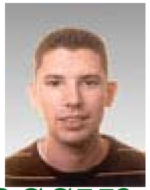
Tomorrow:
Modelling approaches for metamaterials

Towards optical left-handed metamaterials





M. Kafesaki, R. Penciu, Th. Koschny, P. Tassin,


E. N. Economou and C. M. Soukoulis


Foundation for Research & Technology, Hellas (FORTH)
& Univ. of Crete, Greece

Ames Lab & Iowa State University (ISU), USA

Left-handed metamaterials?

Negative electrical permittivity (ε)
Negative magnetic permeability (μ)

Veselago (1968)

⇓

$$n = -\sqrt{\varepsilon\mu}$$

negative

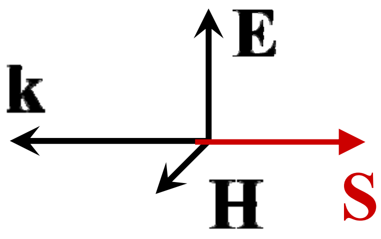
$$e^{ikx} = e^{i(\omega n/c)x}$$

⇓

Novel and unique effects and possibilities

Novel phenomena in left-handed materials (LHMs)

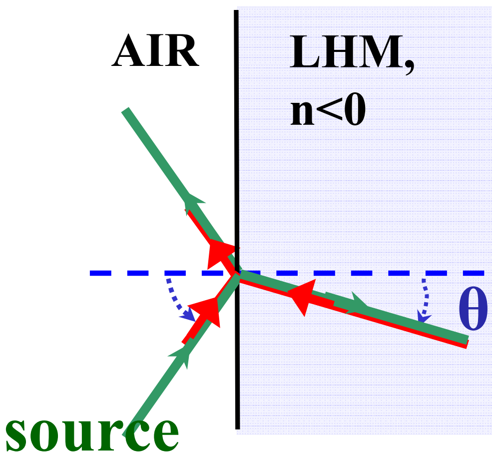
Backwards propagation (opposite phase & energy velocity)



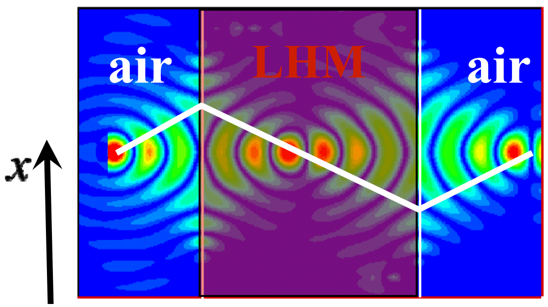
$$\mathbf{k}\times\mathbf{E}=\frac{\omega}{c}\mu\mathbf{H}$$

S=E H

Negative refraction



Flat lenses - "Perfect" lenses (subwavelength resolution) $\Delta x \Delta k_x \geq 1$



- Opposite Doppler effect
- Opposite Cherenkov radiation
- ……

- Interesting physical system
- New possibilities for light manipulation → important potential applications

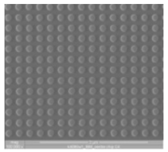
Veselago (1968), Pendry (2000)

Application areas of left-handed materials

New solutions and possibilities in

- **Imaging/microscopy**

- **Lithography**

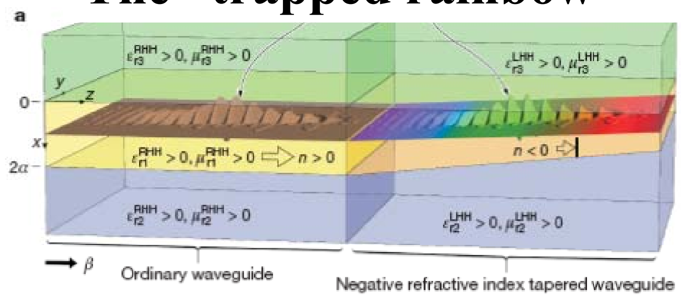


- **Data storage**

Exploiting the subwavelength resolution capabilities of LHMs

- **Communications and information processing (subwavelength guides, optimized/miniaturized antennas & filters, improved transmission lines ...)**

The "trapped rainbow"



Tsakmakidis et. al., Nature 450, 397 (2007)

- **....**

Negative μ and n towards visible

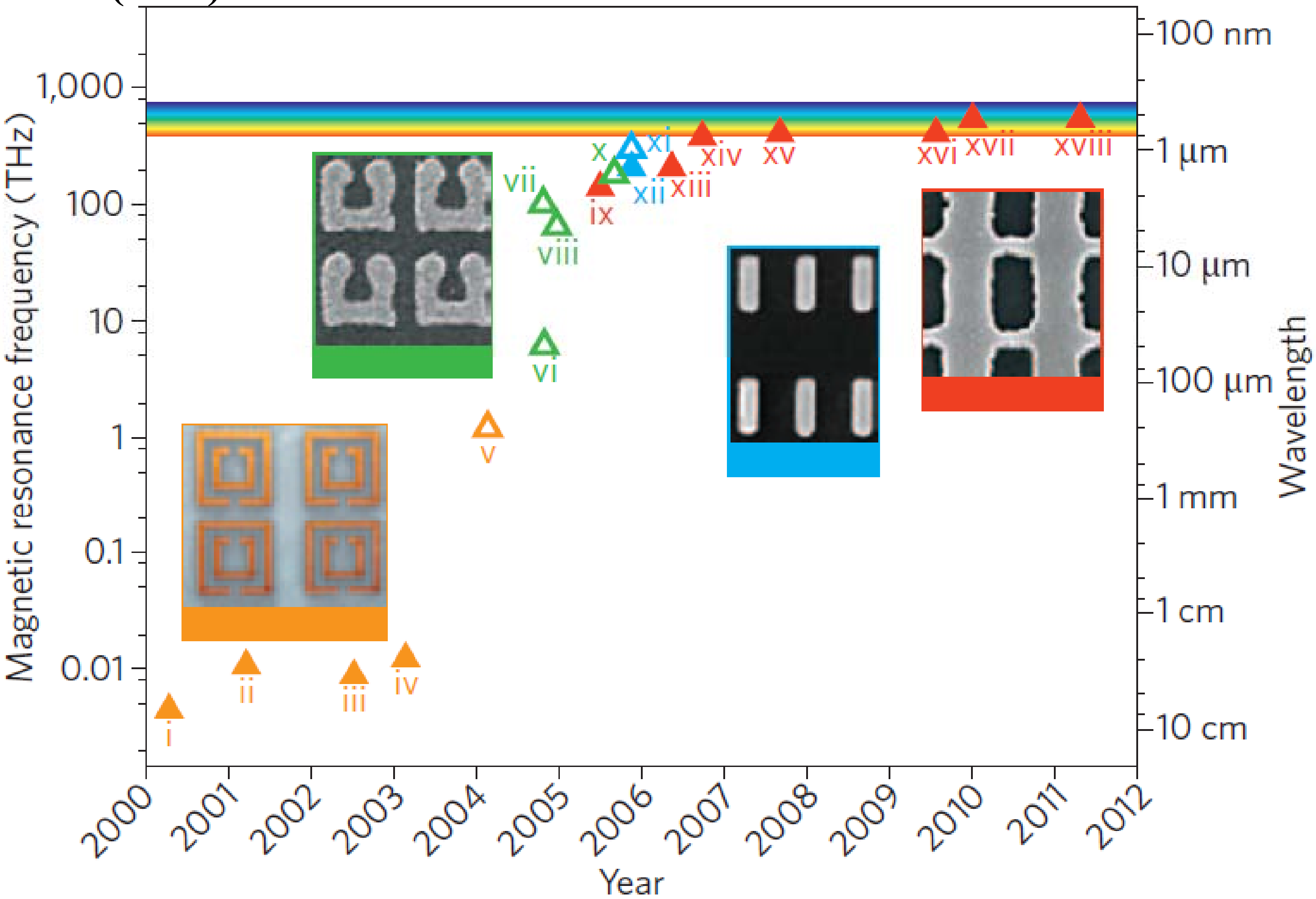
Figure from Soukoulis & Wegener, Nature Photonics **5** (2011)

Fore review, see:

- **Soukoulis et. al., Science 315 (2007)**
- **Shalaev, Nat. Mat. (2007)**
- **Soukoulis & Wegener, Nat. Phot. (2011)**

Leading efforts by

- **Karlsruhe**
- **Purdue**
- **Stuttgart**
- **Berkeley**
- **….**



Hard to achieve optical negative μ

Negative *μ* and *n* towards visible

Fore review, see:

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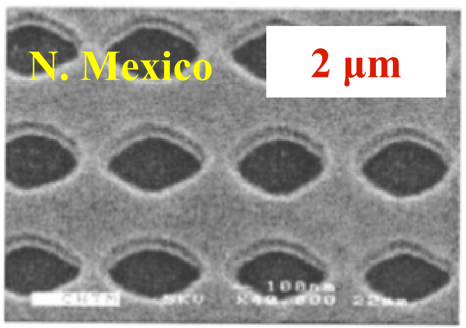
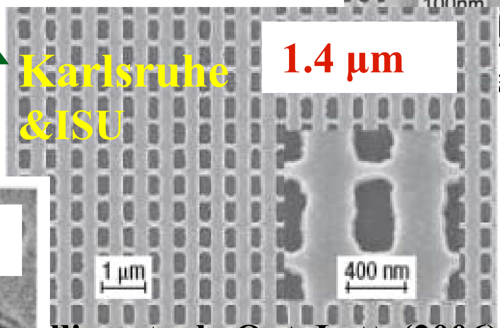
- ….

