Pseudo-Riemannian cones with reducible holonomy group

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By a classical theorem of Gallot (1979), the Riemannian cone over a complete Riemannian manifold M has reducible holonomy group only if the manifold (M, g) is flat. This result was used by Ch. Bär for description of Riemannian spinor manifolds with real Killing spinors.

We are interested in generalization of Gallot's result to the pseudo-Riemannian case. We study the pseudo-Riemannian cone $C(M)$ over a pseudo-Riemannian manifold M with reducible holonomy group $H(C(M))$.

In the case when the holonomy group $H(C(M))$ has a non-degenerate invariant subspace (that is the cone $C(M)$ is locally decomposable as a pseudo-Riemannian manifold) we describe the local structure of M and prove that it has full pseudo-orthogonal holonomy group. Moreover, under extra assumptions that the pseudo-Riemannian manifold M is compact and complete, we prove that M is flat.

Then we study the case when the cone $C(M)$ is indecomposable, but the holonomy group $H(C(M))$ preserves two complementary subspaces. This implies that the cone $C(M)$ has a para-Kähler structure. We prove that para-Kähler structures on $C(M)$ naturally correspond to para-Sasakian structures on M and para-hyper-Kähler structures on $C(M)$ correspond to para-3-Sasakian structures on M.

We consider also the case when $C(M)$ has Lorentzian signature and reducible holonomy group and we describe the structure of the base manifold M.