Nonlinear parabolic problems with a very general quadratic gradient term

A. Dall’Aglio\(^{(\ast)}\), D. Giachetti\(^{(\ast)}\) and S. Segura de León\(^{(\ast\ast)}\)

January 23, 2006

\(^{(\ast)}\) Dipartimento di Metodi e Modelli Matematici - Università di Roma I
Via Antonio Scarpa 16, I-00161 Roma, Italy

\(^{(\ast\ast)}\) Departament d’Anàlisi Matemàtica - Universitat de València
Dr. Moliner 50, 46100 Burjassot, València, Spain

**Keywords:** Nonlinear parabolic problems, gradient term with quadratic growth, convection-diffusion problems, superlinear reaction term, existence and regularity, bounded and unbounded solutions, a priori estimates, logarithmic Sobolev inequalities.

**MSC2000:** 35B33, 35B45, 35K20, 35K55, 35K57.

**Abstract.**

We study existence and regularity of distributional solutions for a class of nonlinear parabolic problems. The equations we consider have a quasi-linear diffusion operator and a lower order term, which may grow quadratically in the gradient and may have a very fast growth (for instance, exponential) with respect to the solution. The model problem we refer to is the following

\[
\begin{cases}
    u_t - \Delta u = \beta(u)|\nabla u|^2 + f(x,t), & \text{in } \Omega \times ]0,T[; \\
    u(x,t) = 0, & \text{on } \partial \Omega \times ]0,T[; \\
    u(x,0) = u_0(x), & \text{in } \Omega;
\end{cases}
\]

with \( \Omega \subset \mathbb{R}^N \) a bounded open set, \( T > 0 \), and \( \beta(u) \sim e^{u_1} \); as far as the data are concerned, we assume \( \exp(\exp(|u_0|)) \in L^2(\Omega) \), and \( f \in X(0,T;Y(\Omega)) \), where \( X, Y \) are Orlicz spaces of logarithmic and exponential type, respectively. We also study a semilinear problem having a superlinear reaction term, problem that is linked with problem (1) by a change of unknown. Likewise, we deal with some other related problems, which include a gradient term and a reaction term together.