Low losses

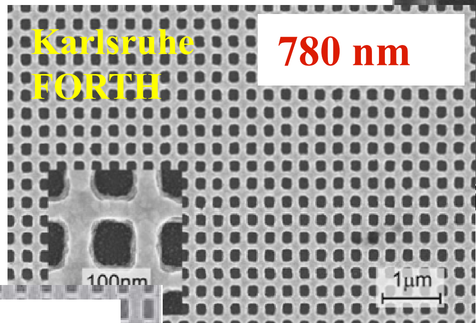
(Re(*n*)/Im(*n*)=3)



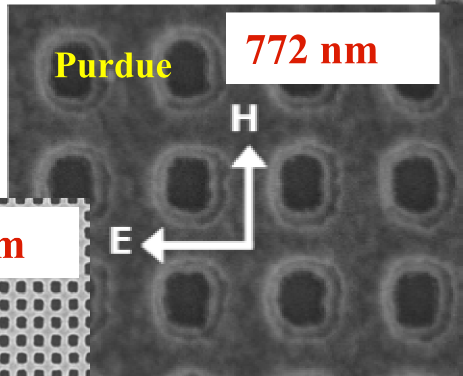
Zhang et. al., PRL (2005)



lling et. al., Opt. Lett. (2006)



et. al., Opt. Lett. (2007)



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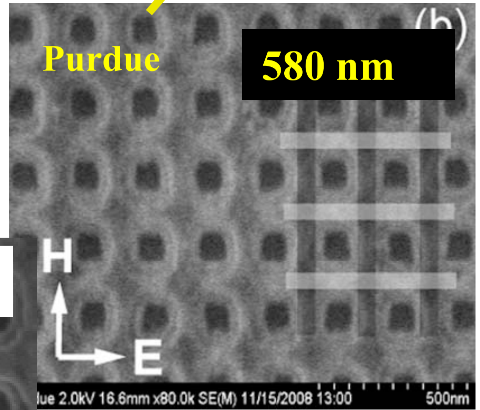


Xiao et. al., Opt. Lett. (2009)

ω

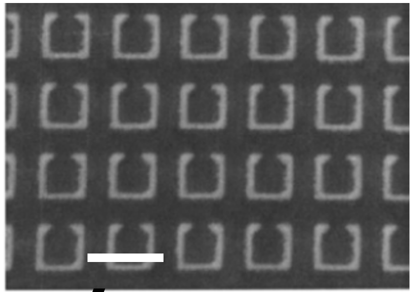
High losses

Single functional layer

Multilayer metamaterials

5-layers SRRs

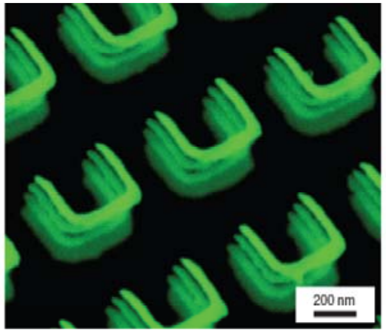
Negative μ at ~6 THz



FORTH, Opt. Lett. 30, 1348 (2005)

4-layers SRRs

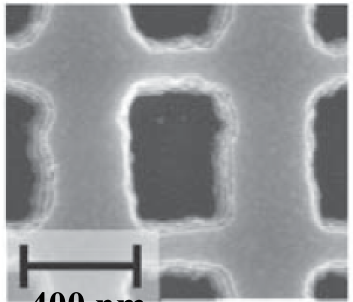
Negative μ at ~120 THz



Stuttgart, Nat. Mat. 7, 31 (2008)

3-layers fishnet

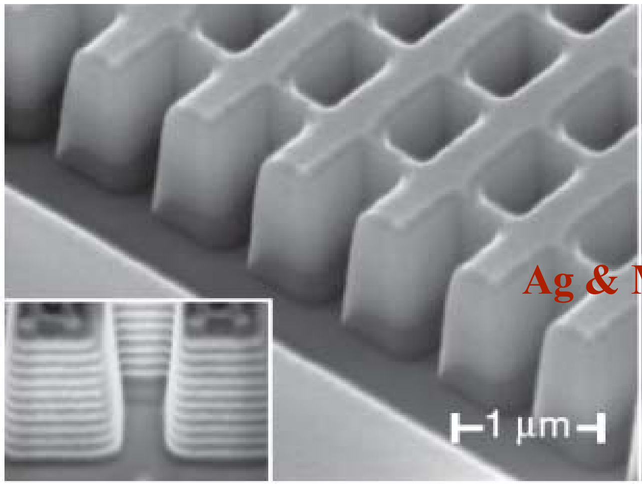
Negative *n* at ~1.5 μm, FOM=2.



Karlsruhe, Opt. Lett. 32, 552 (2007)

10-layers fishnet

Negative index at ~1.7 μm, FOM=3.5

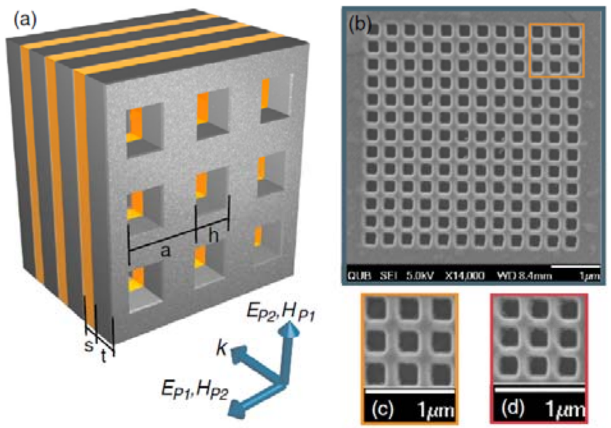


Ag & MgF_2

Berkeley, Nature 455, 376 (2008)

7-layers fishnet

Negative index at ~640 nm, FOM=3.3

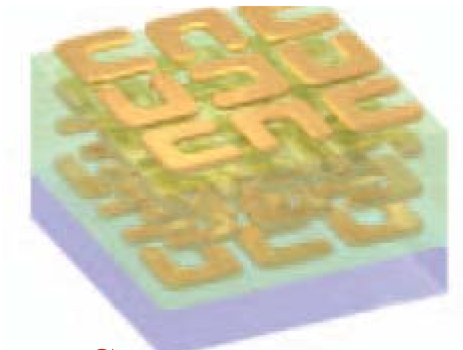


Valencia, Phys. Rev. Lett. 106, 067402 (2011)

Towards 3D-isotropic negative μ and n

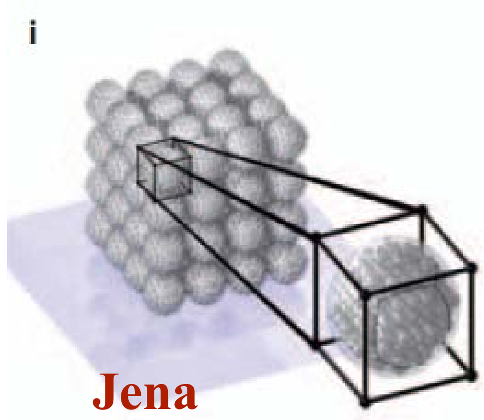
Figure from Soukoulis & Wegener, Nature Photonics **5** (2011)

Multilayer SRR (stereometamaterials)



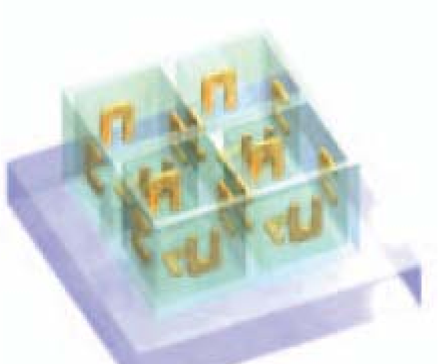
Stuttgart

Caltech



Jena

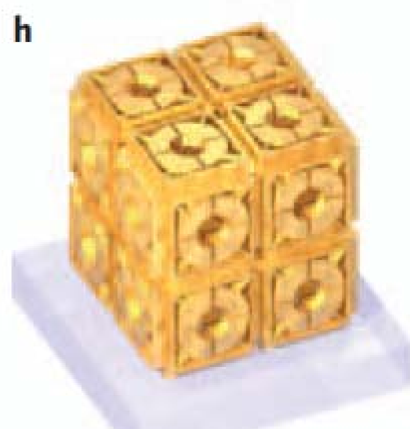
3D-SRR (membrane-projected lithography)



Sandia

Chiral metamaterial (DLW+electroplating)

Karlsruhe



Ameslab

Optical metamaterials: Problems/challenges

High losses

Limited fabrication capabilities

Current procedures:

- **difficult/time-consuming**
- **expensive**
- **unable to produce**
  - **complicated patterns**
  - **large samples**
  - **3D isotropic designs**

Optical metamaterials: Facing the challenges

High losses ⟷

- Analysis & design optimization
- "Good" constituent media (material optimization)
- Gain media?
- Alternative approaches (anisotropic media, chiral media, EIT)

Limited fabrication capabilities

Current procedures:

- difficult/time-consuming
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⟷

- Advancement of fabrication procedures
- New fabrication methods (direct laser writing, nanoimprint lithography)
- New designs/approaches, adapted to fabrication capabilities

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Outline

- **Analysis of wave propagation in optical left-handed metamaterials (OLHMs)**
- **Optimization (design & material) of OLHMs as to achieve high operation frequency and broad-bandwidth**
- **Loss-examination & optimization of OLHMs as to achieve low-losses**
- **Alternatives of metal for OLHMs?**

