Invited conferences

Isogeometric Compatible Discretizations: Fundamentals, Current Research Thrusts, and Future Directions

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Isogeometric compatible discretizations have emerged as an attractive candidate for the numerical solution of partial differential equations arising in applied mechanics and engineering [1, 2, 3, 4]. These discretizations bridge the fields of isogeometric analysis and discrete differential geometry, and in many applications of interest, they conserve important mathematical structure. For instance, in the setting of incompressible fluid flow, isogeometric compatible discretizations satisfy conservation of mass in a pointwise manner, and as a byproduct, they also preserve the underlying geometric structure of the Navier-Stokes equations [5]. In this talk, I will first review the construction of isogeometric discrete differential forms, the basic building blocks of isogeometric compatible discretizations, and I will present numerical examples illustrating their promise in computational electromagnetics and computational fluid dynamics. I will then discuss several current research thrusts, including (i) adaptive methods based on hierarchical spline complexes of discrete differential forms, (ii) optimal multi-level solvers which exploit the geometric structure of isogeometric discrete differential forms, (iii) application of isogeometric discrete differential forms to the numerical solution of boundary integral equations arising in electromagnetics, and (iv) application of isogeometric discrete differential forms to computational fluid-structure interaction. I will conclude by discussing related and future research directions.

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Low Rank Tensor-Product Spline Surfaces and Volumes

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The coefficients of parametric tensor-product spline surface and volume patches form tensors of order 3 and 4, respectively. Patches with coefficient tensors that possess a low tensor rank — that admit a representation as sum of a small number of rank-1 tensors. i.e., of tensor-products of vectors — have been found to be useful for applications in geometric modeling and isogeometric analysis, since they combine memory-efficient representations with computationally efficient algorithms, e.g, for assembling system matrices required for numerical simulation [1]. The talk will describe some of these advantages. In addition we describe and analyze methods for creating low rank tensor-product spline surface and volume patches [2]. In particular, we will discuss the approximation order bivariate tensor-product spline functions of low rank. Based on joint work with N. Dyn, B. Khoromskij, U. Langer, A. Mantzaflaris, D. Mokriš and F. Scholz.

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On Learning Invariants and Representation Spaces of Shapes and Forms

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We study the power of the Laplace Beltrami Operator (LBO) in processing and analyzing geometric information. The decomposition of the LBO at one end, and the heat operator at the other end provide us with efficient tools for dealing with images and shapes. Denoising, segmenting, filtering, exaggerating are just few of the problems for which the LBO provides an efficient solution. We review the optimality of a truncated basis provided by the LBO, and a selection of relevant metrics by which such optimal bases are constructed. Specific example is the scale invariant metric for surfaces that we argue to be a natural selection for the study of articulated shapes and forms.

In contrast to geometry understanding there is a new emerging field of deep learning. Learning systems are rapidly dominating the areas of audio, textual, and visual analysis. Recent efforts to convert these successes over to geometry processing indicate that encoding geometric intuition into modeling, training, and testing is a non-trivial task. It appears as if approaches based on geometric understanding are orthogonal to those of data-heavy computational learning. We propose to unify these two methodologies by computationally learning geometric representations and invariants and thereby take a small step towards a new perspective on geometry processing.

I will present examples of shape matching, facial surface reconstruction from a single image, reading facial expressions, shape representation, and efficient computation of invariant operators and signatures.

The shearlet representation. Directional multiscale analysis of multivariate data

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Sparse representations of multidimensional data have gained increasingly more prominence during the last decade as a variety of applied problems, ranging from biomedical imaging through electronic surveillance and remote sensing, require to process complex multi-dimensional data in an accurate and effective manner. Among such systems, shearlets have been especially successfull as they provide optimally sparse representations for 2D and 3D cartoon-like images, a function class that is useful in many applied problems, offer unique microlocal properties that are useful for the analysis and detection of edges, and have fast algorithmic implementations. In this talk, I will recall the construction of shearlets, illustrate their sparse approximation properties, and discuss the application of the shearlet transform to the geometric characterization of singularities. These properties provide the theoretical underpinning for several state-of-the-art applications from signal processing and inverse problems, including data restoration, computed tomography and feature extraction. I will briefly illustrate some of these applications.

From Superresolution to Cat Videos: Filters, Ideals & SVD

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We start from a deconvolution problem, nowadays popular as "superresolution". Realizing that it can be seen as finding a filter with prescribed kernel, we arrive at an algebraic problem that in turn can be reduced to finding the kernel spaces of a certain sequence of Hankel matrices. This in turn requires the ability to iteratively compute the SVD of an augmented sequence of matrices which yields a nice application to cat videos, another important recent field of research.

Factorization of Dual Quaternion Polynomials. State of the Art and Applications to Computational Kinematics and Mechanism Science

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Quaternions and dual quaternions provide a convenient algebraic structure for modeling numerous geometric objects that obey certain "algebraic side relations". Prominent examples include PH-curves or important Lie groups such as SO(3) or SE(3). But quaternions or Clifford algebraic generalizations are also used in computational geometry, differential geometry, or other mathematical fields.

