
University of Southampton

Computational model for collective interactions

- Assume each meta-atom supports a single mode of current oscillation: 1 dynamic variable per meta-atom

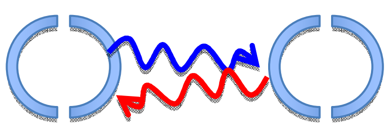
Polarization $\vec{P}_j(\vec{r},t) = Q_j(t)\vec{p}_j(\vec{r})$ $\quad\quad$ $\vec{M}_j(\vec{r},t) = \dot{Q}_j(t)\vec{w}_j(\vec{r})$ Magnetization

- Calculate the scattered electric and magnetic fields from each meta-atom



$\vec{E}_{\mathrm{S,j}}(\vec{r},t)$ $\quad\quad$ $\vec{B}_{\mathrm{S},j}(\vec{r},t)$

- Scattered fields mediate interactions between meta-atom dynamic variables. The incident field drives the system

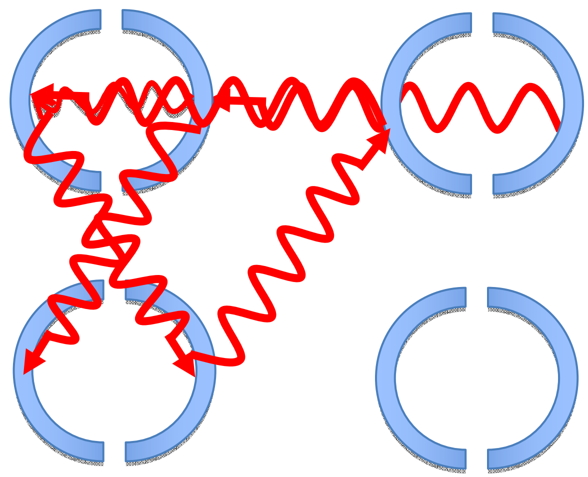


$$\dot{b}_j(t) = F_j + (i\Delta - \tfrac{\Gamma}{2})b_j + \textstyle\sum_{j'\neq j} C_{j,j'}b_{j'}$$

- Meta-material supports collective modes of oscillation: each with a distinct resonance frequency and decay rate

Cooperative response

Uniform eigenmode model breaks down due to
boundary effects, dislocations, defects, disorder …



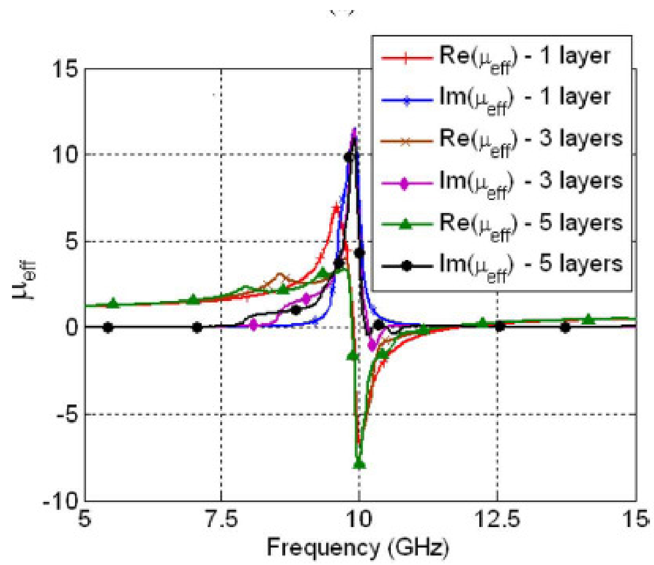
Recurrent scattering:
Wave is scattered more than once
by the same scatterer

A Unique Extraction of Metamaterial Parameters Based on Kramers–Kronig Relationship

Zsolt Szabó, Gi-Ho Park, Ravi Hedge, and Er-Ping Li, *Fellow, IEEE*

IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 58, NO. 10, OCTOBER 2010

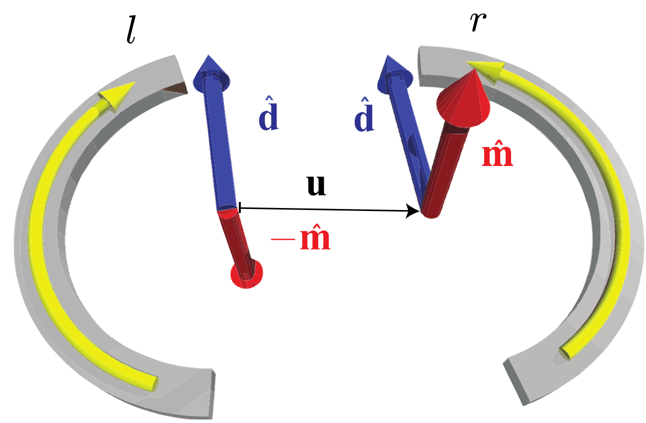
retrieving effective material parameters can fail



Single split ring resonator

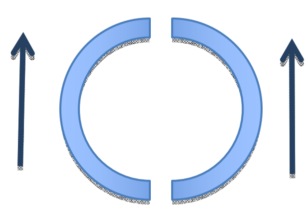
$$\begin{pmatrix} \dot{b}_r \\ \dot{b}_l \end{pmatrix} = \mathcal{C}_{\mathrm{SRR}} \begin{pmatrix} b_r \\ b_l \end{pmatrix} + \begin{pmatrix} f_{r,\mathrm{in}} \\ f_{l,\mathrm{in}} \end{pmatrix}$$

$$\mathcal{C}_{\mathrm{SRR}} = \begin{pmatrix} -\Gamma/2 & i\mathrm{d}\Gamma G - \bar{\Gamma} S \\ i\mathrm{d}\Gamma G - \bar{\Gamma} S & -\Gamma/2 \end{pmatrix}$$



Symmetric oscillation

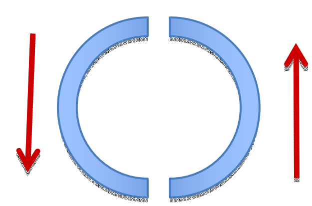
$$c(t) = \frac{1}{\sqrt{2}}\left[b_r(t) + b_l(t)\right]$$