Relevant publications

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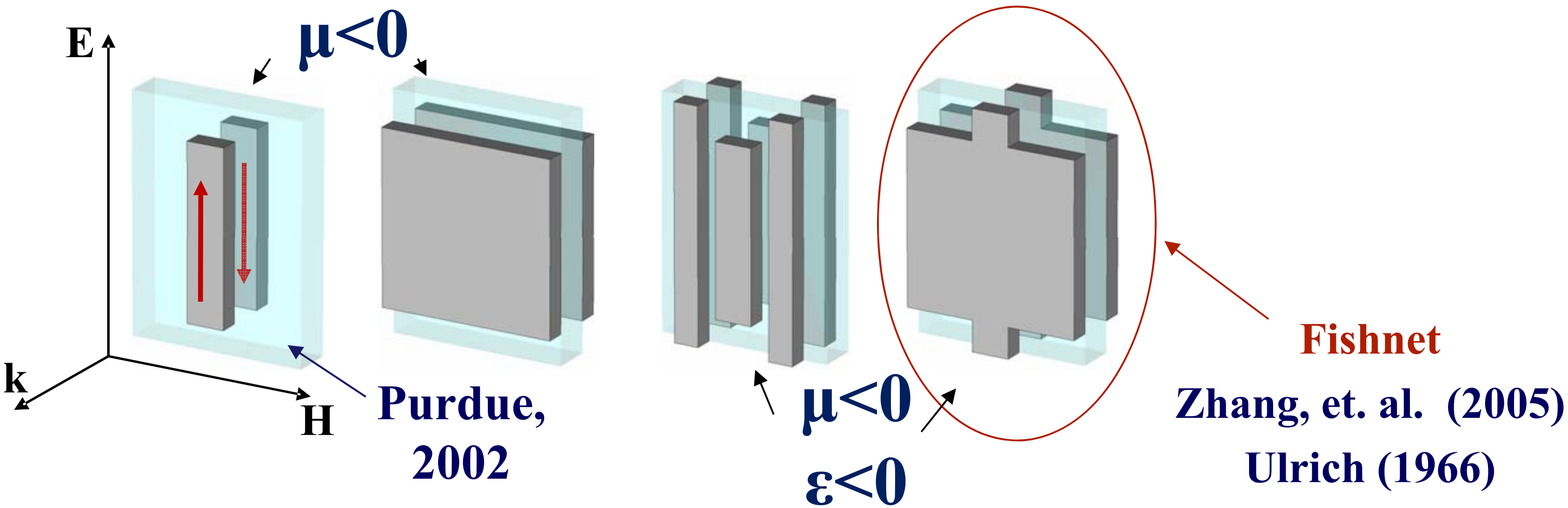
Designs discussed here

Unit cells



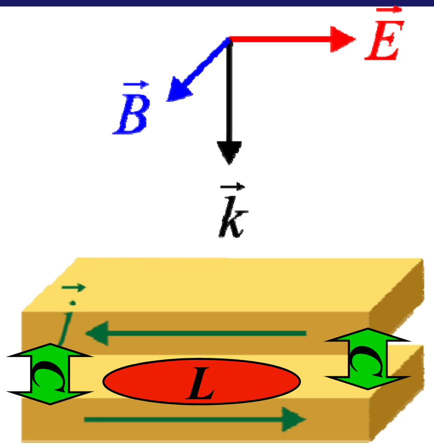
Aim

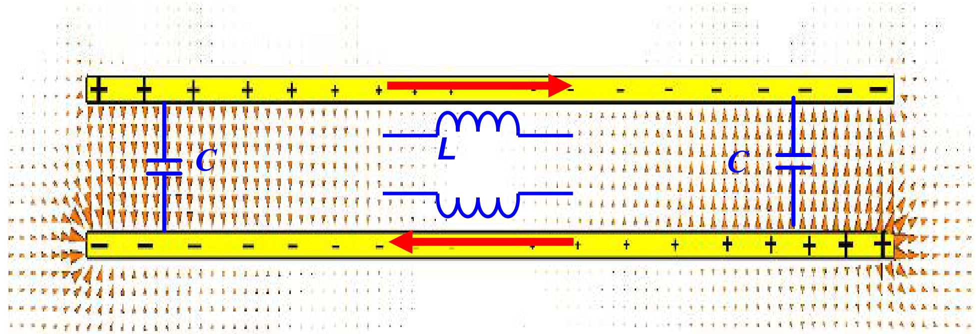
Examine the magnetic response of the designs as they are scaled down targeting optical negative permeability

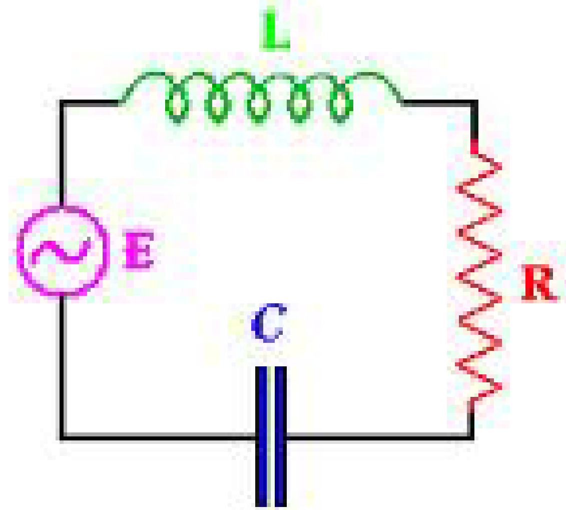
Seek for optimization rules

High frequency
Broad-band
Low-loss

Slab-pair : Effective RLC circuit







Kirchoff's equation

$$L\dot{I} + \frac{1}{C}\int I dt + RI = -\frac{d\phi_{ext}}{dt}$$

$$\phi_{ext} = \mu_0 H_0 A e^{-i\omega t}$$

⇩

Current, I

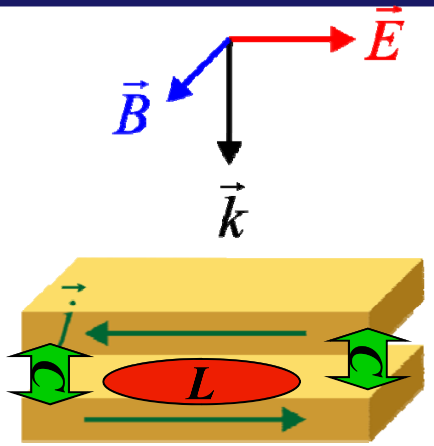
⇩

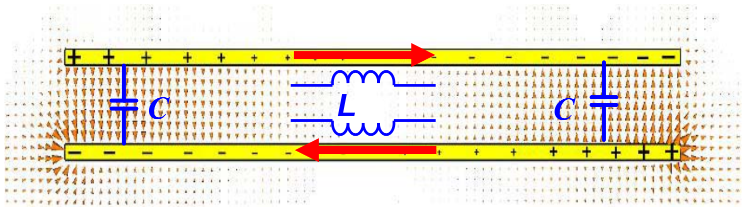
Magnetic moment , m=IA

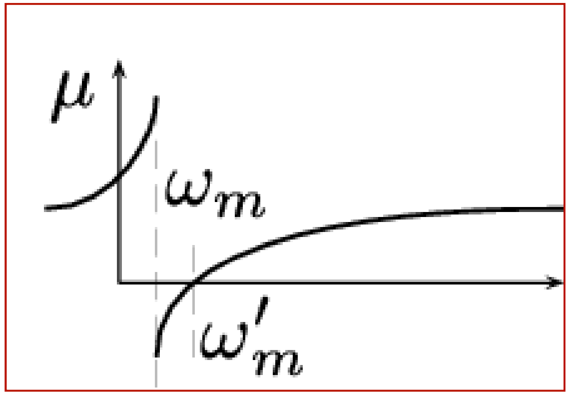
⇩

Magnetization, $M=m/V_{uc}=\mu_0(\mu-1)H$

Slab-pair magnetic response (RLC circuit description)







$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_m^2 + i\omega\gamma}$$

F ~ volume fraction of the resonator within unit cell (determines the resonance strength)

$$\omega_m = \frac{1}{\sqrt{LC}}$$

$$\gamma = \frac{R_{tot}}{L}$$

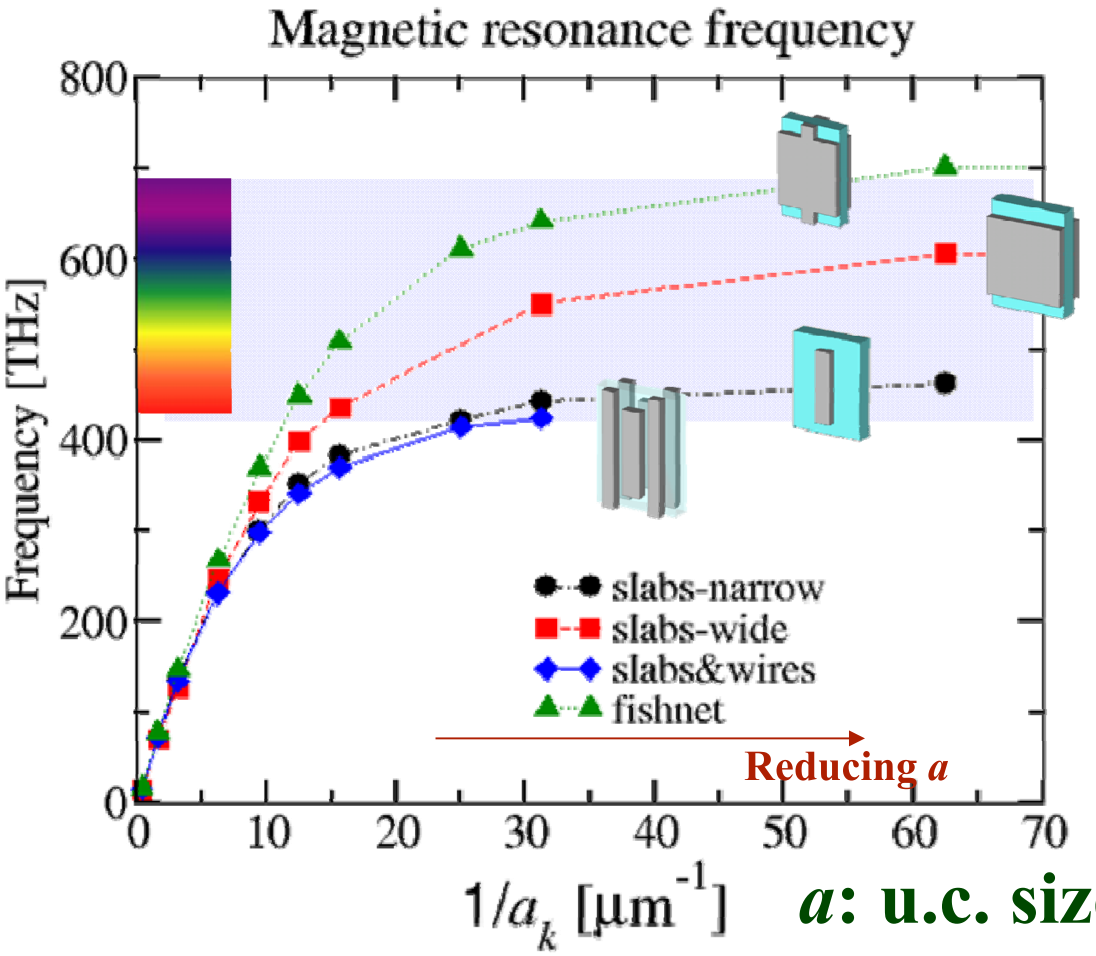
Damping factor (accounts for all-type losses)