In our contribution, we summarize results from a joint research project with Josef Schicho (JKU Linz) on "Algebraic Methods in Kinematics" that are related to the decomposition of dual quaternion polynomials into linear factors. Denote by $\mathbb{DH}[t]$ the ring of quaternions in t with dual quaternion coefficients. Multiplication is defined by the convention that t commutes with all coefficients. If $C \in \mathbb{DH}[t]$ satisfies the Study condition $C\overline{C} \in \mathbb{R}[t]$ ("motion polynomial") it generically admit factorizations of the shape $C = (t - h_1)(t - h_2) \cdots (t - h_n)$ where h_1, h_2, $\ldots$, $h_n \in \mathbb{DH}$ and each linear factor describes a rotation about a fixed axis. Thus, factorizations correspond to decompositions of the rational motion described by C into products of rotations.

We present exact and approximate linkage constructions based on the combination of different factorizations as well as recent results on Kempe theorems in rational cases. They shed some new light on possibilities to mechanically "draw" rational objects but require to also consider non-generic motion polynomials whose factorization theory is not yet fully understood. Finally, we also address factorization of non-motion polynomials where we obtain a decomposition into vertical Darboux motions instead of rotations.

Throughout the presentation we will be able to confirm that Rida Farouki's statement on the "inseparability of algebra and geometry" holds true beyond its original context of PH curves.

An Inverse Eigenvector Problem with Application to Non-Uniform Subdivision Surfaces

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"One of the most important unsolved problems of linear algebra is the *inverse eigenvalue problem* for non-negative matrices: to determine necessary and sufficient conditions that a given n-tuple of complex numbers be the spectrum of an $n \times b$ matrix" [1]. We consider the related but less ambitious constructive problem of actually finding matrices that have some prescribed eigenvectors and eigenvalues. Generally, the desired matrices have a certain structure. For example, consider the matrix

$$M = \begin{bmatrix} a_1 & b_1 & c_1 & 0 \\ a_2 & b_2 & 0 & c_2 \\ a_3 & 0 & b_3 & c_3 \\ 0 & a_4 & b_4 & c_4 \end{bmatrix}.$$

We wish to find a_i, b_i, and c_i — all real and different — such that the eigenvalues of M are $\lambda_1 = 1,\ \lambda_2 = \lambda_3 = 1/2,\ \lambda_4 < 1/2$ and the eigenvectors are also given. If we approach this problem algebraically by studying the characteristic polynomial

$$p(t) = \begin{vmatrix} a_1 - t & b_1 & c_1 & 0 \\ a_2 & b_2 - t & 0 & c_2 \\ a_3 & 0 & b_3 - t & c_3 \\ 0 & a_4 & b_4 & c_4 - t \end{vmatrix} = (t-1)(t-1/2)^2(t-\lambda_4)$$

we must solve several non-linear equations in several variables, and it is challenging to find real solutions, or to assure that real solutions actually exist.

This kind of problem arises in constructing refinement rules for non-uniform subdivision surfaces around extraordinary points [2]. This presentation shows how to solve this problem using a simple geometric insight that avoids having to solve a system of non-linear equations, as would be required if approaching the problem algebraically, and illustrates the real-world application of this problem to non-uniform subdivision surfaces.

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Recent Advances in Isogeometric Analysis: Mortar Methods for NURBS Patch Coupling

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Isogeometric analysis fosters the integration of design and analysis by using the geometry description of the CAD system for the numerical solution of differential equations. The use of tensor-product NURBS surface patches provides great flexibility and is common for the design of surfaces. But it also entails the need for a coupling of nonconforming patches in the numerical analysis. The mortar finite element method allows the matching of different NURBS discretizations at the cross boundaries ("interfaces"), which requires neither additional variables nor empirical parameters. The matching is realized by enforcing a weak continuity condition which can be expressed as an orthogonality relation between the jump across the interface and a test space of splines which live on the interface.

In order to obtain the optimal approximation rate of the numerical method, we choose test spaces of splines with knots according to one of the neighboring patches. Spline spaces with a degree-reduction constraint near the interface vertices were proposed in [1]. We show that dual B-spline bases provide a good tool for the construction of these spaces, for the analysis of an "inf-sup" condition and the discretization of the interface constraints. Their disadvantage, however, relies in the fact that the discretized matching conditions extend over the full interface, hence, are not local.

As alternative test spaces we propose the span of approximate dual B-splines defined in [2]. They also satisfy the uniform inf-sup condition and obtain the optimal approximation rate. Moreover, they provide local discretizations of the patch coupling. Numerical examples are presented for the benchmark example of an elastic plate with hole. They include domain decompositions with T-sections and star-intersections.

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Contributed talks

Adaptive rational interpolation for point values

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Image zooming can be performed using the tensor product of one dimensional interpolatory techniques. Linear techniques, data independent, as the Lagrange polynomial interpolation, lead to fast and efficient algorithms. However, the quality of the zoomed image may not be satisfactory in the regions where the stencils used to construct the interpolatory function contains a singularity of the function. Increasing the order of the interpolatory technique does not solve the problem. In this case the number of stencils that cross the singularity grows, increasing the region of poor accuracy.