Net electric dipole

Antisymmetric oscillation

$$d(t) = \frac{1}{\sqrt{2}}\left[b_r(t) - b_l(t)\right]$$



Net magnetic dipole

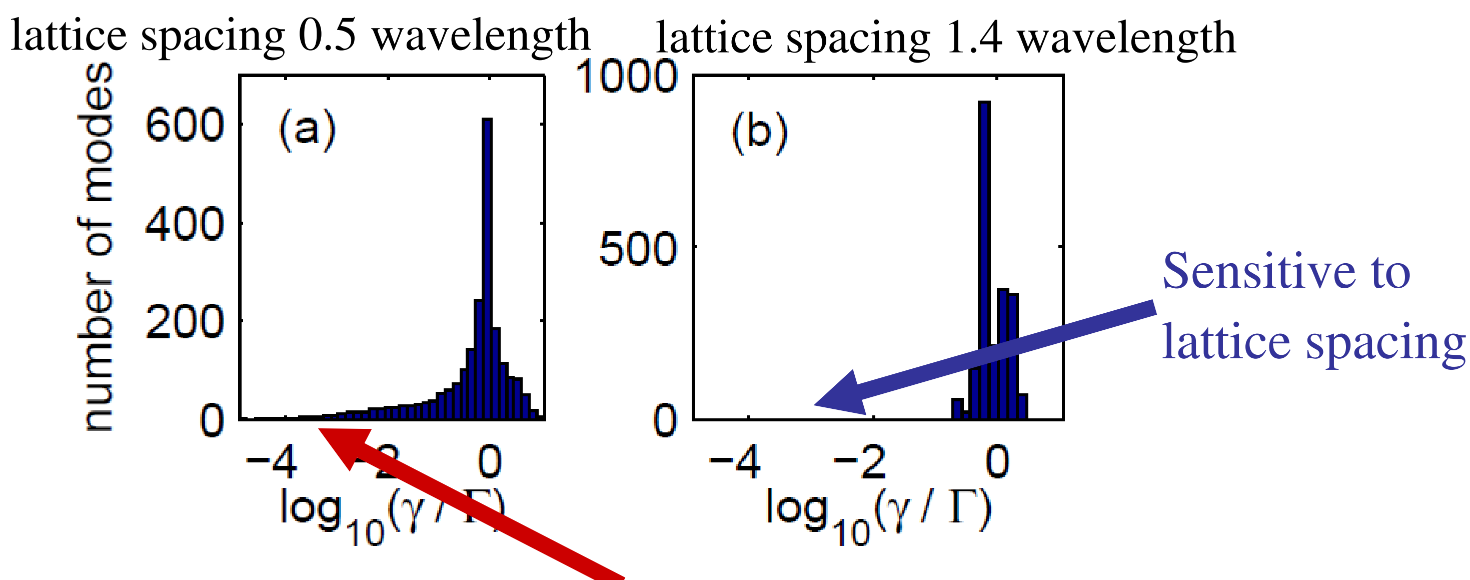
Collective excitation eigenmodes

Split ring resonator (SRR): Two eigenmodes per unit-cell

33×33 array = 2178 modes

An isolated one half of a SRR would radiate at the rate Γ

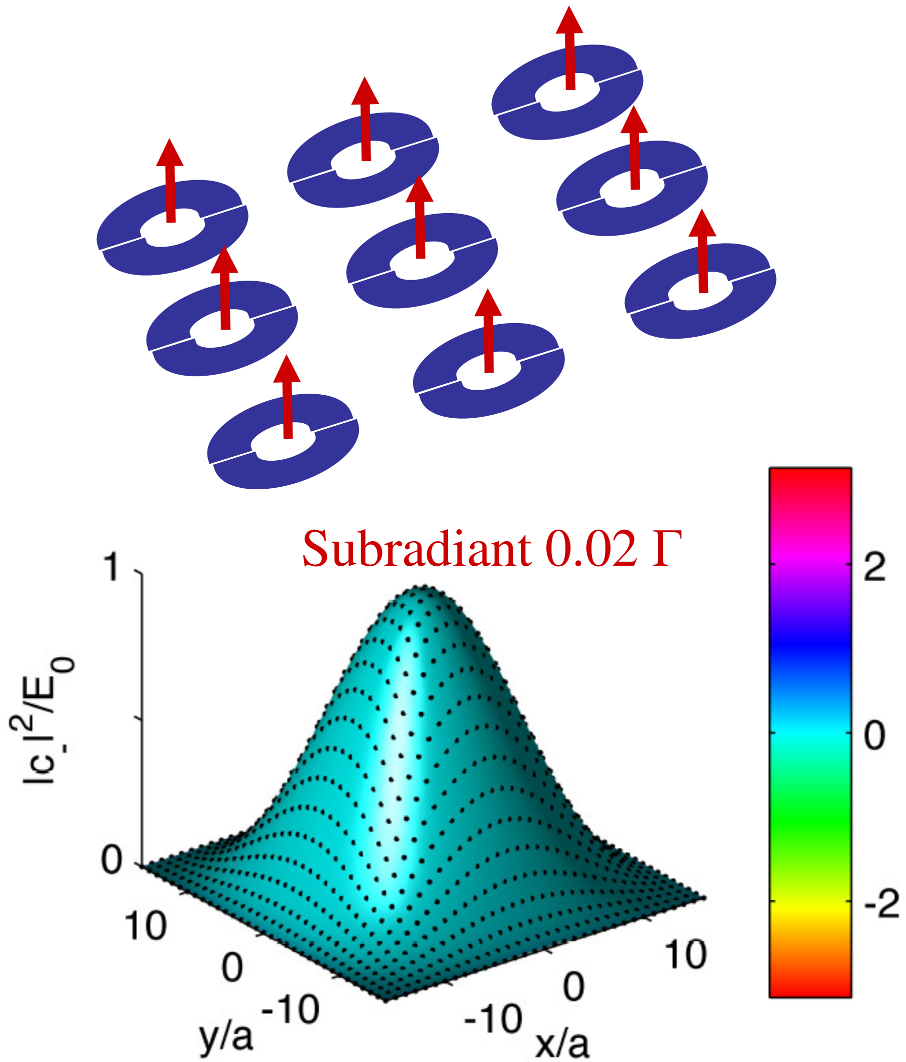
Collective resonance linewidths γ



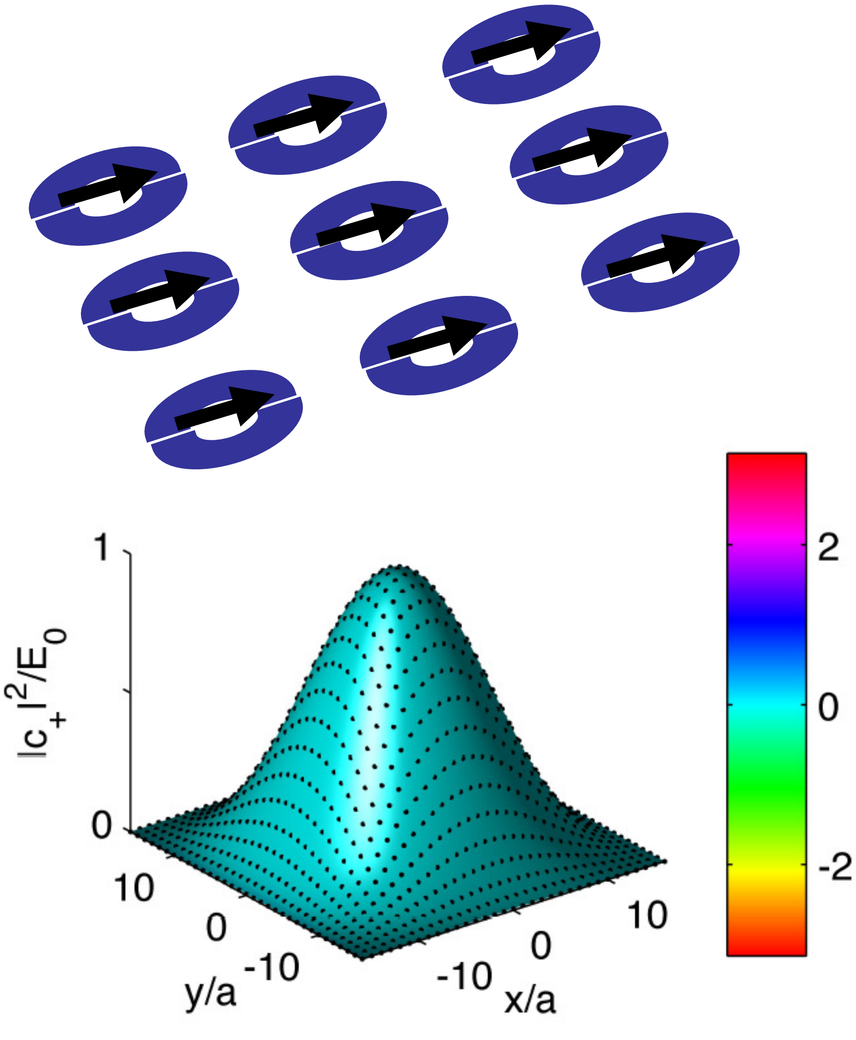
Strongly subradiant modes $10^{-5}\Gamma$

Eigenmodes with uniform phase profiles

All magnetic dipoles oscillating in phase

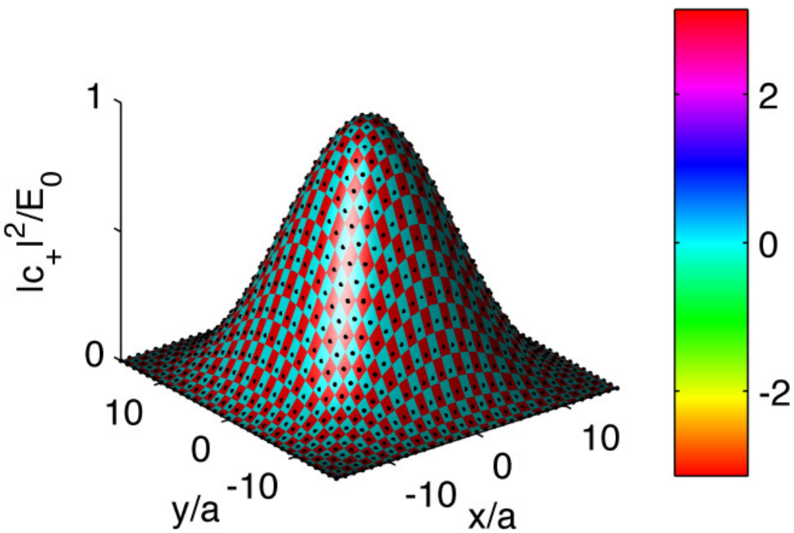


All electric dipoles oscillating in phase

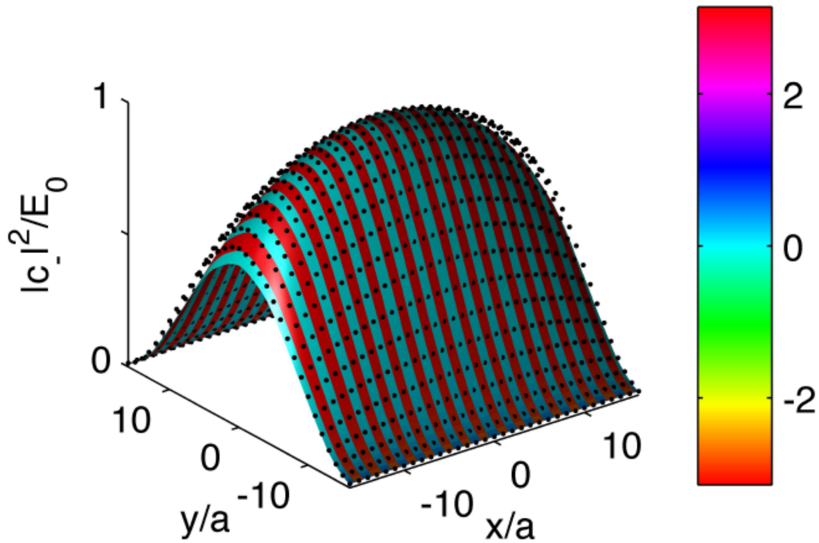


Subradiance and superradiance

Most subradiant $10^{-5}\Gamma$
checkerboard pattern of electric dipoles

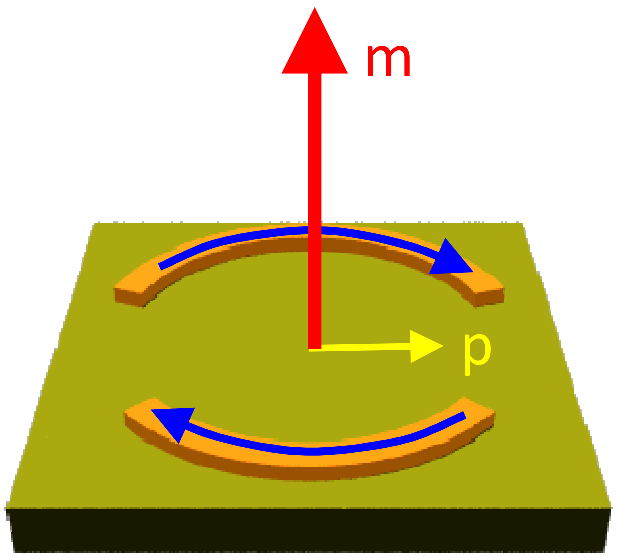


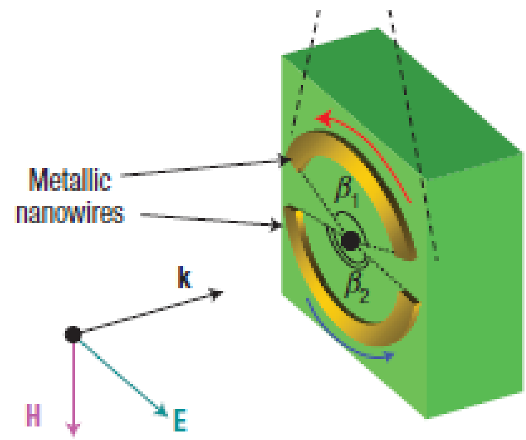
Most superradiant 11 Γ
antisymmetric pattern of magnetic dipoles



Phase-matched with field
propagating in the x-direction

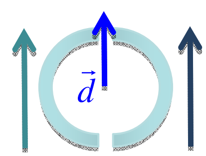
Asymmetric split-ring resonators





Symmetric oscillation

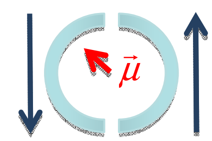
$$c(t) = \frac{1}{\sqrt{2}}[b_r(t) + b_l(t)]$$



electric

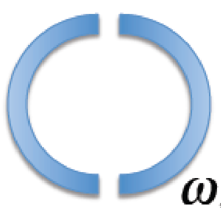
Antisymmetric oscillation

$$d(t) = \frac{1}{\sqrt{2}}[b_r(t) - b_l(t)]$$



magnetic

$$\omega_l = \omega_0 - \delta\omega$$



$$\omega_r = \omega_0 + \delta\omega$$

l - left half ring

r – right half ring

Spectral Collapse in Ensembles of Metamolecules

V. A. Fedotov,[1,*] N. Papasimakis,[1] E. Plum,[1] A. Bitzer,[2,3] M. Walther,[2] P. Kuo,[4] D. P. Tsai,[5] and N. I. Zheludev[1,†]

[1]*Optoelectronics Research Centre, University of Southampton, SO17 1BJ, United Kingdom*
[2]*Department of Molecular and Optical Physics, University of Freiburg, D-79104, Germany*
[3]*Institute of Applied Physics, University of Bern, Sidlerstr. 5, CH-3012 Bern, Switzerland*
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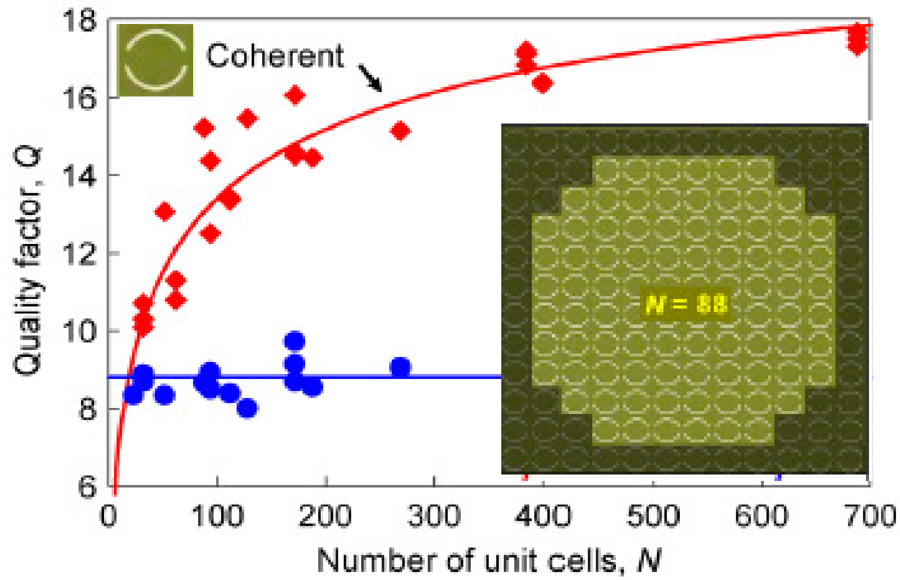
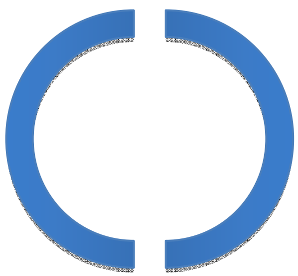
(Received 5 August 2009; revised manuscript received 20 April 2010; published 1 June 2010)

We report on the first direct experimental demonstration of a collective phenomenon in metamaterials: spectral line collapse with an increasing number of unit cell resonators (metamolecules). This effect, which is crucial for achieving a lasing spaser, a coherent source of optical radiation fuelled by coherent plasmonic oscillations in metamaterials, is linked to the suppression of radiation losses in periodic arrays. We experimentally demonstrate spectral line collapse at microwave, terahertz and optical frequencies.