$$R_{tot} = R + R_{rad}$$

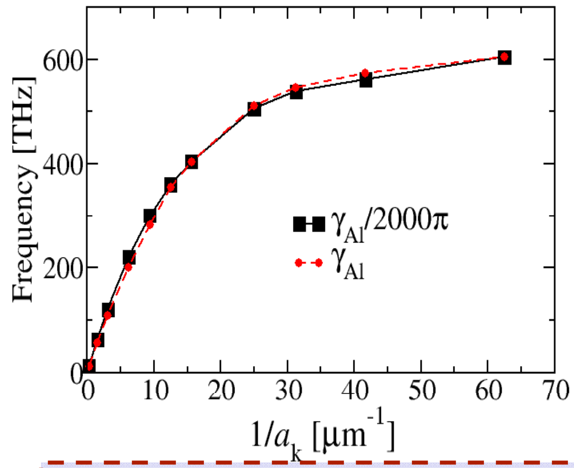
Tassin *et al.*, Nature Photon. **6**, 259 (2012)

Magnetic resonance frequency vs length scale

Al metal, Glass substrate



***a*: u.c. size**

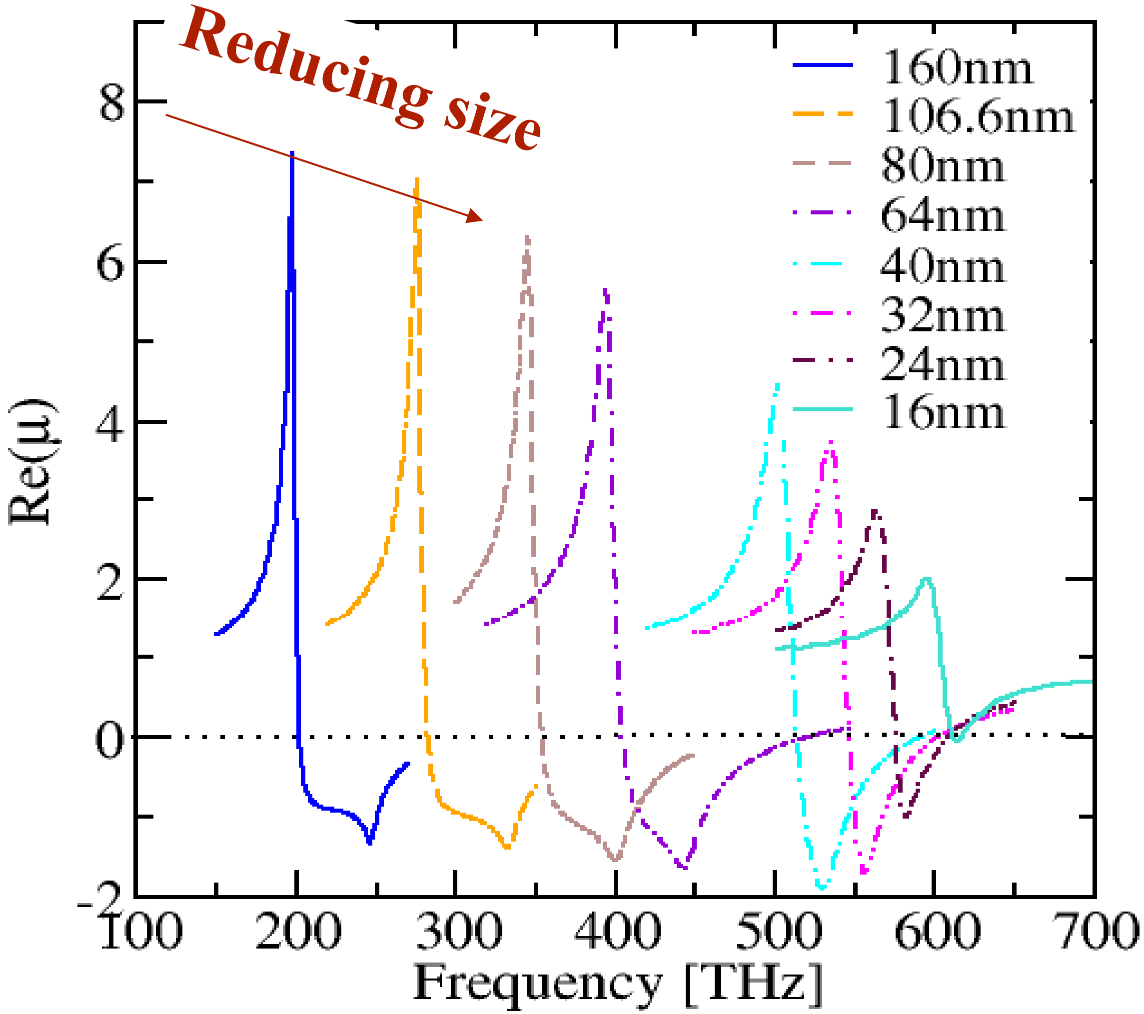


Saturation value independent of ohmic losses

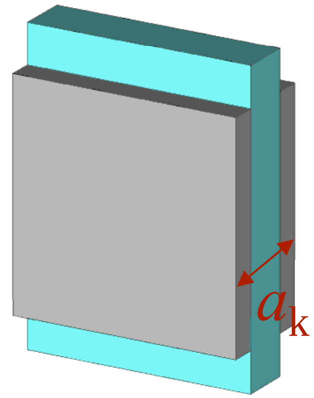
Saturation value depends on design

Saturation of magnetic resonance frequency in small length scales (*a*<500 nm)