The use of non-linear interpolatory techniques, data-dependent, produces an improvement in the visual quality of the zoomed image. The interpolatory technique ENO (Essentially Non-Oscillatory) introduced by A. Harten et al. [2] performs a previous choice of the *stencil*, so that the interpolant is constructed, if it is possible, taking information from regions where the function is smooth.

The WENO (Weighted ENO) interpolatory technique introduced by Liu et al. [4] is an improvement of the ENO technique. This technique builds the interpolant using a convex combination of all the approximations computed with *stencils* containing the interval where we want obtain an approximation. The key lies in a wise assignment of weights to the convex combination. These weights must be chosen so that polynomials corresponding to singularity-crossing stencils should have a negligible contribution to the convex combination.

S. Carrato and G. Ramponi introduced in [1] a non linear rational interpolator of order two. In this talk we will present and analyze non linear rational interpolators of higher order. We will also show numerical experiments comparing the results obtained with bicubic interpolation, WENO interpolation and the rational interpolatory techniques in different applications: interpolating 1D signals and zooming images.

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Quasi-interpolation by C^1 quartic splines on a three-directional mesh

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In [3], the authors proposed a new scheme based on quartic C^1 splines on type-1 triangulations approximating regularly distributed. The novelty of the method (previously used in the quadratic bivariate case [1] and also in a three-dimensional case [2]) was that the quasi-interpolating splines were directly determined by setting the Bernstein–Bézier coefficients of the splines to appropriate combinations of the given data values instead of defining the approximating splines as linear combinations of compactly supported bivariate splines. The associated quasi-interpolation operator provided by the proposed scheme reproduces the cubic polynomials and the quasi-interpolating splines interpolate the values at the vertices of the triangulation.

We revisit this problem to obtain a family of quasi-interpolation operators that reproduce the space $\mathbb{P}_3$ of cubic polynomials in order to define an operator that minimizes the associated quasi-interpolation error. This is done by using interpolatory masks and minimizing the error for quartic monomials. We also consider the construction of near-best quasi-interpolation operators.

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More on C^1 Surfaces over Quadrilateral Meshes

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SMART2014 is at the origin of our book in print [1]. Our original results on C^1 Bézier surfaces over general quadrilateral meshes met then a strong echo in the IGA community. Thus we were encouraged to edit and publish this book, a first step in solving approximations of PDEs over complex engineering domains without surface or mesh reedition. In this spirit a recent breakthrough is that given a function defined on such a mesh, whose graph surface also has a B-spline representation, the resulting C^1 basis functions can explicitly be constructed along the edges while using higher order B-splines inside each patch, see [2].

A natural application of this paradigm is the numerical solution of fourth-order PDE problems such as the biharmonic equation, using the isogeometric method. In our presentation we shall try to answer the following questions: how can we exploit in real life problems these constructions and can it be implemented easily?

To give a partial answer to the first question we propose to use the ideas of Babuska and Vogelius [4] on Geometric Domain Reduction. By considering (quasi)-cylindrical domains (in CAD often referred to 2.5 dimensional domains), we can reduce the study of such 3D solids to a collection of 2-D problems by the use of a projection onto a space of polynomials defined over the "thickness". This approach leads to the natural construction of plate models. It can also be sued for the construction of flows of limited thickness and generalize Helle-Shaw equations [3], (injection between two plates), and lubrification, while keeping the structure nearly $2D$. Hence the IgA method can be extended to simulations that are defined on 2.5 domains and still retain their $3D$ details.

For the second question we shall review the possibility of NOT constructing the above mentioned basis, but rather use Lagrange multipliers type methods.

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Subdivision of Point Distributions

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The subdivision procedure is usually defined on a mesh. This is theoretically equivalent to the alternative definition over a partition of the parametric domain which allows to consider nonuniform measures on the elements of the partition in defining the refinement schemes. The motivation for such a generalization is that in practice the surfaces (or the curves, or the volumes) to be represented are often known only by a point cloud. In it, the distribution of the points is not exactly uniform and in several instances there are even "holes" with no points. The latter is an issue that is not easy to resolve by the standard subdivision schemes.

In this talk the subdivision is considered as a two-step procedure. The first one aims at establishing the *measure* on the refined elements of the partition. It is then used in the second step for the calculation of the refinement scheme and the *positions* of the refined elements. As usual, the refinement scheme for the positions is based on polynomial reproduction and the only difference with the standard theory is the replacement of the Lebesgue measure with the one found in the first step.

The local mass and the first few local moments are quantities used for the subdivision of the point distributions in the first step. The scheme is defined to preserve certain test measures in replacement of the standard polynomial reproduction property. Some basic schemes on quadrilaterals, triangles, and intervals will be presented.

The Computation of the Degree and Coefficients of Two Bivariate Polynomials in the Bernstein Basis

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Parametric Bézier curves and surface patches are defined in the Bernstein basis and their intersections can be computed algebraically. Typically, the intersection algorithms reduce to the computation of the roots of univariate or bivariate polynomials. Classical root finding methods such as Newton's method fail to accurately compute polynomial roots of high multiplicity, but these roots are associated with smooth curve and surface intersections. These roots can, however, be computed from a square-free factorisation of the given polynomial, and this leads naturally to a sequence of greatest common divisor (GCD) computations.