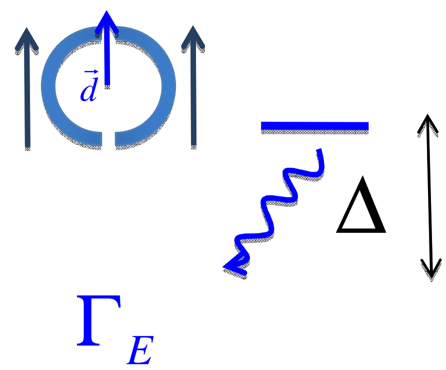
V. Fedotov *et al*, PRL **104**, 223901 (2010).

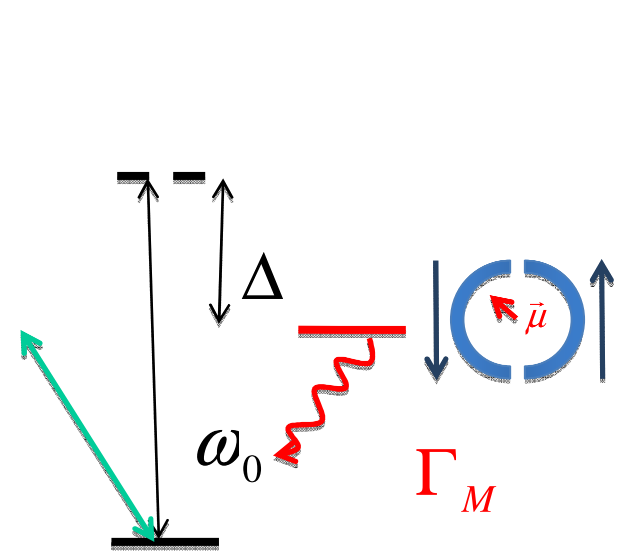


Symmetric case



$\omega_l = \omega_0$ $\omega_r = \omega_0$

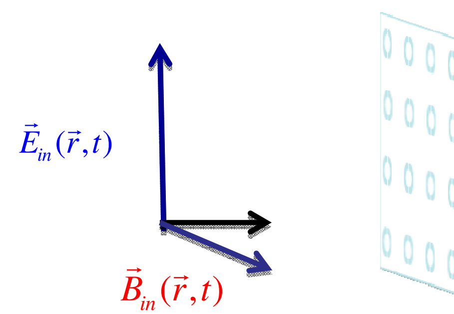




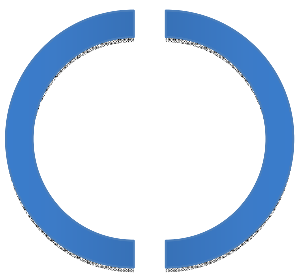
$$\frac{d}{dt}\begin{pmatrix} c(t) \\ d(t) \end{pmatrix} = \begin{pmatrix} -i\Delta - \Gamma_c & 0 \\ 0 & i\Delta - \Gamma_d \end{pmatrix} + \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

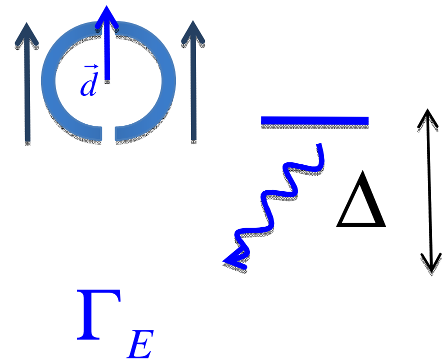
eigenstates

typically only electric dipole mode driven

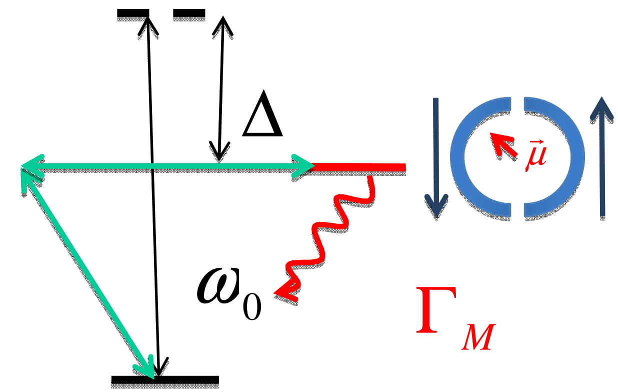


Asymmetric case





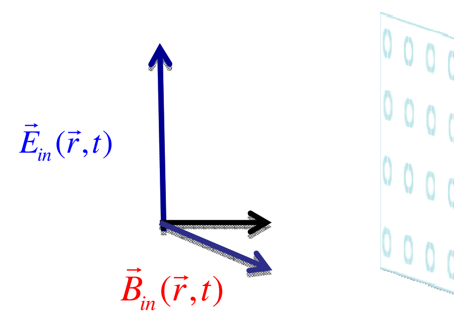
$$\omega_l = \omega_0 - \delta\omega \qquad \omega_r = \omega_0 + \delta\omega$$



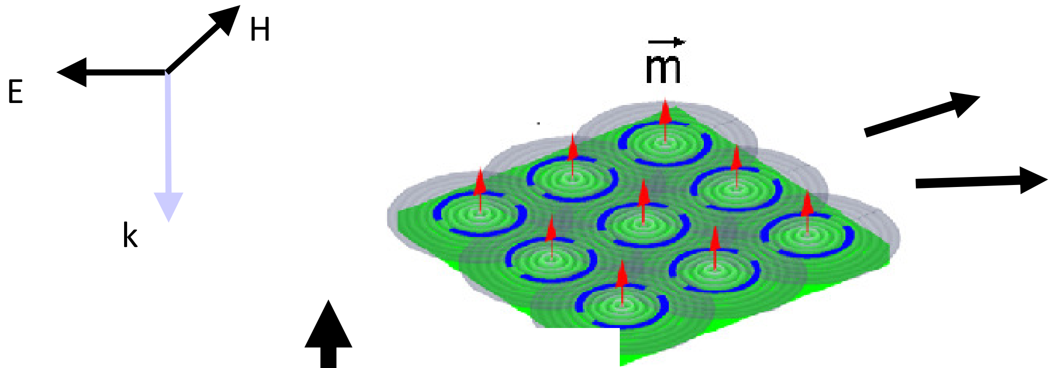
$$\frac{d}{dt}\begin{pmatrix} c(t) \\ d(t) \end{pmatrix} = \begin{pmatrix} -i\Delta - \Gamma_c & -i\delta\omega \\ -i\delta\omega & i\Delta - \Gamma_d \end{pmatrix} + \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$$

Coupling between states by asymmetry

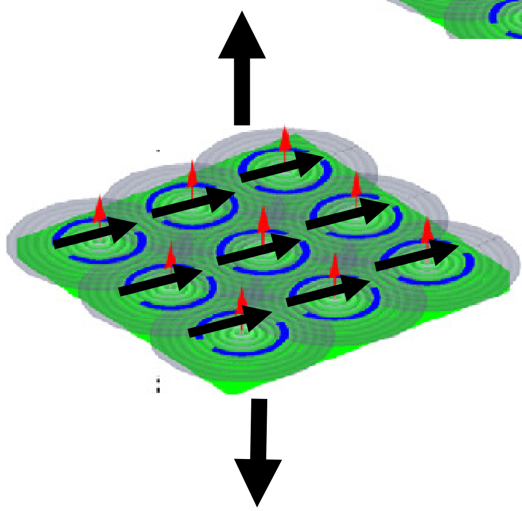
electric dipole mode drives the magnetic dipole mode



Collective response by phase-matched uniform modes



weak radiation
on the plane



electric dipoles radiate strongly
out of the plane

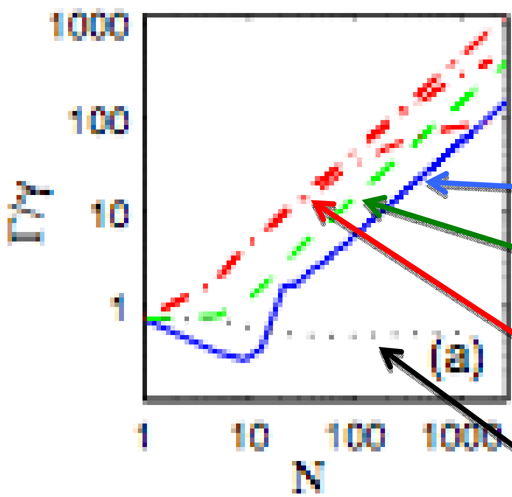
⇒ change in transmission

Uniform collective electric dipole mode phase-matched
with incident plane-wave

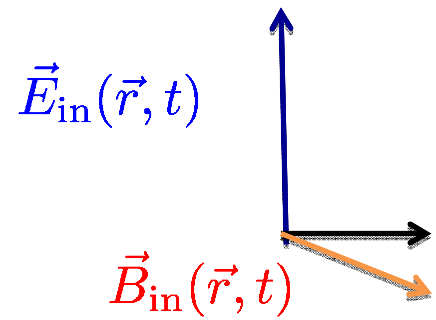
Split-ring asymmetry drives weakly radiating uniform collective
magnetic dipole mode ⇒ over 98% of excitation can be driven to the magnetic mode

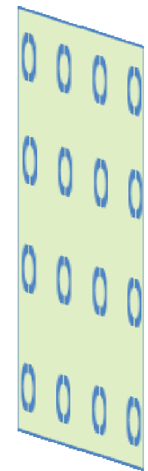
Lifetime of the phase-matched magnetic mode and system size