Magnetic permeability by scaling down the structures



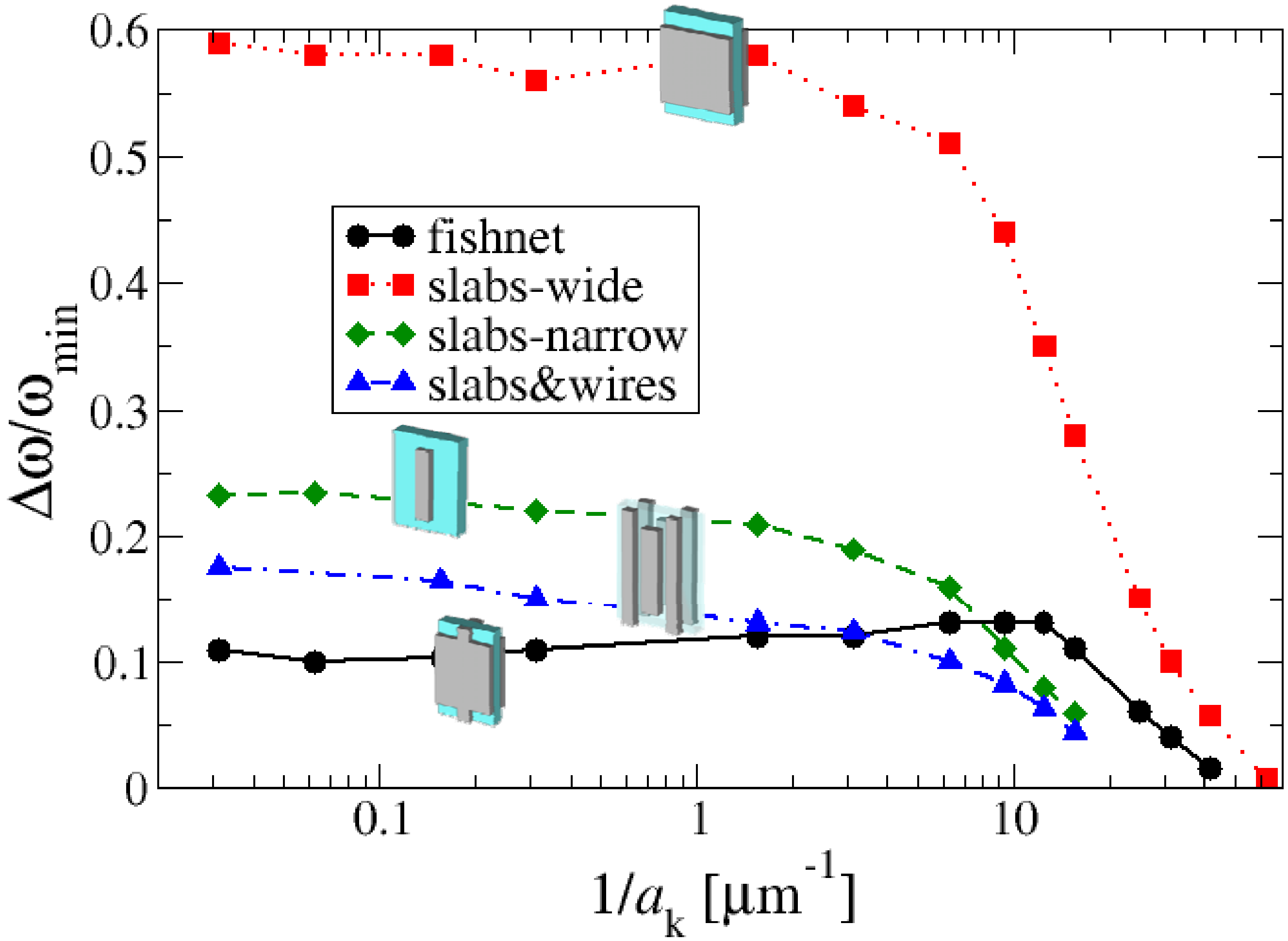
Al metal
Glass substrate



Weakening of magnetic resonance at small scales

μ ultimately does not reach negative values

Spectral width of negative μ regime



Al metal

Glass substrate

Spectral width depends on geometry

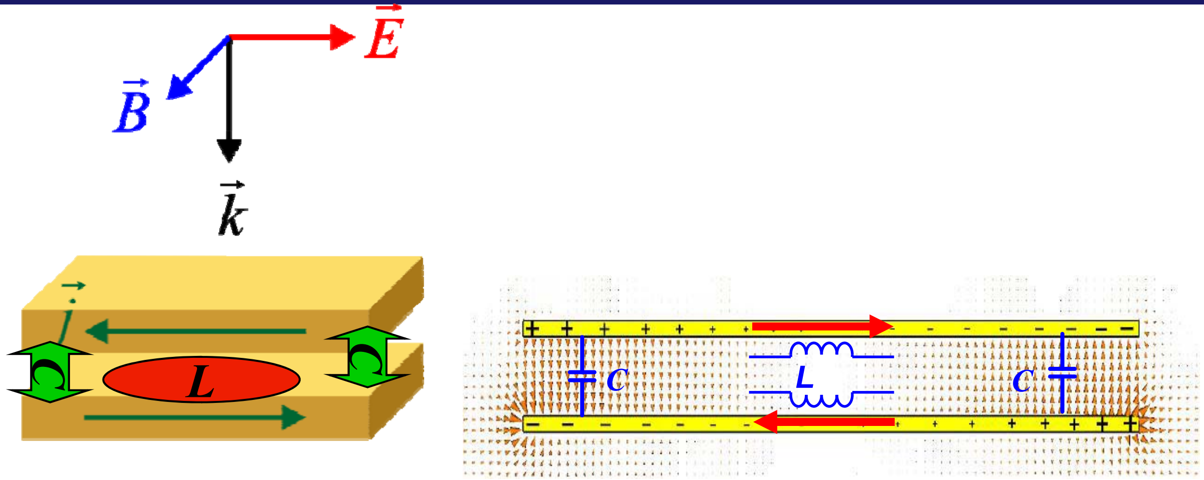
Spectral width only slightly affected by metal loss

Even in the absence of ohmic losses

$\Delta\omega/\omega_{min}$: constant at larger scales

tends to zero for smaller scales

Slab-pair magnetic response (RLC circuit description)



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$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\gamma = \frac{R_{tot}}{L}$$

Damping factor (accounts for all-type losses)

$R_{tot} = R + R_{rad}$

Tassin *et al.*, Nature Photon. **6**, 259 (2012)

Slab-pair total resistance

$$R = \rho \frac{l}{S}$$

$$\rho = \frac{\gamma_m - i\omega}{\varepsilon_0 \omega_p^2}$$

For free electrons (Drude metals)

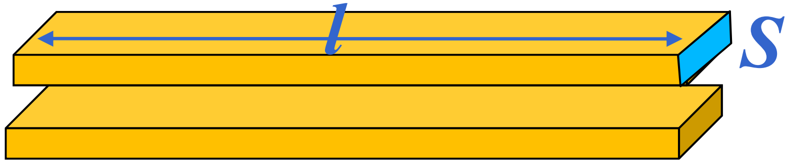
ω_p= metal plasma frequency

γ_m=metal collision frequency

Material dependent + Geometry dependent

$$\mathrm{Re}(R) = R_{ohm}$$
$$\mathrm{Im}(R) = -\omega L_e$$

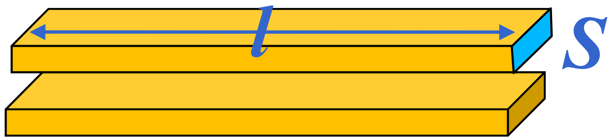
$$R = \rho \frac{l}{S} = \frac{\gamma_m}{\varepsilon_0 \omega_p^2} \frac{l}{S} - i\omega \frac{1}{\varepsilon_0 \omega_p^2} \frac{l}{S} = R_{ohm} - i\omega L_e$$



Inductive term (electrons kinetic inductance) due to electrons inertia *("Difficulty" to accelerate finite mass particles with such high rates)*

Solymar, Economou, Shevts, Tretyakov, …

High frequency magnetic permeability



$$\mu(\omega) = 1 - \frac{F'\omega^2}{\omega^2 - \omega_m^2 + i\omega\gamma'}$$

Kinetic inductance factor

$$\omega_m = \frac{1}{\sqrt{LC(1+\xi)}}$$

$$\xi = \frac{L_e}{L}$$

$$F' = F\frac{1}{1+\xi}$$

$$\xi \sim \frac{1}{a^2}$$

F: filling ratio

$$L_e = \frac{1}{\varepsilon_0 \omega_p^2}\frac{l}{S}$$

For uniform scaling:

$$L_e \sim \frac{1}{a}$$

$$\mathrm{Re}(R) \sim \frac{1}{a}$$

Pronounced role in small scales

a: lattice constant

$\sigma = i\varepsilon_0 \frac{\omega_p^2}{\omega^2 + i\gamma_m}$

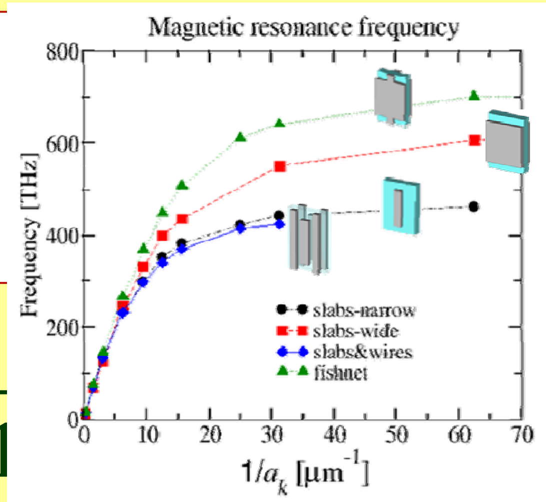
Explaining the observed response

***a*: u.c. size**

$$L \propto a,\ C \propto a,\ L_e \propto 1/a, \xi \propto 1/a^2$$

Magnetic resonance frequency saturates to $\omega_{m\text{-}max}$
-dependent on shape
-independent of metal losses

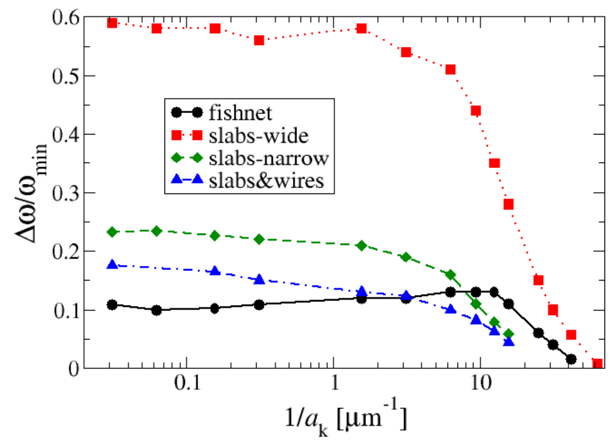
$$\omega_m = \frac{1}{\sqrt{(L+L_e)C}} \sim \frac{1}{\sqrt{c_1 a^2 + c_2}} \to \text{const.}$$



Strength parameter F' becomes proportional to area →
-Weakening of magnetic resonance
-Vanishing of negative μ regime <u>even if the absence of ohmic losses</u>

$F' = F$ … 1 … 2

Upper bound of negative μ regime $\frac{\omega_m'}{\omega_m} = \frac{1}{\sqrt{1-F'}}$