Methods for the computation of the GCD are well defined for polynomials in the power basis, but significantly less work has been done on equivalent methods for polynomials in the Bernstein basis. This paper will consider the computation of the GCD of two arbitrary bivariate polynomials in Bernstein form, using a structure-preserving matrix method. This is an extension of the structure-preserving matrix method used to compute the GCD of two univariate polynomials in the Bernstein basis, but the structures of the matrices are more involved and additional non-trivial theoretical issues must be addressed.

Sylvester matrices of bivariate polynomials, expressed over a triangular domain and in the tensor-product form, in the Bernstein basis, are developed and it is shown they are significantly more complex than their equivalents in the power basis. The Sylvester matrices lose their partitioned Toeplitz structure, and the combinatorial terms associated with the coefficients of polynomials in Bernstein form cause entries of the matrices to span several orders of magnitude, which can cause numerical problems. These problems are more severe for bivariate Benrstein polynomials than univariate Bernstein polynomials because the combinatorial terms are either larger, or more combinatorial terms must be considered. The effects of this can be mitigated by preprocessing the polynomials and considering alternative forms of the Sylvester matrices and the sequence of Sylvester subresultant matrices. The inclusion of these preprocessing operations yields improved results for the degree and coefficients of the GCD. The entries of the kth Sylvester subresultant matrices are dependent on k, but a matrix transformation is described such that the kth Sylvester subresultant matrix can be obtained from the $(k-j)$th, which allows the sequence of subresultant matrices to be constructed efficiently.

The presentation includes a discussion of the theoretical developments and the results of computational experiments.

Adaptivity with THB-splines in isogeometric analysis

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Hierarchical spline spaces are a major tool in the field of local refinement techniques for isogeometric analysis. In particular, a fully automatic adaptive isogeometric method (AIGM) based on THB-splines [3] was recently developed and analyzed [1, 2]. The theoretical framework exploits the properties of the truncated basis of hierarchical spline spaces, together with admissible mesh configurations, in order to prove the efficiency and reliability of a simple residual-based error indicator, as well as the optimality of the AIGM.

We present the analysis of the behaviour of the AIGM on a selection of numerical examples, which confirm the effectivity of the estimator and the optimal convergence of the method. The results also show the influence of meshes characterized by different classes of admissibility. The tests were performed by integrating the adaptive scheme in the hierarchical version of the Matlab/Octave package GeoPDEs presented in [4].

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Refinable functions and Iterated Function Systems

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In this poster we present some links between Invariant measures of Iterated Function Systems and refinable functions [1]; we focus on existence and regularity of these objects.
The importance in applications of real functions solving a refinement equation has led many authors to study their existence and properties. A partial list of these applications covers fractal interpolation, subdivision in Computer-Aided Geometric Design (CAGD) and attractors of Iterated Functions Systems (IFS). In many of these applications only the existence of an integral operator is required so that L^1 regularity of refinable functions has been explored widely, moreover the existence of refinable distributions has been introduced.
In parallel, the study of Iterated Function Systems considers *invariant measures* satisfying a balance equation that is easily seen to be the dual of a functional refinement condition: the density of an IFS measure is a refinable function. The scope of this poster is to outline the translation language between the two approaches, and to expose the main results that can be useful to carry from one side to the other.
Finally, few significant examples are proposed, taken from [1, 2, 3].

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PUM interpolation with variable subdomains and anisotropic radial kernels

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The Radial Basis Function Partition of Unity Method (RBF-PUM) is an efficient and accurate computational technique used to solve big interpolation problems [4]. The basic idea of PUM consists of decomposing the domain into several subdomains forming a covering of the original domain, then constructing a local RBF interpolant on each subdomain. When the data distribution is quite uniform, the interpolation scheme can effectively work with subdomains of hyperspherical shape and each having the same size [1]. If data are instead very irregularly distributed, a method that allows to automatically select variable subdomain sizes and optimal RBF shape parameters is essential [2]. In some particular cases, however, for instance to deal with *track data*, the choice of using hyperspherical PU subdomains and isotropic RBFs is not completely suitable. For this reason, in this work we propose a new RBF-PUM interpolation scheme that is based on the use of variable subdomains of elliptical shape and anisotropic radial kernels [3]. This method is also developed to detect optimal values of shape parameters by using a cross-validation technique.

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Smoothness of anisotropic wavelet frames and subdivision

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We present a detailed regularity analysis of anisotropic wavelet frames and subdivision. In the univariate setting, the smoothness of wavelet frames and subdivision is well understood by means of the matrix (joint spectral radius) approach. In the multivariate setting, this approach has been extended only to the special case of isotropic refinement with a dilation matrix all of whose eigenvalues are equal in the absolute value. The general anisotropic case has resisted to be fully understood: the matrix approach can determine whether a refinable function is continuous, but its Hölder and higher regularity remained mysteriously unattainable.

We show first how to compute the exact Hölder regularity of a refinable function. In the anisotropic case, our expression for its exact Hölder exponent reflects the impact of the variable moduli of the eigenvalues of the corresponding dilation matrix. In the isotropic case, our results reduce to the well-known facts from the literature. We present an efficient algorithm for determining the finite set of the restricted transition matrices whose spectral properties characterize the Hölder exponent of the corresponding refinable function. We next analyze the higher regularity of refinable functions and illustrate our results with several examples.