Inverse decay rate of the phase matched magnetic mode



Number of split rings in the lattice





Lattice spacing

$\frac{\lambda}{4}$

$\frac{3}{8}\lambda$

$\frac{\lambda}{2}$

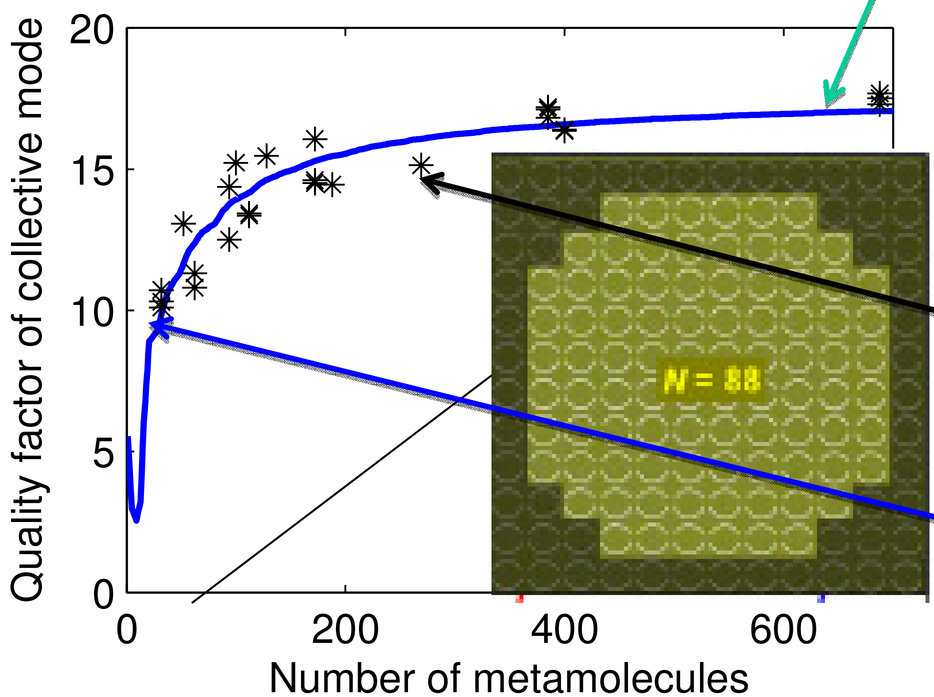
$\frac{3}{2}\lambda$

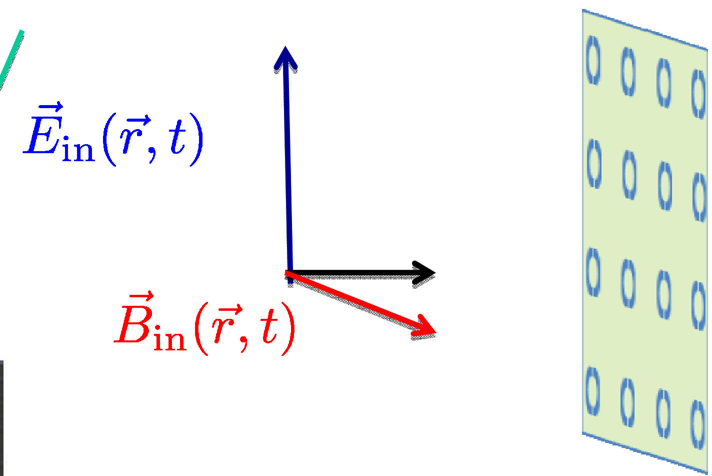
Linewidth narrowing

Q-factor: Theory vs experiment

Saturation is due to ohmic losses

Transmission Resonance Quality Factor





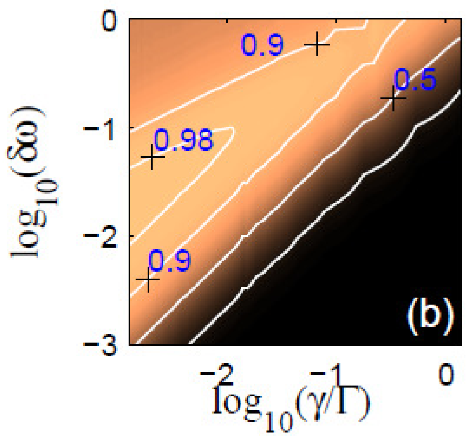
Experimentally measured quality factor: V. A. Fedotov *et al,* Phys. Rev. Lett. **104**, 223901 (2010)

Theoretical quality factor of collective phase matched mode calculated from model

Excitation probability of magnetic mode

Phase-coherent magnetic collective mode

Phase-matched with incident plane-wave

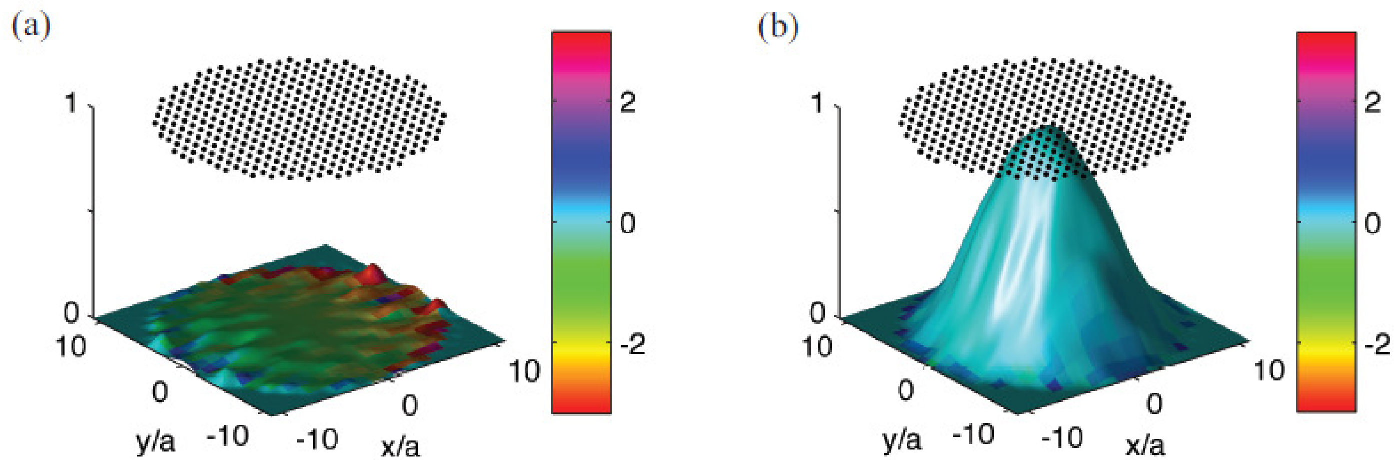


Split-ring asymmetry drives weakly radiating collective magnetic dipole mode

$\gamma \ll \Gamma \quad \delta\omega \gtrsim \gamma$

98% overlap with target mode

Collective mode



Exploiting collective interactions

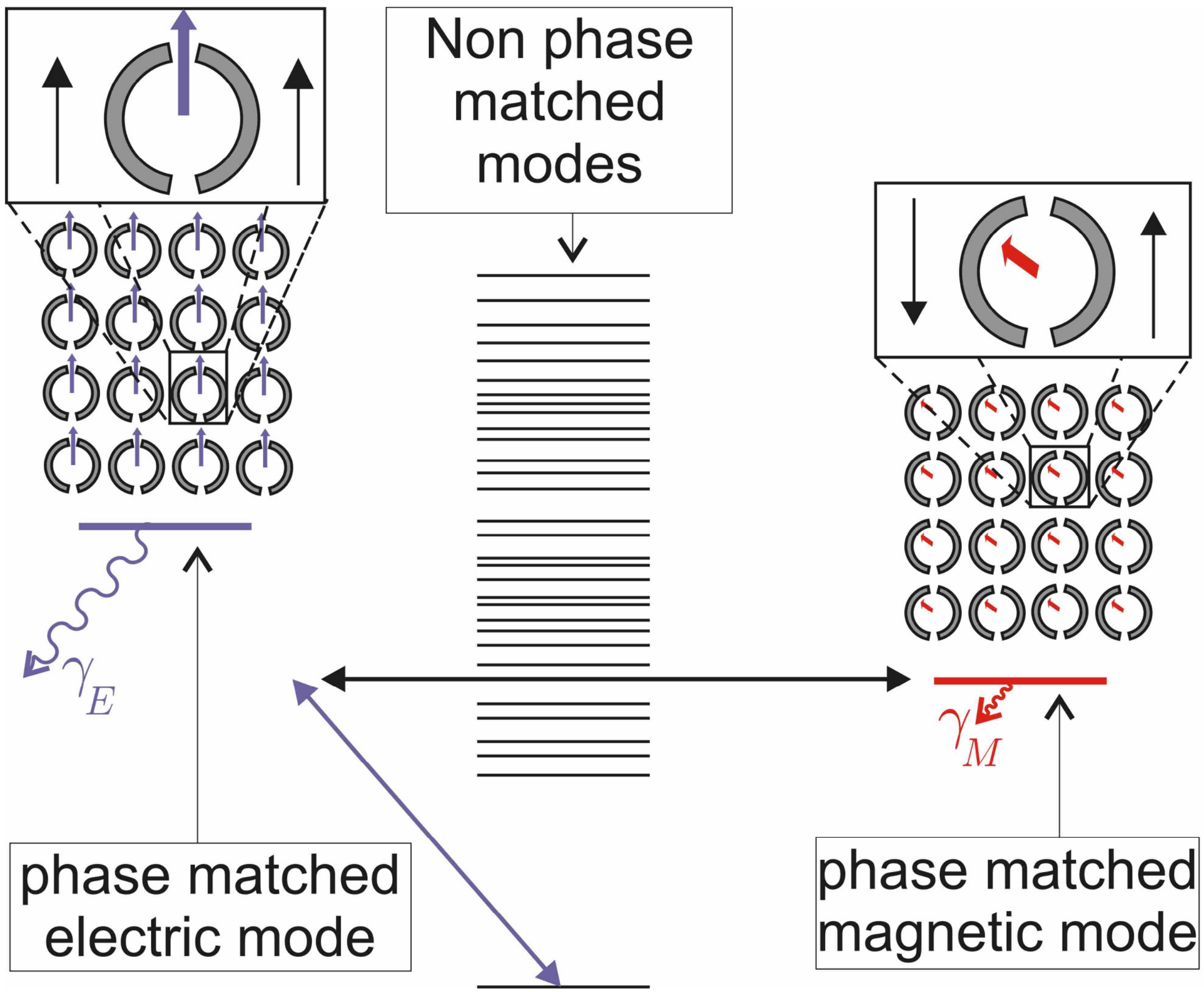
Coupling between the collective modes EIT-like
but not a single-resonator effect

Excitations in the presence of disorder in the positions of the resonators exhibit complex structure

Excitation of a superposition of collective modes that exhibit subwavelength structures – coherent control

Highly localized excitation can transfer energy to a collective excitation and subsequently to a low-divergence free-space light beam

Collective phase-matched modes: Cooperative Asymmetry Induced Transparency

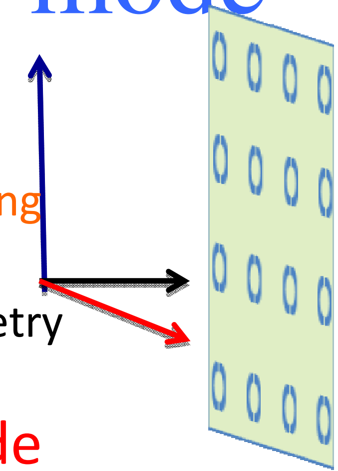


When the collective magnetic mode decays slowly, the magnetic mode can be excited at the expense of the electric mode

$$\dot{c}_E = \left[i\omega_E - \frac{\gamma_E}{2}\right] c_E - i\delta\omega\, c_M + F(t)$$

$$\dot{c}_M = \left[-i\omega_M - \frac{\gamma_M}{2}\right] c_M - i\delta\omega\, c_E$$

Incident field driving

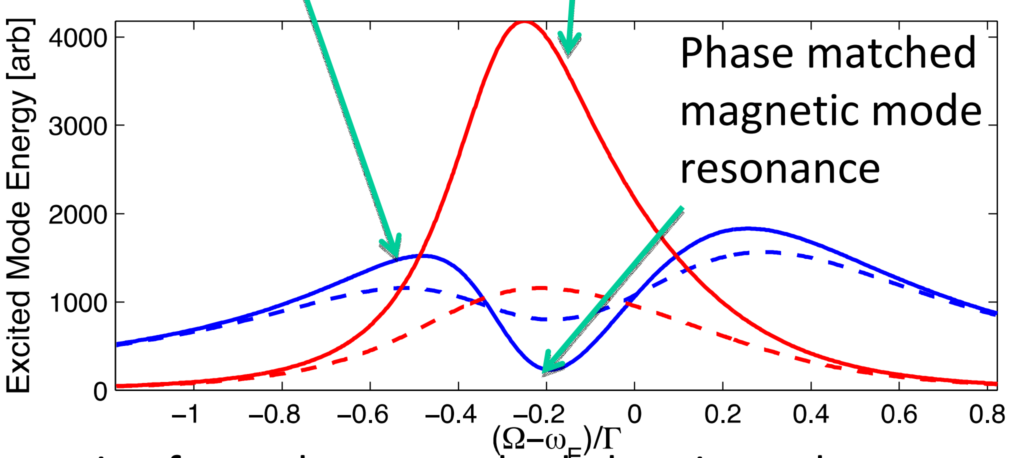


Collective mode decay rates

Mode coupling due to asymmetry

Energy in phase matched electric mode

Energy in phase matched magnetic mode



Detuning from phase matched electric mode resonance

$\gamma_M = 0.146\,\Gamma$

$\gamma_E = 1.23\,\Gamma$ solid

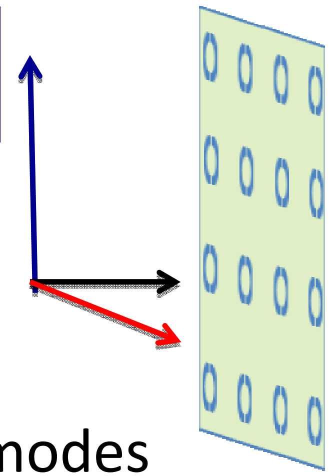
$\omega_M = \omega_E - 0.20\,\Gamma$

$\gamma_M = 0.5\,\Gamma$ dashed

¡ = Single meta-atom decay rate

Phase-matched electric mode is unexcited in the transparency window

Results from simplified two mode model -------------------- dashed lines
Results of numerical simulations of the full meta-material – solid lines