For high frequency magnetic metamaterials

High

$$\omega_m = \frac{1}{\sqrt{LC(1+\xi)}}$$

for

High operation frequency

Broad-band

High

$$\mu = \mu_0[1 - \frac{F'\omega^2}{\omega^2 - \omega_m^2 + i\omega\gamma'}]$$

$$F' = F\frac{1}{1+\xi}$$

F: filling ratio

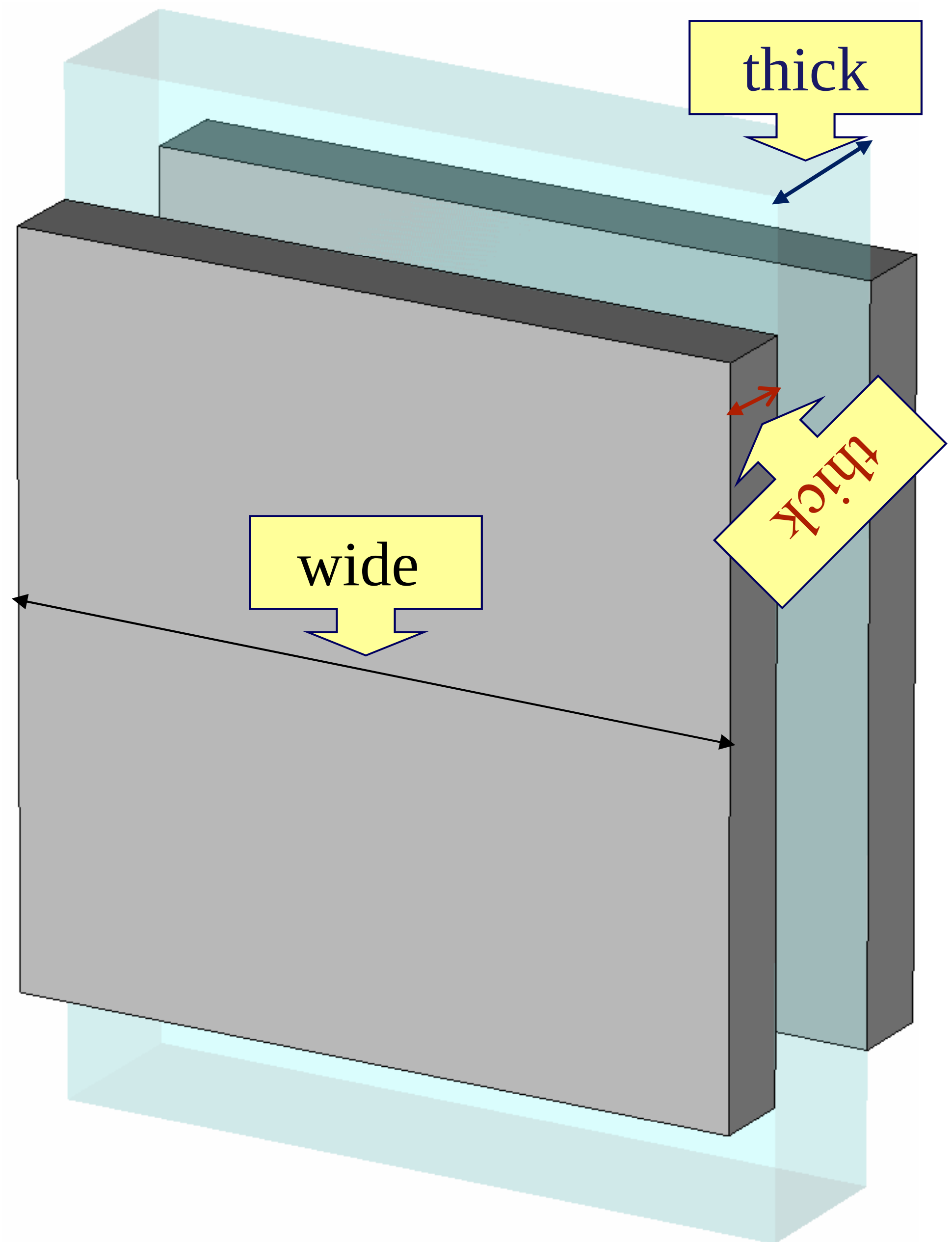
Requirements

- **Small capacitance, *C***
- **Large structure filling ratio, *F***
- **<u>Small L_e/L (=ξ)</u>**

- **Material requirements**
- **Geometry requirements**

$$\xi = \frac{1}{\varepsilon_0 \omega_p^2} f(geometry)$$

Optimizing slab-pair-based systems



Requirements

- **<u>Thick</u> & wide slabs**
- **"Thick" separation layer**
- **"Metal" of high ω_p**

What about losses?

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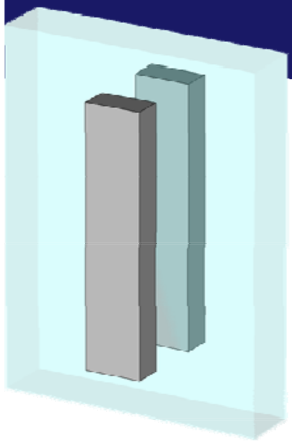
Dissipative loss



$$\Pi = \frac{\text{Dissipative loss } (=\mathrm{Re}(R)\,|\,I\,|^2)}{\text{Incident power}}$$

$$\sim \frac{F\tilde{\omega}^4\zeta}{[\tilde{\omega}^2(1+\xi)-1]^2+\tilde{\omega}^2\zeta^2}$$

$$\tilde{\omega}=\omega/\omega_0$$

$$\omega_0=\frac{1}{\sqrt{LC}}$$

F: filling ratio

Kinetic inductance factor (dimensionless)

$$\xi=\frac{L_e}{L}=-\mathrm{Im}(R)\frac{1}{\tilde{\omega}}\sqrt{\frac{C}{L}}==G(\text{geometry})\frac{1}{\omega}\times\mathrm{Im}(\rho)$$

Dissipation factor (dimensionless)

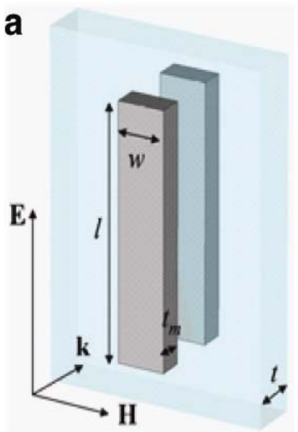
$$\zeta=\mathrm{Re}(R)\sqrt{\frac{C}{L}}=\frac{1}{Q}=G(\text{geometry})\times\mathrm{Re}(\rho)$$

$$\mathrm{Re}(\rho)=\frac{\gamma_m}{\varepsilon_0\omega_p^2}$$

Loss depends only on ξ, ζ, F

Tassin

Magnetic permeability in dimensionless quantities



$$\mu(\omega) = 1 - \frac{F\tilde{\omega}^2}{\tilde{\omega}^2(1+\xi) - 1 + i\tilde{\omega}\zeta}$$

$$\tilde{\omega} = \omega / \omega_0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

F: filling ratio

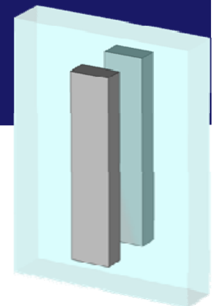
$$\xi \sim \mathrm{Im}(R) \sim G(\mathrm{geometry})\frac{1}{\omega} \times \mathrm{Im}(\rho)$$ **Kinetic inductance factor**

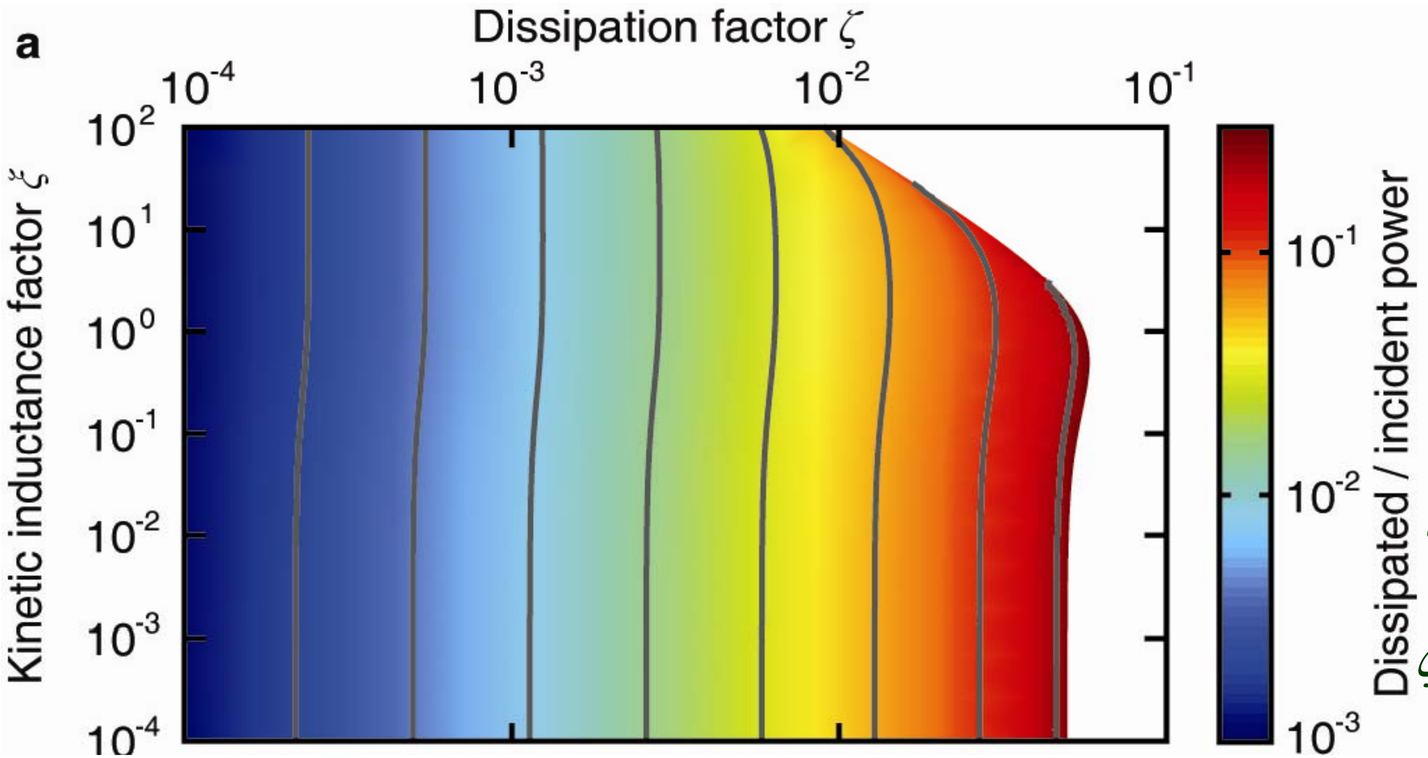
$$\zeta \sim \mathrm{Re}(R) \sim G(\mathrm{geometry}) \times \mathrm{Re}(\rho)$$ **Dissipation factor**

Permeability also depends only on ξ, ζ, F

Tassin *et al.*, Nature Photon. **6**, 259 (2012)

Dissipative power at ω_1: $\mu(\omega_1)$=-1





$$\Pi \sim \frac{F\tilde{\omega}^4\zeta}{[\tilde{\omega}^2(1+\xi)-1]^2+\tilde{\omega}^2\zeta^2}$$

$$\xi \sim \mathrm{Im}(R) \sim G(\mathrm{geometry})\frac{1}{\omega}\times \mathrm{Im}(\rho$$

$$\zeta \sim \mathrm{Re}(R) \sim G(\mathrm{geometry})\times \mathrm{Re}(\rho$$

Dissipation depends "only" on ζ → *ζ : good quantity to quantify losses (loss figure-of-merit) and compare conductors*

For low-loss metamaterials small dissipation factor is required → Small Re(R) → Small Re(ρ)

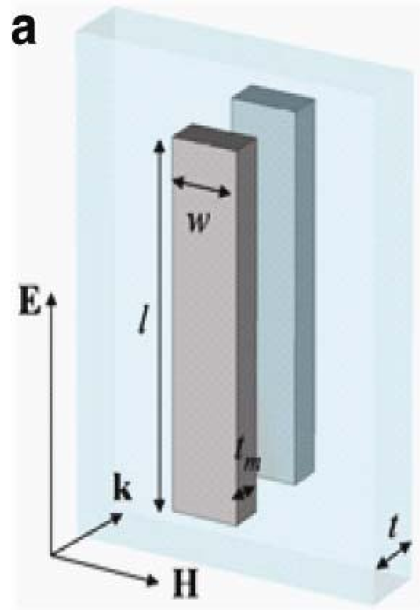
FOMs for high-quality magnetic metamaterials?