Bounding the Lebesgue constant for a rational Hermite interpolant at equidistant nodes

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It is well known that for the interpolation of a function at equidistant points, the family of rational interpolants introduced by Floater and Hormann compares favourably with classical polynomials. Indeed, recent results show that in this setting, the Lebesgue constant associated to these interpolants grows logarithmically, in contrast to the exponential growth experienced by polynomials. A similar behaviour occurs for polynomial interpolants in the case of Hermite interpolation, where also the first derivatives of the interpolant are prescribed at the nodes. In this talk we show how to extend the construction of Floater and Hormann to the Hermite setting and study the growth of the Lebesgue constant for this rational interpolant, which turns out to be bounded from above by a constant for any number of interpolation nodes. Our numerical examples not only confirm this remarkable result, but also show that the Lebesgue function is equal to 1, except close to the end points of the interpolation interval, where it behaves like 2^{2d}, with d being the degree of the locally interpolating polynomials in the construction of Floater and Hormann.

Three-point ternary Hermite subdivision scheme with C^2 regularity

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In this talk we show how to construct and interpolatory Hermite subdivision schemes of small supports and high regularity by increasing the arity of the scheme. It turns out that, leaving the binary domain, it is indeed possible to get ternary interpolatory Hermite subdivision schemes with higher regularity than their binary counterpart. Indeed, we are able to construct a 3-point ternary interpolatory Hermite schemes with C^2 regularity. To get it we extend to the Hermite situation a geometric construction that allow us to reinterpret the definition of the ternary interpolatory scalar 3-point subdivision scheme presented in [1]. The regularity of the vector limit function is proved with a Taylor factorization of an extended scheme adaptation to the ternary case of the results in [2].

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Rational RBF stable interpolation for large data sets

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Rational polynomial interpolation is particularly suitable for approximating functions that oscillate or with steep gradients. However, they are not easy to implement for complex shapes of the domain and their extension to high dimensions is still a challenging problem.
Following the idea presented in [3], we consider a rational Radial Basis Function (RBF) interpolation technique. Such rational RBF approximation however might suffer from instability due to the ill-conditioning of the interpolation matrix [1, 2]. Therefore, we here develop a stable computational technique of the RBF interpolants, based on the so-called Variably Scaled Kernels (VSKs). The study reveals that the method is robust enough to accurately fit data coming from applications, such as Earth's topography. Moreover, when Compactly Supported RBFs (CSRBFs) are used, it enables us to increase the sparsity of the kernel matrices and at the same time to maintain a good accuracy. Furthermore, since a global interpolation method cannot handle truly large sets of points, an efficient implementation via the Partition of Unity (PU) method is carried out as suggested in [4].

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Two Interpolation Properties of Dual 2n-point Subdivision Scheme

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This paper introduces two interpolation properties of dual 2n-point subdivision schemes when n tends to infinity. One points out the limit curves of dual 2n-point subdivision schemes interpolate all their initial control vertices and newly inserted vertices. The other indicates that when the initial control points form an equilateral polygon, the resulting limit curve of dual 2n-point subdivision scheme approaches a circle as n tends to infinity. To this end, we construct the recursive relation for the generating function of dual 2n-point subdivision schemes and then design repeated local operations for the dual 2n-point subdivision schemes. Some numerical examples illustrate the validity of our theoretic analysis.

Non-linear Blending of Linear Subdivision Schemes

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In recent years, several nonlinear subdivision schemes have been proposed which can be expressed as a nonlinear blending between linear interpolatory subdivision schemes [2, 3]. The purpose of this blending is usually to achieve certain shape preservation properties, which linear schemes do not have, while ensuring that the approximation properties of the new scheme are as high as possible.

In [2, 3], the nonlinear schemes proposed can be written as a nonlinear perturbation of the basic 2-point interpolatory subdivision scheme, and their shape preservation properties are directly related to the design principles of the nonlinear blending. The convergence of the new schemes is usually fairly straightforward. On the other hand, their stability (Lipschitz stability) tends to be harder to analyze. In [2, 3], stability is studied using a novel tool stated in [4] that relies on a generalized differential calculus for the nonlinear subdivision rules that define the new scheme.

In this work we propose a nonlinear blending between the basic three-cell linear subdivision scheme [5] and Chaikin's subdivision scheme. The new scheme seeks to improve the performance of the nonlinear version of the three-cell scheme proposed in [1], which can also be written as a nonlinear perturbation of the cell-replicating scheme (which does not converge in the classical sense). By blending with Chaikin's scheme, we can prove convergence of the new nonlinear scheme in the clasical sense. Stability is also analyzed using the generalized differential.

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High smoothness of non-linear Lane Riesenfeld algorithms

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On this talk we consider non-linear Lane Riesenfeld algorithms, refining numbers, obtained from the linear ones by replacing the (linear) arithmetic means by non-linear averaging rules. In a previous work we proved that one can use almost any smooth averaging rule at any step of the algorithm, and obtain C^1 limits, if the scheme converges, and if the corresponding linear scheme is C^1.