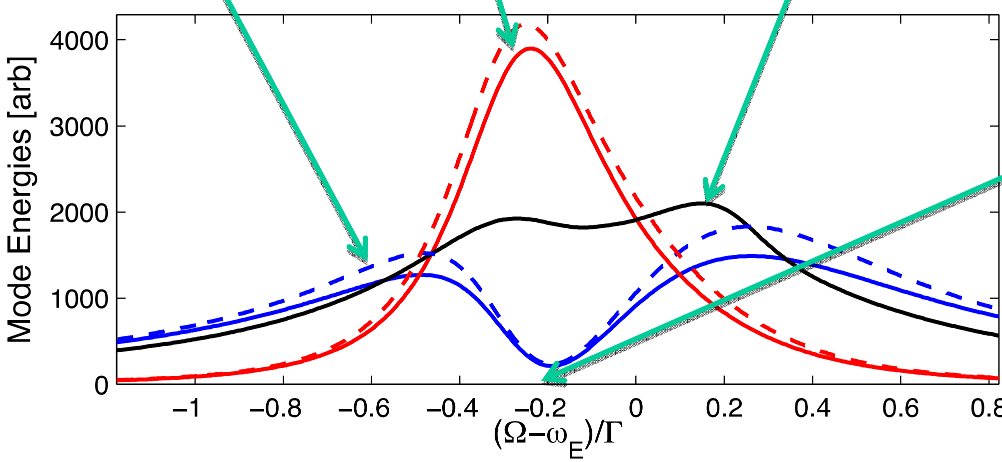


Energy in phase matched electric mode

Energy in phase matched magnetic mode

Energy in all other collective modes

Phase matched magnetic mode resonance



$$\gamma_M = 0.146\,\Gamma$$

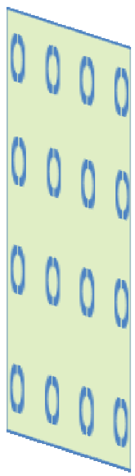
$$\gamma_E = 1.23\,\Gamma$$

$$\omega_M = \omega_E - 0.20\,\Gamma$$

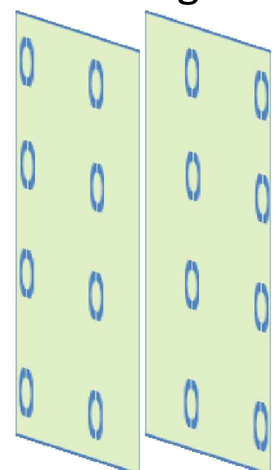
Γ = Single meta-atom decay rate

Changing geometry shifts the transmission resonance

Square lattice



Every other column vertically shifted by 0.1 wavelengths

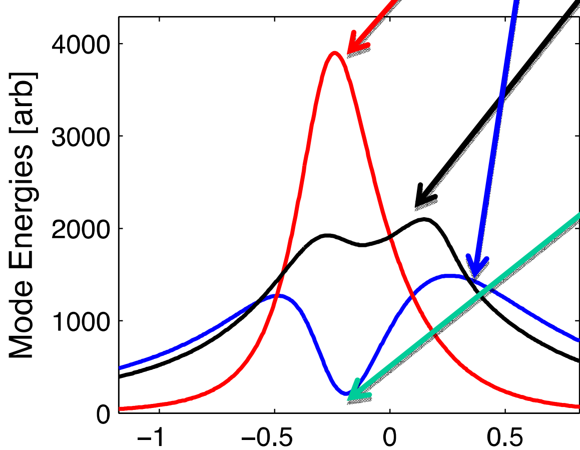


Energy in phase matched electric mode

Energy in phase matched magnetic mode

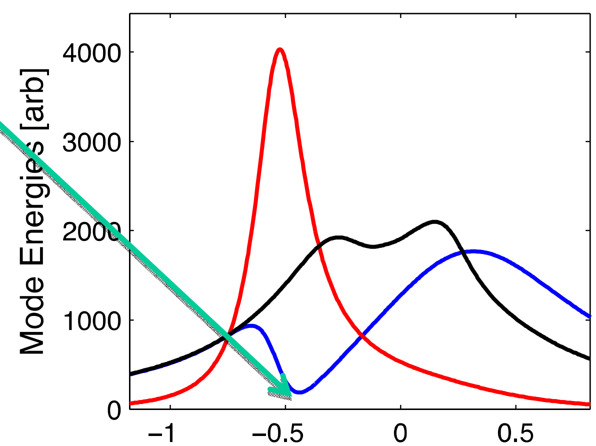
Energy in all other collective modes

K. Aydin *et al,* Optics Express **12**, 5896 (2004)



Phase matched magnetic mode resonance

Nanoelectromechanical switchable photonic metamaterials

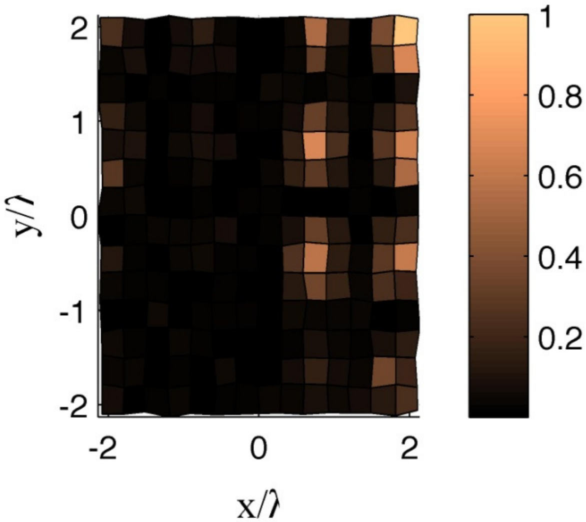


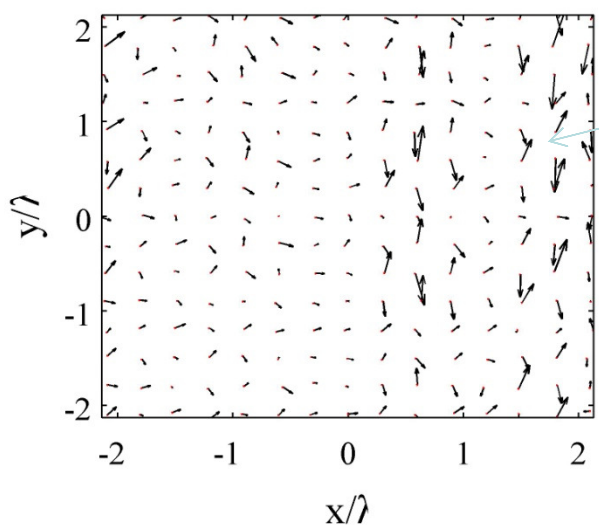
Effects of disorder

Displacing resonators by a Gaussian stochastic noise
Increasing disorder leads to potential localization of collective modes.