Loss figure-of–merit: Dissipation factor ζ

For low-loss metamaterials small dissipation factor ζ (~small Re(R)) is required

$$\zeta \sim \mathrm{Re}(R) \sim \frac{1}{t_m} \times \mathrm{Re}(\rho)$$

t_m: metal thickness



$$\mathrm{Re}(\rho) = \frac{\gamma_m}{\varepsilon_0 \omega_p^2}$$

Freq. saturation figure-of-merit: Kinetic inductance factor ξ

For high-freq. metamaterials small ξ (~small Im(R)) is required

$$\mathrm{Im}(\rho) = -\frac{\omega}{\varepsilon_0 \omega_p^2}$$

$$\xi \sim \mathrm{Im}(R) \sim \frac{1}{t_m} \times \frac{\mathrm{Im}(\rho)}{\omega}$$

$$\tilde{\omega}_m = \omega_m / \omega_0 = 1/\sqrt{(1+\xi)}$$

Material-dependent part: Re(ρ), Im(ρ)

Good quantities to compare conducting materials

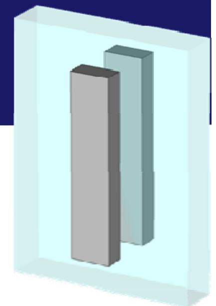
Outline

- **Analysis of wave propagation in optical left-handed metamaterials (OLHMs)**
- **Optimization (design & material) of OLHMs as to achieve high operation frequency and broad-bandwidth**
- **Loss-examination & optimization of OLHMs as to achieve low-losses**
- **Alternatives of metal for OLHMs?**

Relevant publications

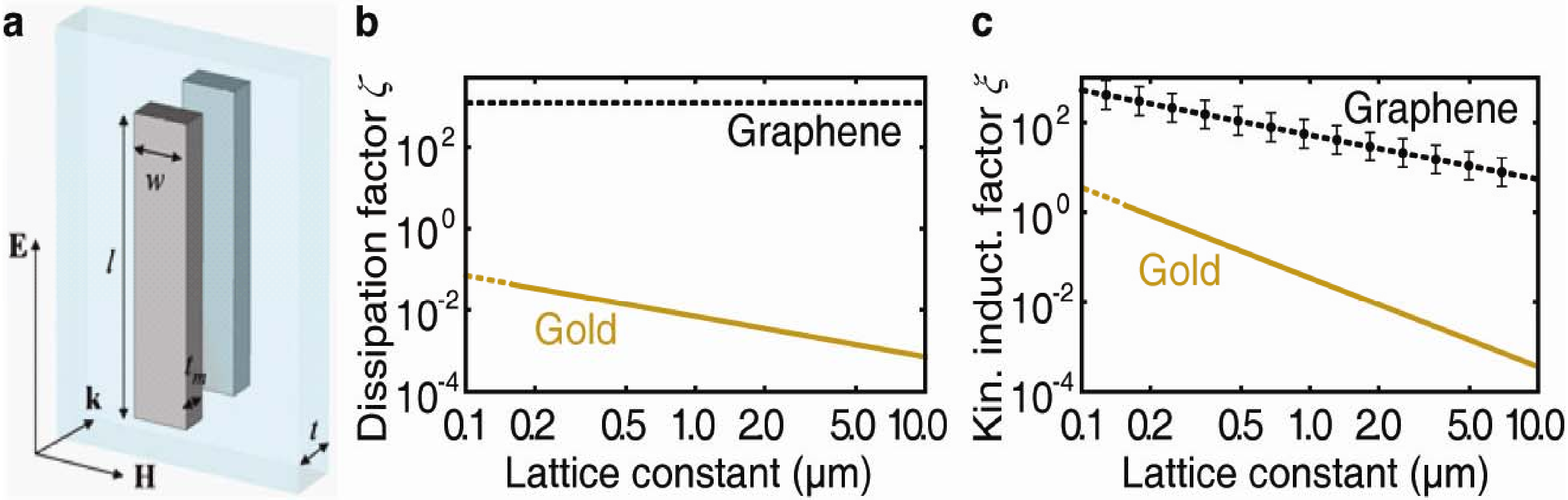
- **R. S. Penciu, M. Kafesaki, Th. Koschny, E. N. Economou, and C. M. Soukoulis**, *Magnetic response of nanoscale left-handed metamaterials,* Phys. Rev. B **81** (2010)
- **Ph. Tassin , Th. Koschny M. Kafesaki and C. M. Soukoulis,** *Graphene superconductors and metams: What makes a good conductor for metamaterials,* Nature Photonics **6**, 259 (2012)

Comparing gold with graphene



At IR

Graphene conductivity from Li et al, Nature Phys. 4, 532 (2008)



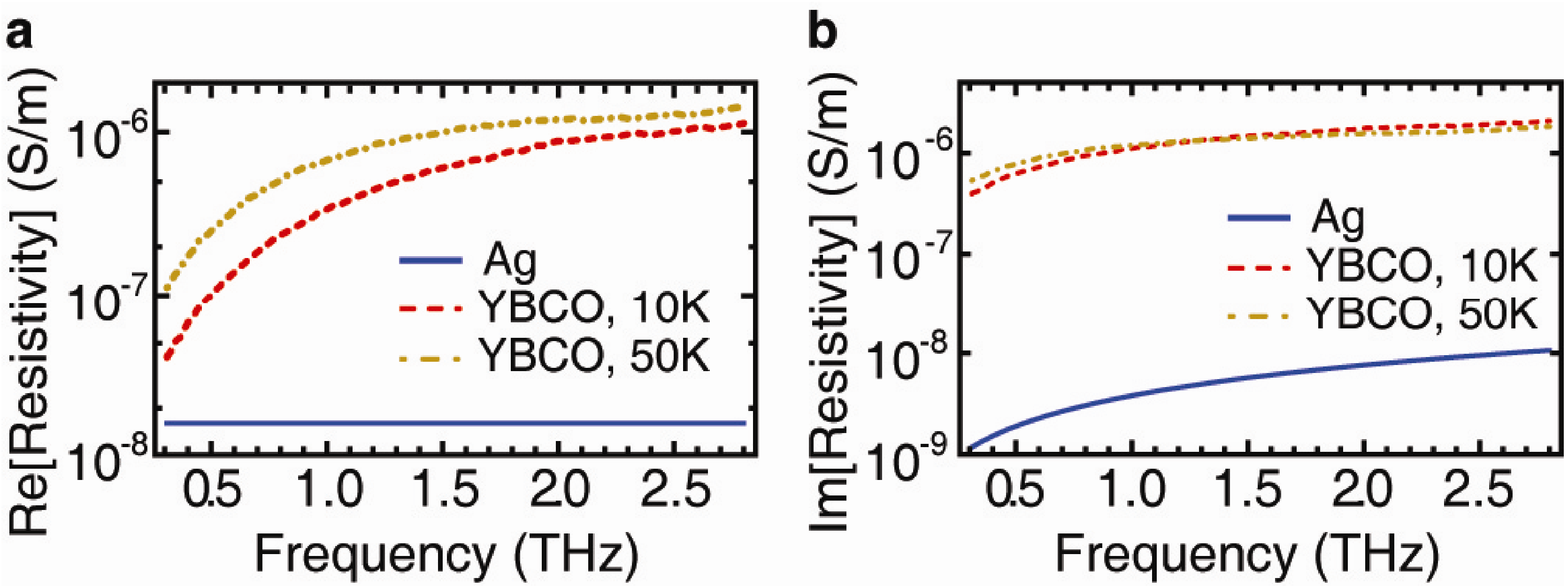
Graphene performs worse than gold

***Reason*?: Small thickness→ large *R*→ weak total current**

Tassin *et al.*, Nature Photon. **6**, 259 (2012)

Comparing silver with YBCO

Resistivity from experimental data



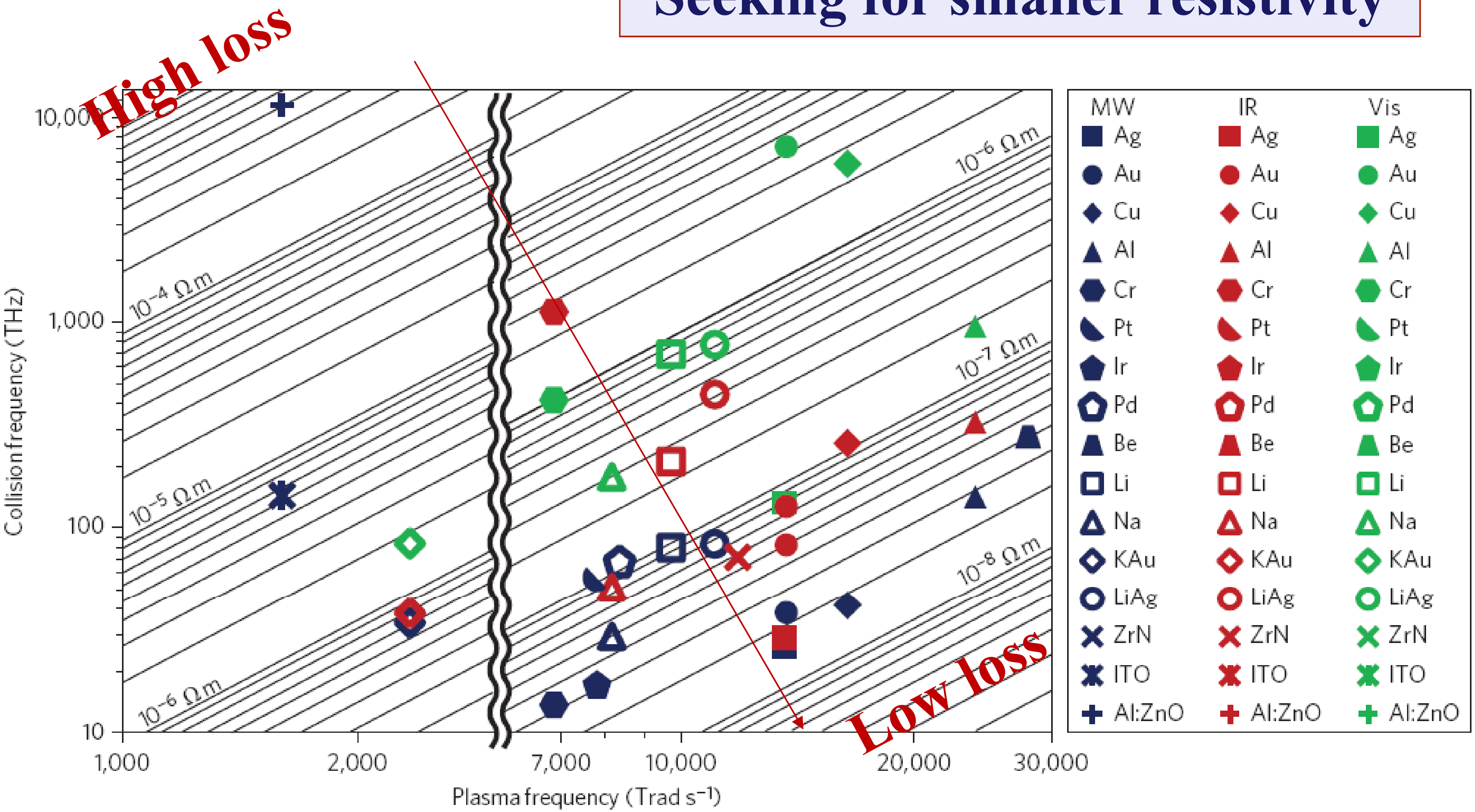
High-Tc superconductors perform worse than silver