In this talk higher smoothness of these schemes is discussed, and the smoothness derived in case of a converging scheme equals the smoothness of the corresponding linear scheme, under certain conditions on the averaging rules. The main tools of analysis are tools for linear non-uniform schemes, presented in a previous work. This analysis imposes stricter conditions on the changes between "neighborin" averaging rules than needed, as demonstrated by our numerical experiments.

Efficient quadrature rules for applications in IGA-BEMs based on spline quasi-interpolation

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When dealing with Isogeometric Boundary Element Methods (IGA-BEMs), one of the main issues is to compute efficiently the singular integrals arising from the boundary integral formulation. Therefore, the development of new ad hoc quadrature schemes suitable to tackle singular kernels is of great interest. In this talk we present a new approach based on a *quasi-interpolation* (QI) scheme. The idea behind the method was introduced in [3] for regular integrals. Starting from a base-case, when the integral I consists of a function f and a singular kernel K, we first approximate f with a suitable spline quasi-interpolant σ_f, obtained from a variant of the Hermite discrete QI spline introduced in [2]. The integral I is then evaluated by computing the so called *modified moments* for B-splines, already introduced in [1].
When the integrand is given by a product of an arbitrary function f, a B-spline B and a kernel K, we can use two approaches. In the first one, the whole product fB is approximated as in the base-case. In the second one, only f is approximated with σ_f and then, the product between σ_f and B is expressed in B-form by using spline product formulas (cf.[4]). At this stage, this is a base-case and it can be handled as described above.
We show the achieved accuracy of the suggested methods with several examples for weakly and strongly singular kernels arising in IGA-BEMs. The new schemes are versatile, as the quasi interpolant spline allows to choose any degree keeping low the computational costs.

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Optimal spline spaces for L2 n-widths

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Recently there has been renewed interest in using splines of maximal smoothness, i.e. smoothness C^{d-1} for splines of degree d, as finite elements for solving PDEs. This is one of the main ideas behind isogeometric analysis. This raises the issue of how good these splines are at approximating functions of a certain smoothness class, especially with respect to approximation in the L^2 norm.

In this talk we study various classes of functions in $H^r(0,1)$ and show that they admit optimal spline subspaces of arbitrarily high degree $d \geq r-1$.

On the Convergence of One-Parameter 2n-Point Interpolatory Subdivision Schemes

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Parameter intervals of convergence with respect to $2n$-point interpolatory subdivision schemes have been established in the literature, especially in the $n = 2$ and $n = 3$ cases. We present here a method based on the computation of zero of a certain polynomial to obtain a useful parameter convergence interval for general integer values of n.

Box-spline isogeometric collocation methods: spectral analysis

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Isogeometric Analysis (IgA) is a well established paradigm for the numerical treatment of PDEs [1]. In its original formulation, IgA is based on tensor-product B-splines and their rational version NURBS (Non-Uniform Rational B-Splines). However, B-splines/NURBS are not a requisite ingredient of IgA. Various alternative dicretization spaces, ranging from generalized splines to subdivision surfaces, have been considered in the frame of the isogeometric approach. In particular, box-splines on three-directional meshes offer some interesting advantages in IgA mainly due to the regularity of their structure and to the flexibility of their support [1, 3].

Collocation methods have been extensively used in IgA. Their major advantage over Galerkin-type methods is the minimal computational effort with respect to quadrature. This property leads to extremely easy and fast constructions of the corresponding linear systems.

In this talk we present some preliminary results on isogeometric collocation methods based on box-splines on three-directional meshes. In particular, we focus on the (strong form) of boundary conditions and on the spectral properties of the resulting collocation matrices.

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On the ellipses/ellipsoids separation problem

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The problem of detecting when two moving ellipsoids (or ellipses) overlap is of interest to robotics, CAD/CAM, computer animation, etc., where ellipsoids (and ellipses) are often used for modelling (and/or enclosing) the shape of the objects under consideration (see for example the references in [2]). By analysing symbolically the sign of the real roots of the characteristic polynomial of the pencil defined by two ellipsoids in the space (or two ellipses in the plane) $\mathcal{A}$ and $\mathcal{B}$ given by $X^T AX = 0$ and $X^T BX = 0$ we derive a new closed formulae characterising when $\mathcal{A}$ and $\mathcal{B}$ overlap, are separate and touch each other externally (see [1, 3, 4] for similar approaches). These conditions are defined by a minimal set of polynomial inequalities depending only on the entries of A and B, need only to compute the characteristic polynomial of the pencil defined by A and B, $f(\lambda) = \det(\lambda A + B)$, and do not require the computation of the intersection points between them. Compared with the best available approach dealing with this problem (see [3]), the formulae here presented is more efficient since it involves a smaller set of polynomials.

This characterisation provides a new approach for exact collision detection of two moving ellipsoids (or ellipses) since the analysis of the univariate polynomials (depending on the time) in the previously mentioned formulae provides the collision events between them (see [2, 3]).

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Cubic B-splines on Powell–Sabin triangulations

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A Powell–Sabin triangulation splits every triangle of the initial triangulation into six smaller triangles. It is used to simplify the definition and analysis of spline spaces, which in general depend on the geometry of the underlying triangulation. The construction of splines on Powell–Sabin triangulations is well-studied, not only for the standard degree 2 (see [4]), but also for higher degrees and orders of smoothness (see e.g. [5]). Recently, C^1 cubic splines with and without additional smoothness constraints have been considered for possible use in approximation theory and geometric modelling (see [2, 3, 6]).