$$\frac{\text{standard deviation}}{\text{lattice spacing}} = 0.05$$

Energy of excited magnetic modes

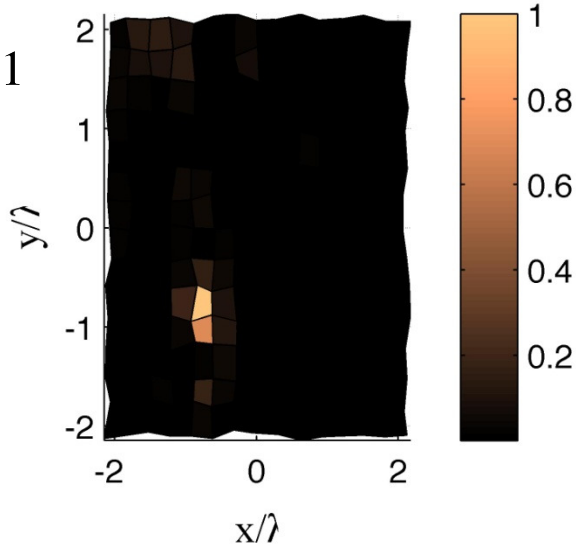


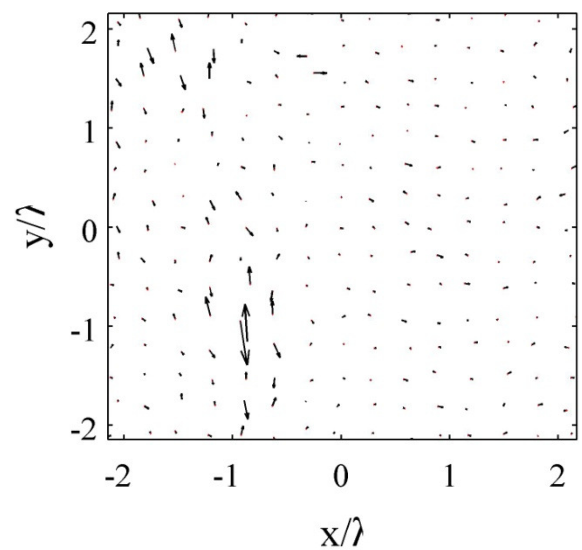


Magnitude and phase of excited magnetic modes

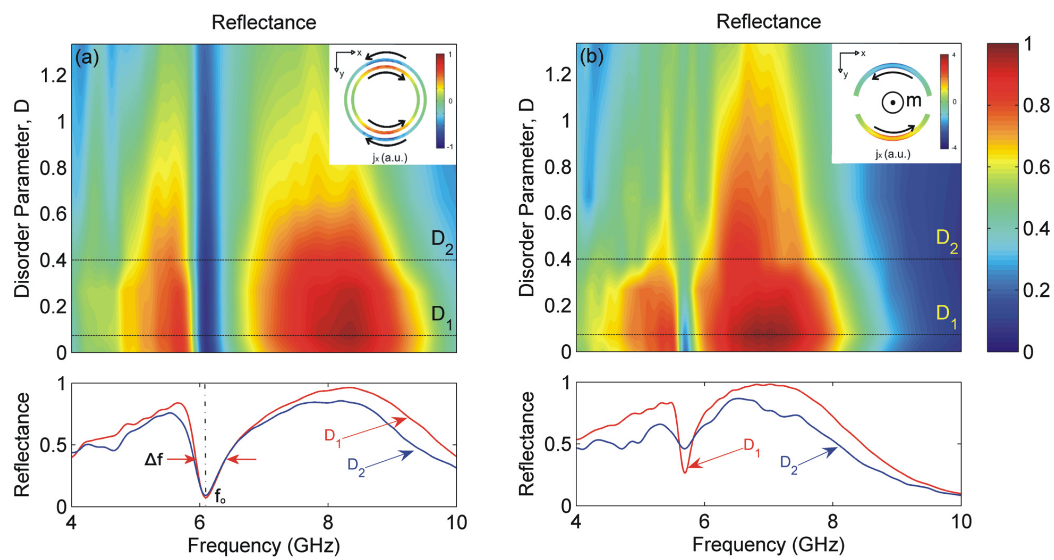
Disorder

$$\frac{\text{standard deviation}}{\text{lattice spacing}} = 0.1$$

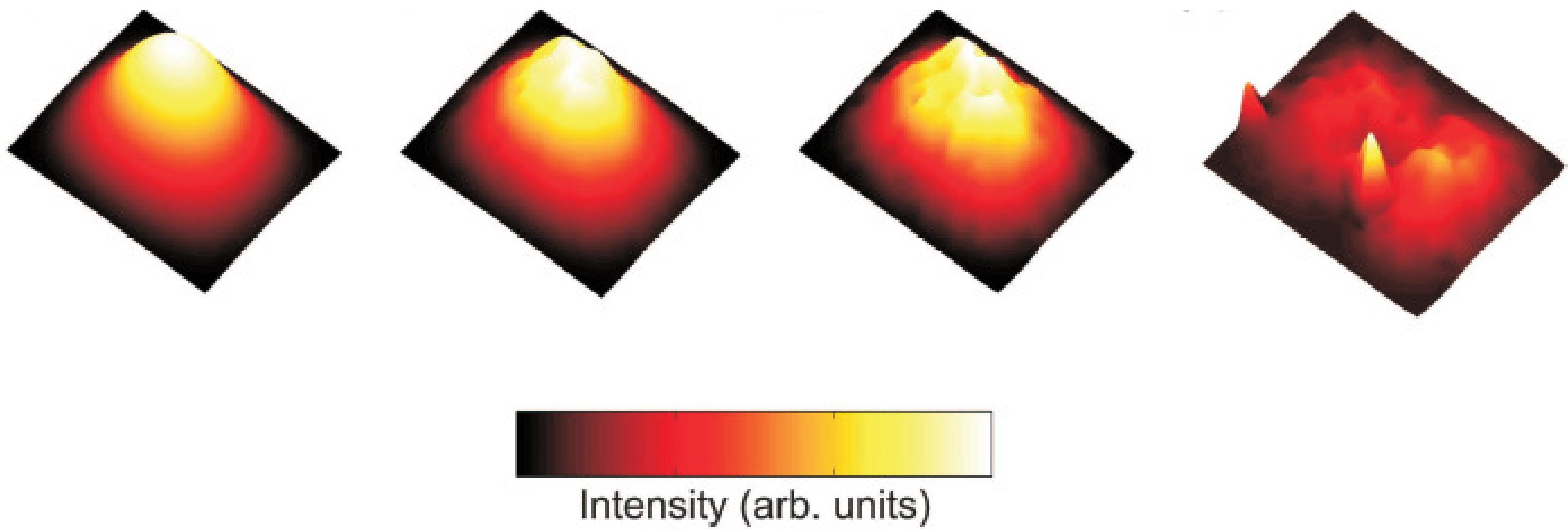




Comparison to disorder experiments by Papasimakis et al



Disorder



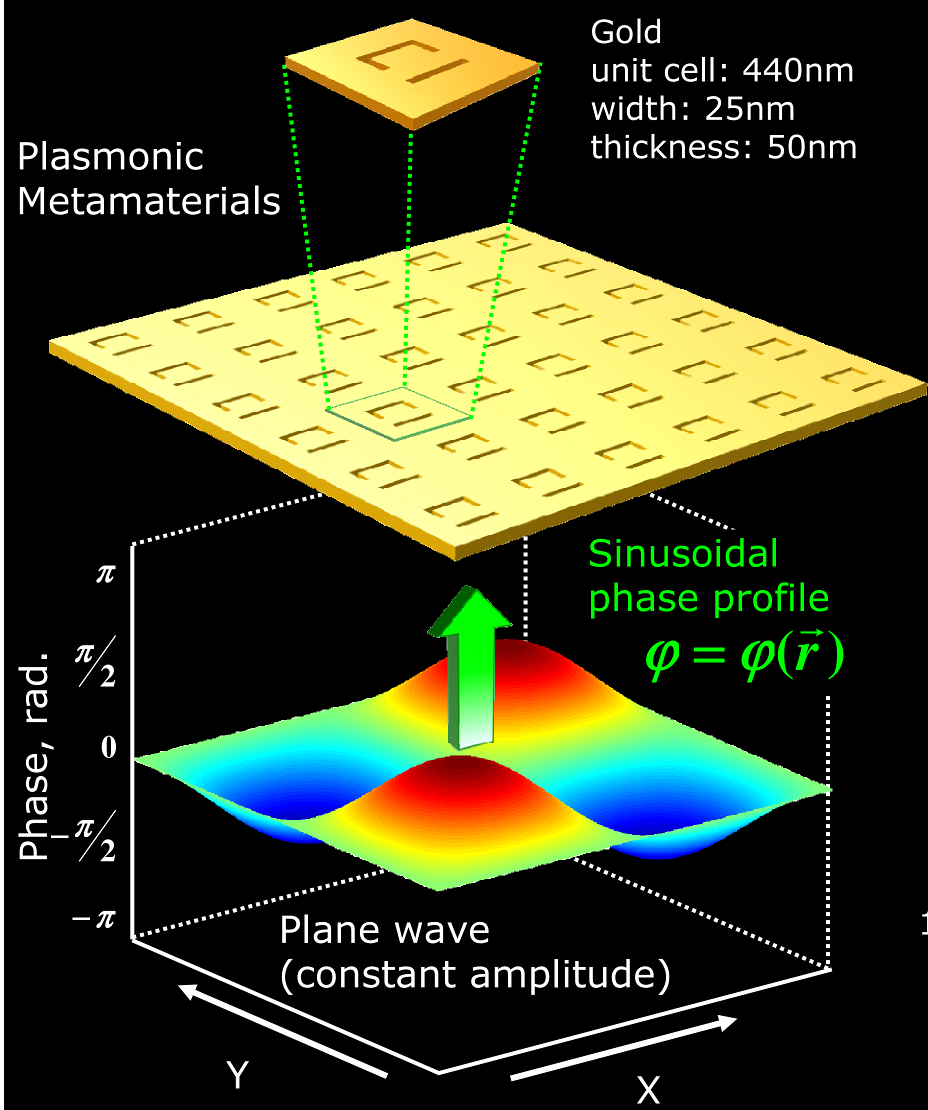
Subwavelength control

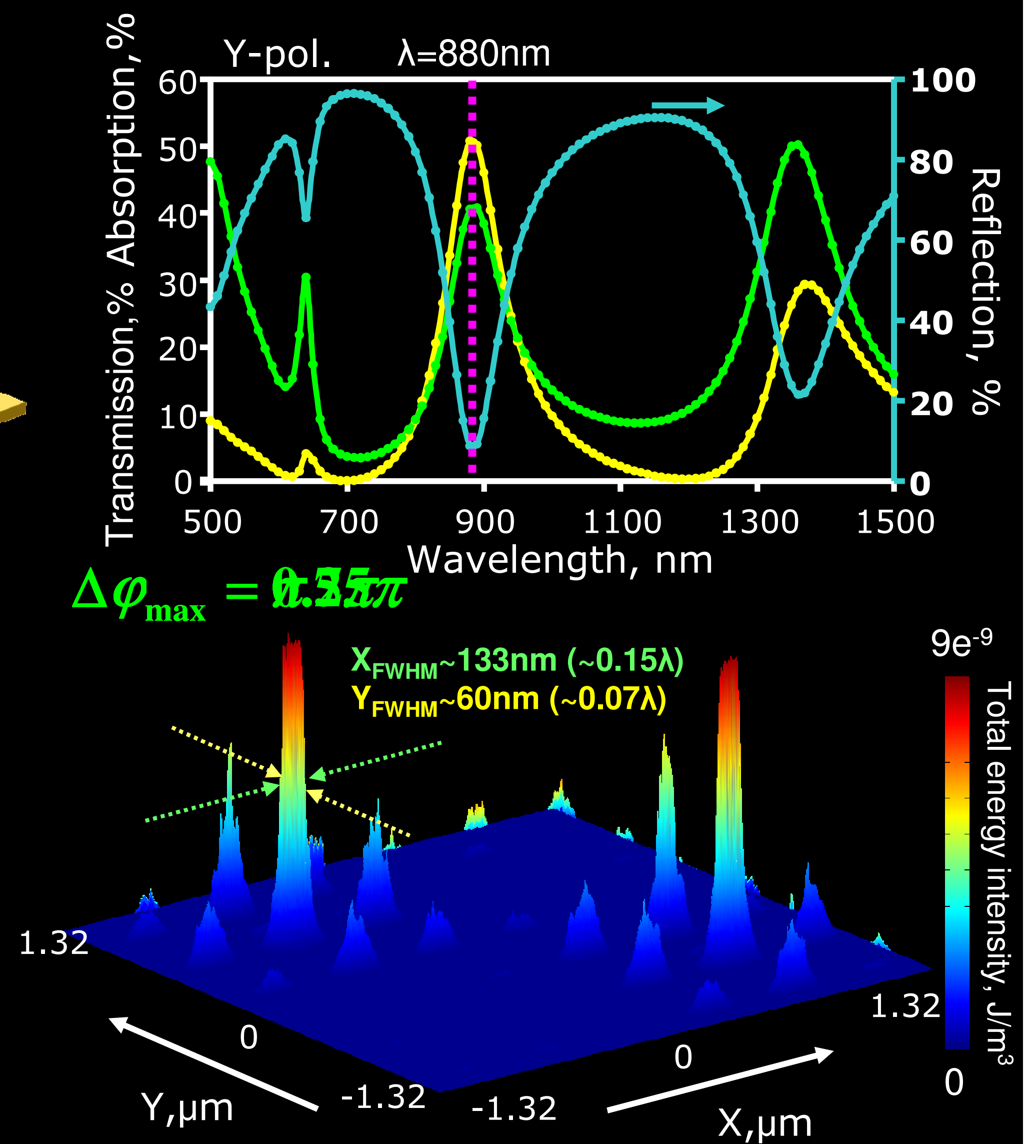
Precise control and manipulation of optical fields on nanoscale
one of the most important and challenging problems in nanophotonics

Tailoring phase modulation of ultrashort optical pulses (Stockman etc.)

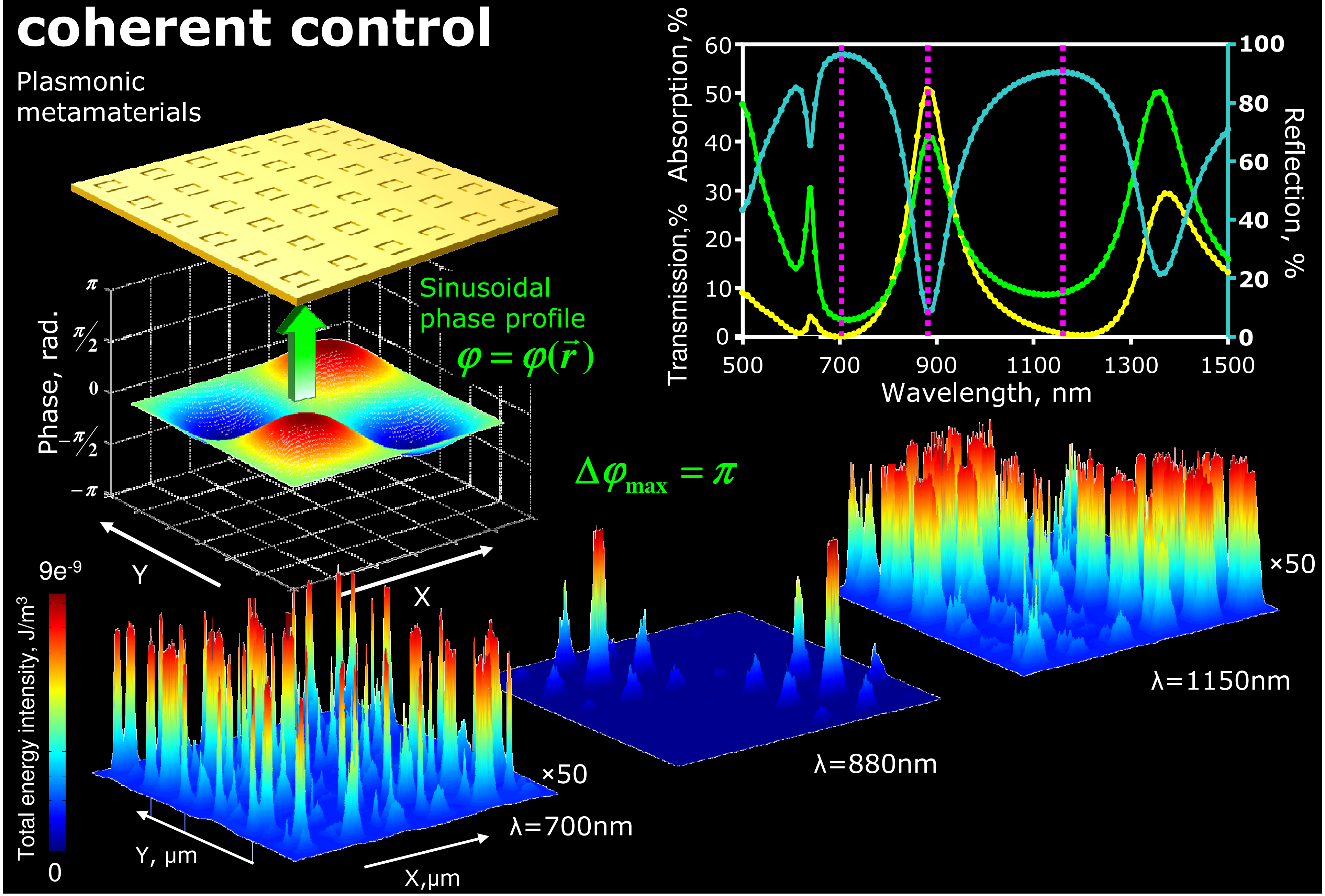
Collective interactions and cw fields

Nanoscale field localization by exciting linear combinations of eigenmodes





Nanoscale field localization with coherent control



see also A. Sentenac and P.C. Chaumet, Phys. Rev. Lett. **101**, 013901 (2008).