***Reason*?: No complete screening of free electrons by Cooper pairs in high frequencies**

Tassin *et al.*, Nature Photon. **6**, 259 (2012)

Comparing losses in different conductors

Seeking for smaller resistivity



Blue: microwaves, Red: infrared, Green: visible

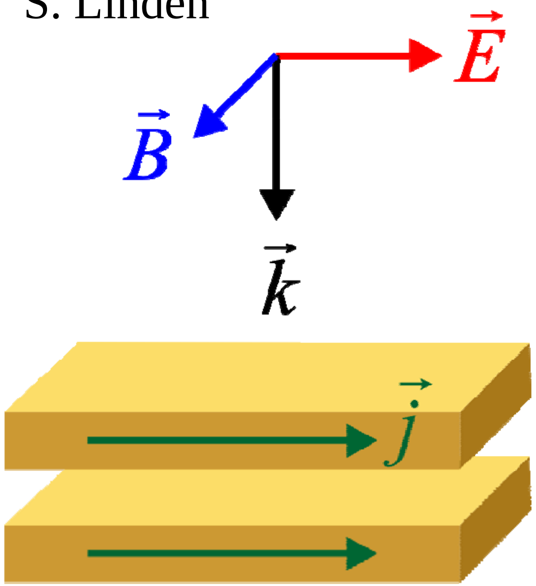
Tassin *et al.*, Nature Photon. **6**, 259 (2012)

Still silver is best at IR and visible!

Slab-pair electric resonance

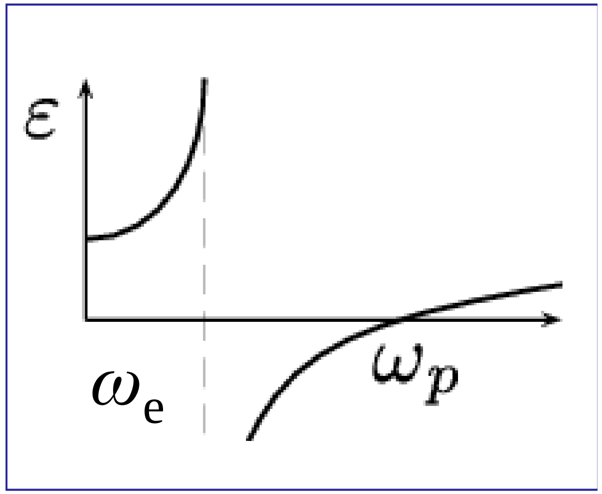
Apart of anti-symmetric current mode (magnetic dipole response)

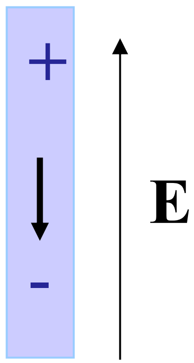
Picture by
S. Linden



Also a
symmetric current mode
(electric dipole response)

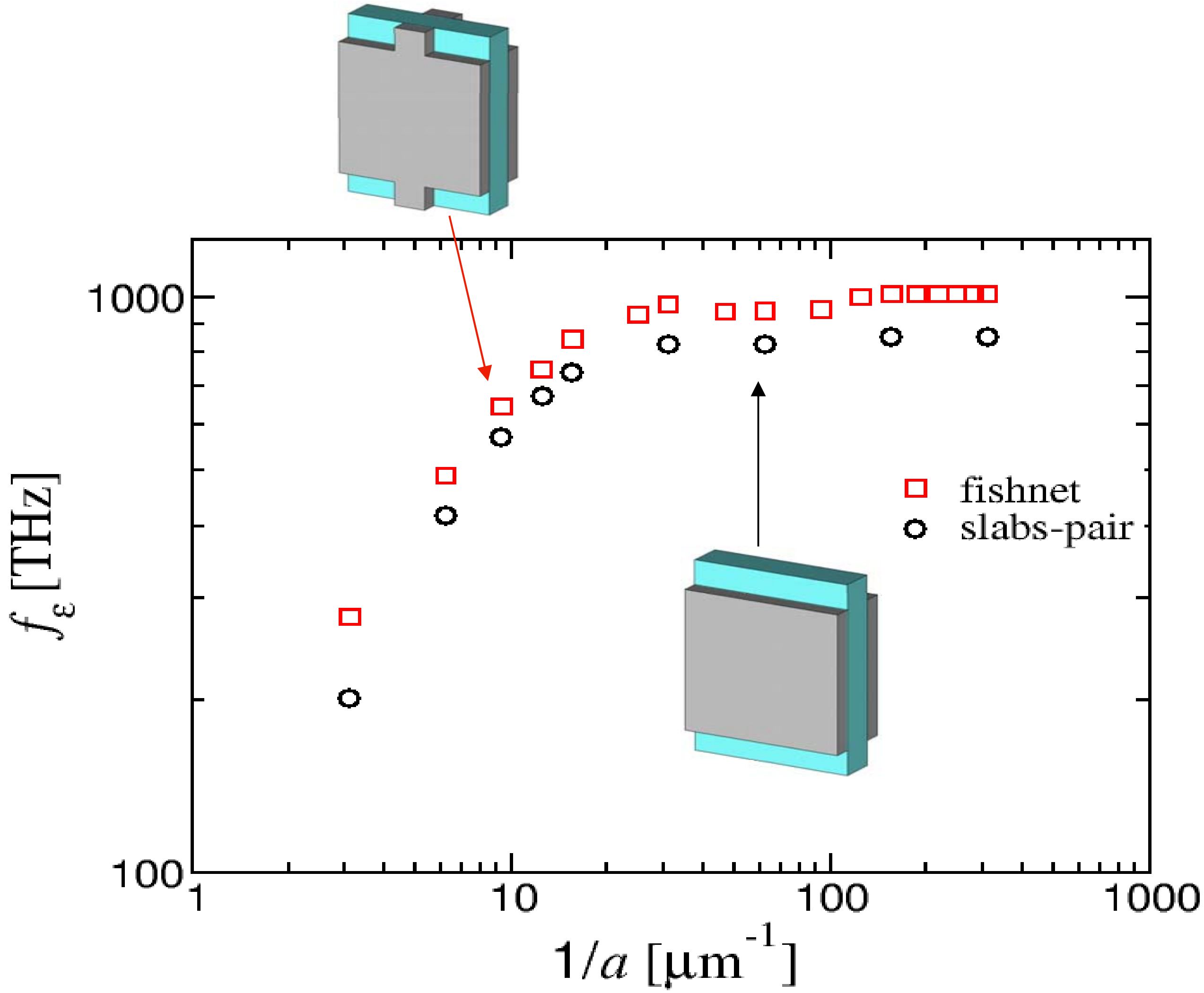
$$\varepsilon(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_e^2 + i\omega\gamma}$$





$$m\frac{\partial^2 \mathbf{x}}{\partial t^2} = -e\mathbf{E} + m\omega_e^2\mathbf{x}$$

Electric dipole resonance in small length scales?

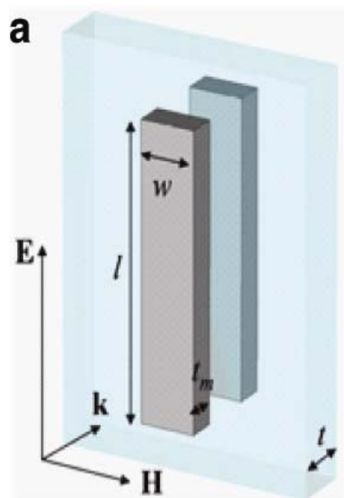


Resonance saturates in small scales

For high-frequency, low-loss quasistatic plasmon resonance

For high-frequency: small kinetic inductance factor ξ (~small Im(ρ)) is required

For low-loss: small dissipation factor ζ (~small Re(ρ)) is required



Summary/Conclusions

- Magnetic metamaterial behaviour towards optical regime (nm scale):
  - magnetic resonance frequency saturates
  - permeability resonance becomes weaker
  - negative permeability regime vanishes
- RLC circuit description and metal dispersive respo
  can account for all the above effects and can lead to
  figures of merit and design rules for high frequency
  magnetic metamaterials
- Optimized high-frequency response requires metals
  small Re and Im part of resistivity

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