In this talk, we present a new B-spline representation (studied in [1]) for the space of C^1 cubic splines. The construction is based on lifting particular triangles and line segments from the domain. The resulting B-splines form a locally supported stable basis and a convex partition of unity. The coefficients of any cubic spline represented in this form can be neatly expressed by the means of the blossom, and the Bernstein–Bézier form of such a spline can be computed in a stable way. Finally, this new B-form allows a representation of classical C^1 quadratic Powell–Sabin splines and C^1 cubic Clough–Tocher splines in a unified context.

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Adaptive Spline Technologies for Aircraft Engine Design

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Industrial products are usually designed within Computer Aided Engineering (CAE) systems based on the B-spline technology and its non-uniform rational extension (NURBS). To overcome the limitations of their tensor-product structure, we invested in the industrial integration of recently developed generalizations: The truncated hierarchical B-splines (THB-splines) and the patchwork B-splines (PB-splines). The talk will introduce the use of THB-splines and PB-splines within the multi-disciplinary design of aircraft engine components at MTU Aero Engines AG:

First, for an adaptive surface fitting framework to reconstruct CAD models from (optical) measured point data where they lead to significant improvements with respect to the quality of the resulting geometric shape compared to the existing tensor-product spline technology. This enables us to transfer automatically the shapes of manufactured and operated parts back into the CAE systems.

Second, we use the adaptive technology for the simulation-based deformation of CAD models describing the shape of the mechanical part of interest. Therefore, the displacement field, as the result of a finite element simulation, is approximated in a first step by an adaptive trivariate spline function. The resulting function is then used to deform the related CAD model with the help of the Parasolid® modeling kernel. This framework solves the fundamental engine design problem of optimizing the parts for performing in hot working conditions while a cold, unloaded CAD model is required for manufacturing.

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Projective Equivalence Detection of Rational Surfaces

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It is known, that proper parameterizations of rational curves in reduced form are unique up to bilinear reparameterizations, i.e., linear transformations of the domain given by the projective line. Recently, this was used to detect equivalences and symmetries of rational curves with respect to various groups of transformations (projective, affine, similarities, Euclidean) that form an equivalence relation, see e.g. [1, 2].

We generalize this result to surfaces, i.e., we investigate surfaces given by a rational parameterization. We show that a birational base-point free parameterization of a surface is unique up to linear transformations of the domain and we use this insight to find all projective equivalences between two given input surfaces. In particular, we formulate a polynomial system of equations, whose solutions specify the projective equivalences, i.e., the projective transformations and the linear reparameterizations associated with them. We use basic linear algebra to reduce the number of unknowns of this system before solving it.

Furthermore, we investigate how this system simplifies for the special case of affine equivalences for polynomial surfaces and how we can use our method to detect projective symmetries of surfaces. Moreover, the method provides an alternative approach to [3, 4] for classifying quadratic surfaces. Our experimental examples show that for the general case the assumptions on the surfaces are fulfilled, i.e., for randomly chosen coefficients the surfaces are birational and base-point free.

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Duality for frames and multivariate wavelet frame construction

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The duality principle, ultimately a statement about adjoint operators, is a universal principle in frame theory. We take a broad perspective on the duality principle and discuss how the mixed unitary extension principle for MRA-wavelet frames can be viewed as the duality principle in a sequence space. This leads to a construction scheme for dual MRA-wavelet frames which is strikingly simple in the sense that it only requires the completion of an invertible constant matrix. Under minimal conditions on the multiresolution analysis our construction guarantees the existence and easy constructability of multivariate non-tensor product dual MRA-wavelet frames of compactly supported wavelets. These can for instance be of interest for multiscale representations of surfaces.

Subdivision algorithms on Riemannian manifolds

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In this talk we study subdivision rules which produce limit curves by refining discrete data. In particular, we are interested in convergence results of refinement schemes on Riemannian manifolds.

One approach to obtain convergence statements is to use so-called proximity conditions to receive results for 'dense enough' input data. For certain classes of subdivision schemes and/or special kinds of manifolds convergence results could be proven which apply to all input data. We extend one of these results to subdivision schemes without sign restrictions on Cartan-Hadamard manifolds.

We show how to extend linear subdivision rules to Cartan-Hadamard manifolds using the Riemannian center of mass. To obtain a convergence result we prove a contractivity condition which only depends on the mask of the scheme.

Mesh Quadrangulation via L_p Compressed Modes Surface Partitioning

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Quad meshes, i.e. meshes made entirely of quadrilaterals, have been widely used for many years in CAD and simulation, because a number of tasks are better suited to quad meshes, especially semi-regular ones, than to triangle meshes. This work focuses on a quad mesh generation approach based on a preliminary 0-genus surface partitioning [1]. In particular, we propose the following two steps:

1. Surface partitioning:
   In this step, a 0-genus patch-based partition of the boundary of an object (surface) is obtained via L_p Compressed Modes. This novel $L^2(\Omega)$ orthonormal basis represents the salient parts of the object using sparser and more naturally localized functions w.r.t the eigenfunctions of the Laplace Beltrami operator (Manifold Harmonics). We enforce the original variational model proposed in [2] by introducing an L_p penalization term, with $0 < p < 1$, which allows for a better control over the accuracy of the shape approximation.

