On the regularity of solutions to some classes of variational problems.

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Abstract

In the talk I will present the problem of the higher differentiability, i.e., the existence of second (weak) derivatives, of solutions to some classes of variational problems of the kind of minimising

$$\int_{\Omega}[l(|\nabla v(x)|)+f(x)v(x)]dx$$

with prescribed boundary conditions.

We will consider the cases where $l(t)$ is either $\frac{1}{p}t^p$ or e^{t^2} or, to the opposite, the case of very slow growth, where, for t large, $l(t)$behaves as $t\ln\ln\ldots\ln(t)$. We will discuss the problems arising in each case.

No previous knowledge of the subject is required.

The nodal set of solutions to sublinear, discontinuous and singular equations

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We are concerned with the nodal set of solutions to equations of the form

$$-\Delta u = \lambda_+ \left(u^+\right)^{q-1} - \lambda_- \left(u^-\right)^{q-1} \quad \text{in } B_1$$

where $\lambda_+, \lambda_- > 0$, $q \in (0,2)$, $B_1 = B_1(0)$ is the unit ball in $\mathbb{R}^N$, $N \geq 2$, and $u^+ := \max\{u, 0\}$, $u^- := \max\{-u, 0\}$ are the positive and the negative part of u, respectively. This class includes sublinear, discuontinuous cases, such as the two-phases *unstable obstacle problem (*$q = 1$*) as well as singular equations.* Notice that the right hand side is not locally Lipschitz continuous as function of u, and precisely has sublinear character for $1 < q < 2$, and a discontinuos character for $q \leq 1$.

We consider the two phases problem treating simultaneously the cases $q \leq 1$ and $1 < q < 2$, proving the following main results: (a) an a priori $\mathcal{C}^{1,\alpha}$ estimate on the solutions; (b) the finiteness of the vanishing order at every point and the complete characterization of the order spectrum; (c) a weak non-degeneracy property; (d) regularity of the nodal set of any solution: the nodal set is a locally finite collection of regular codimension one manifolds up to a residual singular set having Hausdorff dimension at most $N-2$ (locally finite when $N = 2$) and a partial stratification theorem.

Ultimately, the main features of the nodal set are strictly related with those of the solutions to linear (or superlinear) equations, with two remarkable differences. First of all, the admissible vanishing orders are can not exceed the critical value $2/(2-q)$. Moreover, at the threshold $2/(2-q)$, we find a multiplicity of homogeneous solutions, yielding the *non-validity* of any estimate of the $(N-1)$-dimensional measure of the nodal set of a solution in terms of the vanishing order.

The proofs are based on Almgren's and Weiss type monotonicity formulæ, blow-up arguments and the classification of homogenous solutions.

This are joint works with Nicola Soave