PRL **104**, 203901 (2010) PHYSICAL REVIEW LETTERS week ending 21 MAY 2010

Resonant Metalenses for Breaking the Diffraction Barrier

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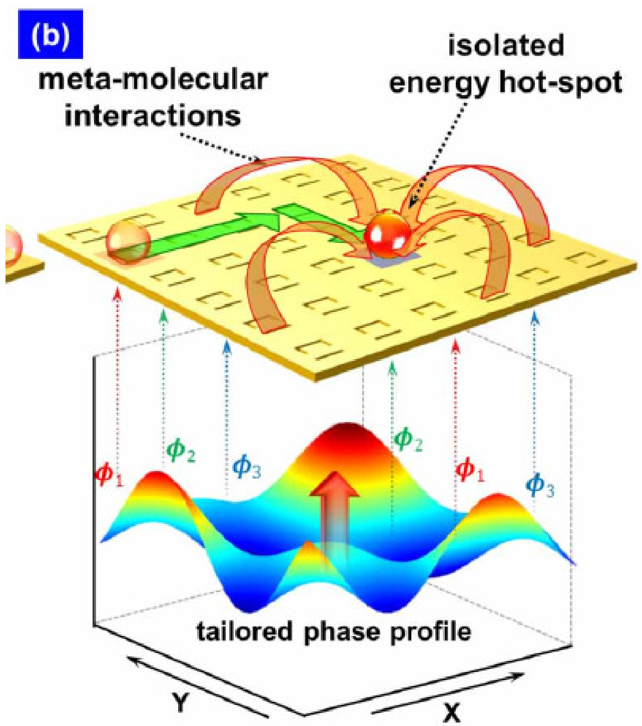
(Received 8 January 2010; revised manuscript received 14 April 2010; published 18 May 2010)

Transfer of sub-diffraction evanescent fields to propagating fields by a resonator array

Experimental observation

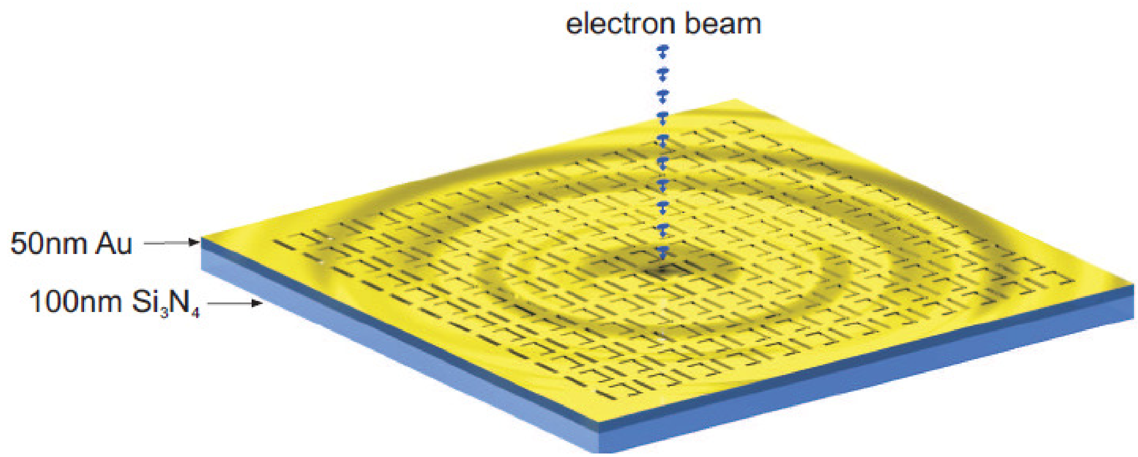
T. S. Kao, E. T. F. Rogers, J. Y. Ou, and N. I. Zheludev*



Localized excitation by electron beam

Highly localized excitation can transfer energy to a collective excitation and subsequently to a low-divergence free-space light beam

Excite only 1-4 unit-cell resonators in an ASR metamaterial array



The phase-uniform collective magnetic mode can assume dominant role of total excitation energy

High degree of spatial coherence

Highly directed emission pattern and narrow resonance linewidth

Role of phase-coherent magnetic mode

Excite resonantly collective magnetic mode

(a) Excite only a single resonator:

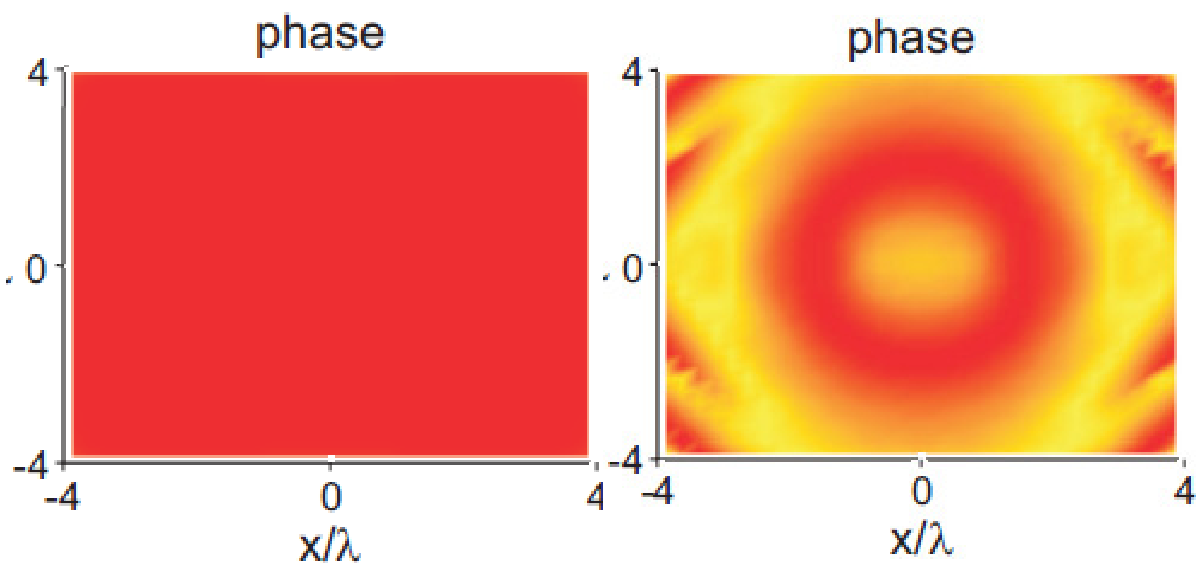
70% of steady-state response in collective magnetic mode

(b) Excite four resonators:

85% of steady-state response in collective magnetic mode

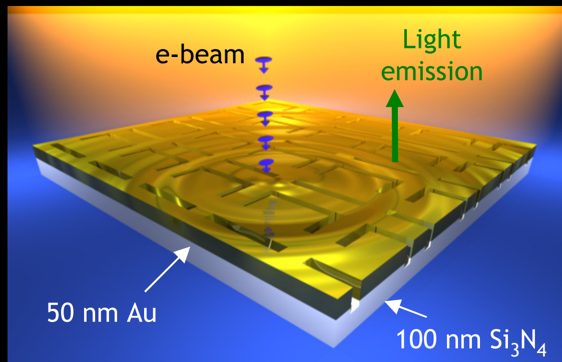
phase variance 2.2 degrees

If detuned 8 linewidths off-resonant, 45% and 24 degree phase variance

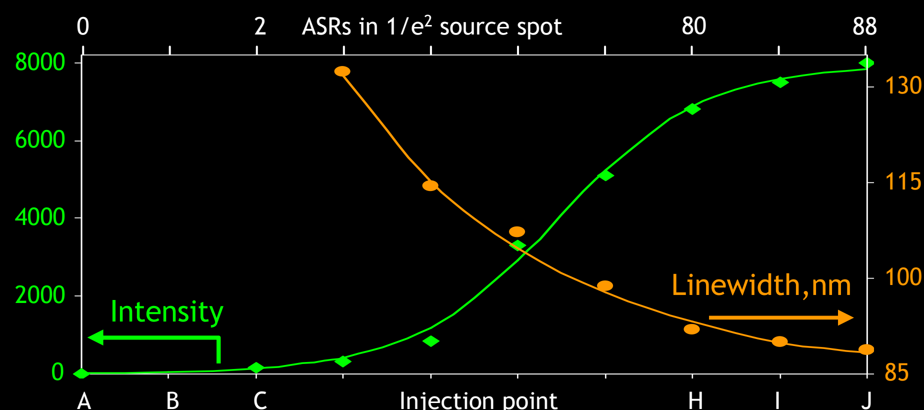


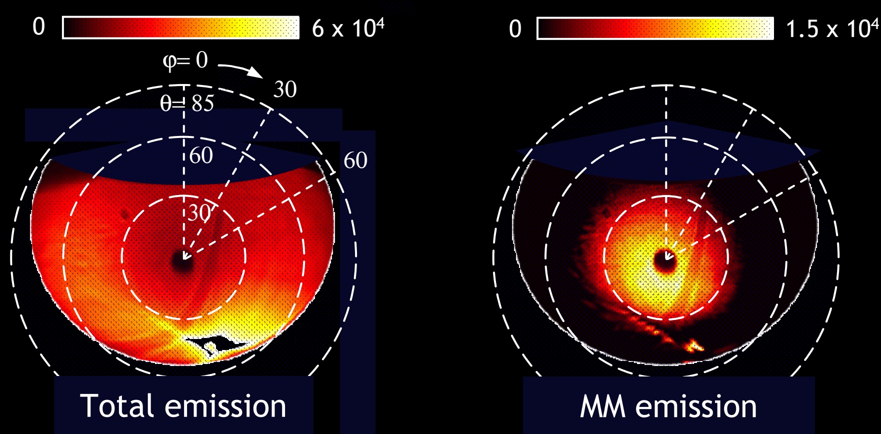
G Adamo et al.

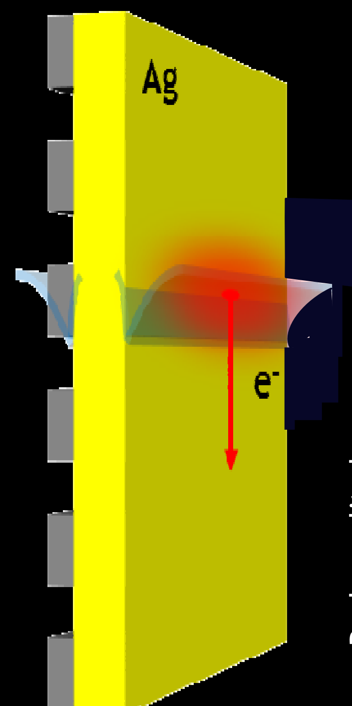
Electron-Beam-Driven Metamaterial Light Sources

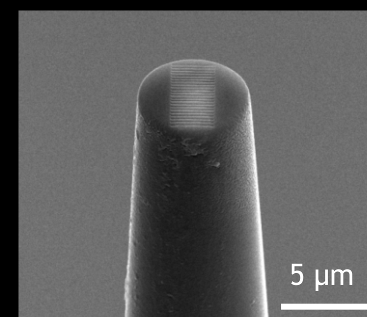


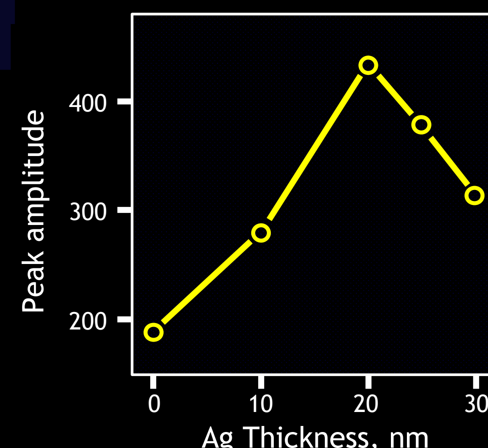
- Collective (coherent) light emission from plasmonic metamaterials
- Smith-Purcell emission enhancement through Surface Plasmons





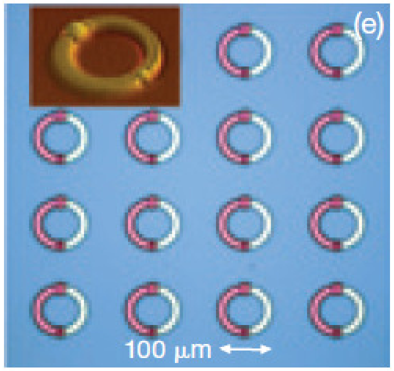






Nonlinear and quantum systems

SQUID rings



Suppress ohmic losses

Introduce nonlinearity in the system

Current through SQUID ring

$$I_j = \frac{\Phi_{\text{ext},j} - \Phi_j}{L},$$

total external magnetic flux ($\Phi_{\text{ext},j}$)

flux trapped in squid loop (Φ_j)

$$I_j = \frac{dQ_j}{dt} + \frac{Q_j}{CR} + I_c \sin(2\pi f_j)$$

$$f_j = \Phi_j / \Phi_0$$

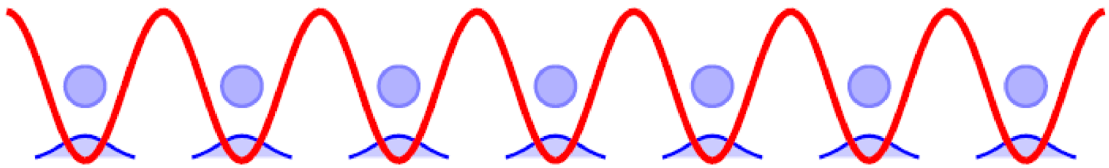
$$\Phi_0 = h/2e$$

$$\frac{Q_j}{C} = \frac{d\Phi_j}{dt}$$

Nonlinear relationship for the flux

$$\frac{d^2 f_j}{d\tau^2} + \gamma \frac{df_j}{d\tau} + \beta \sin(2\pi f_j) = f_{\text{ext},j} - f_j$$

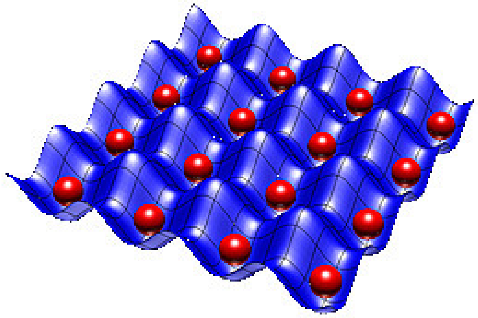
Optical lattice



- Atoms trapped in periodic optical potential
- AC Stark effect of off-resonant lasers

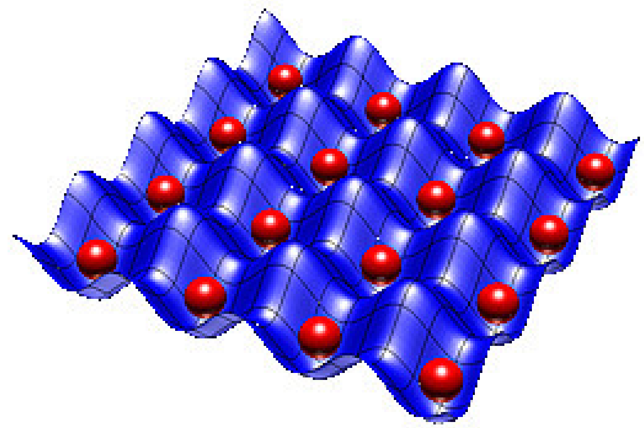
Optical lattices as a
platform to exploit
collective interactions

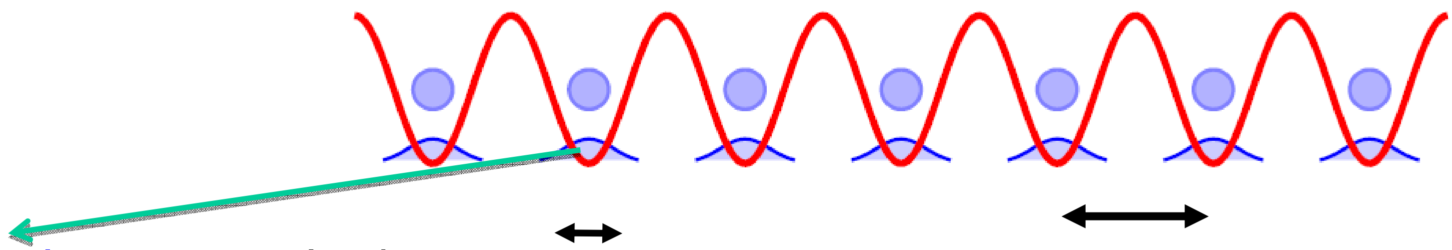
Atomic dipole-dipole interactions
mediated by scattered EM fields



"Quantum metamaterial"

- Prepare a 2D optical lattice
- one atom per site
- Atoms can be reasonably well localized
- Positions of dipoles fluctuate due to zero-point vacuum fluctuations



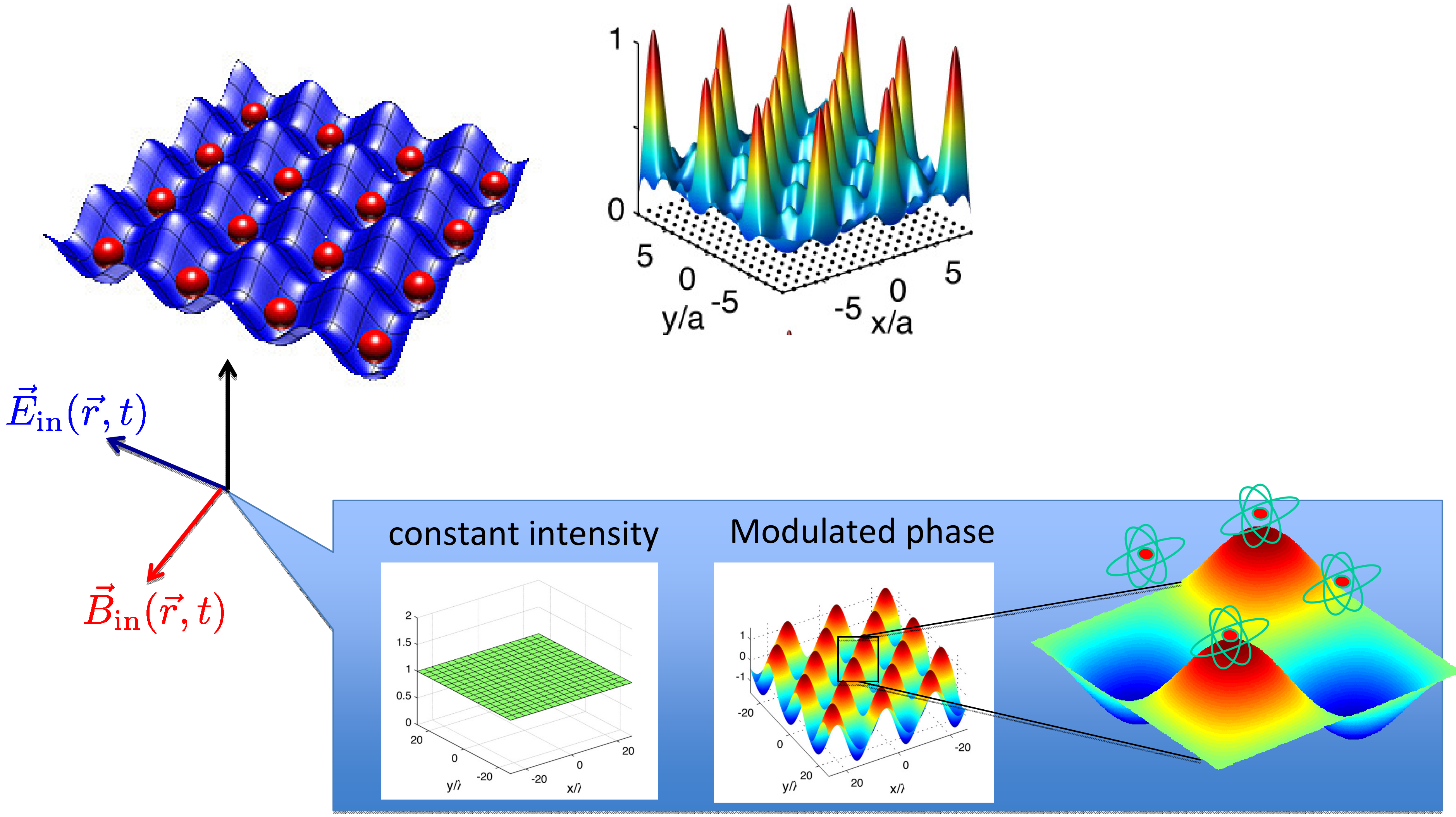


$$|\phi_{\mathbf{R}}(\mathbf{r})|^2 \propto \exp(-r^2/\ell^2)$$

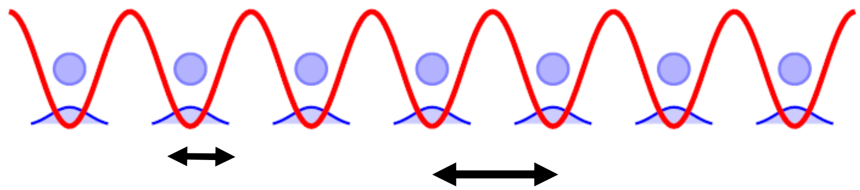
size of Wannier Function ℓ

lattice spacing d

Cooperative localization



Quantum effects and confinement

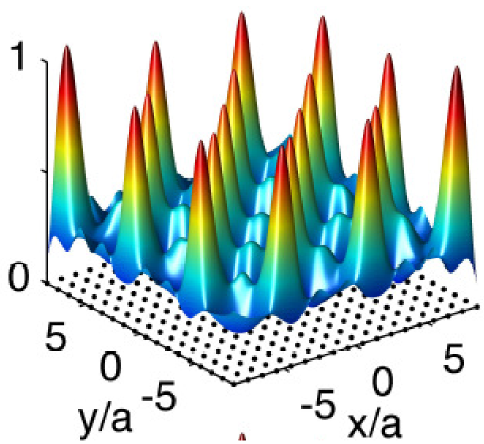


size of Wannier Function ℓ

lattice spacing d

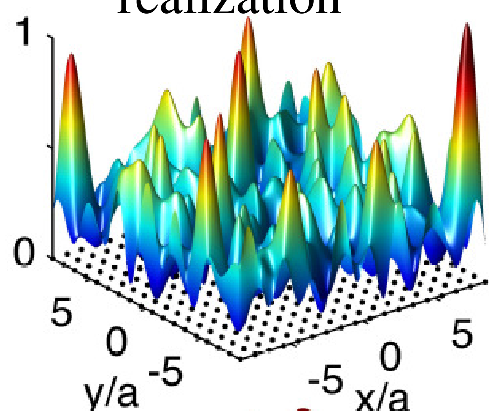
$$|\phi_{\mathbf{R}}(\mathbf{r})|^2 \propto \exp(-r^2/\ell^2)$$

$\ell = 0$



$\ell = 0.12$ d

single stochastic realization



$\ell = 0.12$ d

Ensemble average of many realizations