2. Quad mesh evolution:
   In the second step, we use Lagrangian surface evolution with tangential redistribution in order to create the quadrangulation of the given surface. Starting with the 0-genus partitioning from the previous step, initial quad surfaces matching the partition boundaries are created and then evolved towards the object's boundary. Lagrangian-type evolution is supported by an ad-hoc tangential redistribution which helps to preserve good properties of quad elements in the quadrangulation.

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On computing the mock-Chebyshev interpolation

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There is a famous example known as Runge Phenomenon published by C. Runge in 1901: polynomial interpolation of a function f, using equidistant interpolation points on $[-1, 1]$, could diverge on parts of this interval even if f is analytic everywhere on the interval. Among all techniques that have been proposed to defeat this phenomenon in the literature of approximation theory, there is the mock-Chebyshev interpolation on a grid: a subset of $(n + 1)$ points from an equispaced grid with $O(n^2)$ points chosen to mimic the non-uniform $n + 1$-point Chebyshev-Lobatto grid.

We present a new algorithm for computing the mock-Chebysev nodes using distance between every two consecutive points. The complexity of the algorithm is $O(n)$, where $n+1$ is the number of the Chebyshev nodes on the interval $[-1, 1]$. We also discuss bivariate generalization of the mock-Chebyshev nodes to the Padua interpolation points in $[-1, 1]^2$. Numerical results are also provided.

Non-stationary Subdivision Schemes with Locally Different Tension Parameters

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In this talk, we introduce a class of non-stationary subdivision schemes which can locally control the shape of the limit curve. We set a locally different tension parameter at each edge of the initial control polygon, which forces the proposed schemes to be non-uniform. After each step of refinement, the tension parameters are updated in a way that can reproduce and blend locally different analytic curves. We analyze the convergence and smoothness of our non-uniform schemes by examining their asymptotic properties. Numerical examples are presented in order to illustrate the advantages of the proposed scheme in geometric modeling.

An isogeometric immersed model with application to linear elasticity and THBox-splines

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An accurate representation of computational domains in numerical PDE simulation is one of the key features in the design of isogeometric methods. The use of an immersed boundary approach in isogeometric analysis allows us to construct complicated single patch domains by suitably combining trimming with geometry mappings.

In this talk we present an isogeometric immersed model to solve linear elasticity problems. The model does not need additional degrees of freedom in the final system of equations and the corresponding matrix is symmetric. The method is free of user defined penalties and stabilization parameters. Flexible and adaptive geometry representations are achieved by employing hierarchically nested splines spaces. In particular, we focus on truncated hierarchical box splines (THBox-splines) defined over regular triangulations. Several numerical examples demonstrate the optimal convergence of the adaptive scheme.

Accelerated Guided Refinement for Modeling and Analysis

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We present a new approach for constructing high quality multi-sided caps, that fill the holes in B-spline surfaces and are suitable for isogeometric analysis. The approach leverages two ingredients. First, a guide surface is created that expresses the design intent for the multi-sided surface region. This guide does not need to match the boundary data of the multi-sided hole but serves to efficiently create the contracting rings of a cap in the manner of subdivision surfaces. The second ingredient is *accelerated* subdivision: three guided rings shrink the multi-sided region leaving a gap of a size that Catmull-Clark subdivision produces only after 8 refinement steps. The construction of the accelerated contracting rings is based on the characteristic map of [1].

However, our goal is not to create subdivision surfaces. The remaining microscopic gap is filled by a simple, yet high quality, finite cap. The layers of accelerated guided rings are C^2 connected tensor-product pieces, hence refine as in the regular case including the T-junctions. This enables adaptive analysis functions on the surfaces.

The approach allows for constructions of degree bi-6, bi-5, bi-4, bi-3 to adjust continuity and quality levels.

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Overlapping Multi-Patch Structures in IGA

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Isogeometric analysis (IGA) is a novel computational approach, which connects the technology of computer aided design (CAD) with numerical simulation via finite element analysis. In IGA, the same basis functions (typically tensor-product B-splines) are used for the describing the geometry and for the numerical analysis.

Several tensor-product spline patches are required in order to represent more complicated domains. The number of patches, which is required to represent a domain, can be kept low by using *trimmed* patches. These patches, however, cause difficulties for the numerical simulation, in particular related to numerical integration and imposition of boundary conditions. To avoid these difficulties, we will explore the use of overlapping spline parameterizations for IGA.

As an example we consider the Poisson problem on a two-patch domain, which is represented by two overlapping patches. Consequently, the problem is divided into two sub-problems, which are coupled appropriately, using an interpolation approach. While this approach is related to the iterative *Additive Schwarz Domain Decomposition Method (ASDDM)* (see e.g., [1]), the resulting system can also be solved directly in a single step. We will show that the use of overlapping patches helps to avoid trimming and significantly simplifies the domain parametrization problem.

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Subdivision Surfaces and Clough-Tocher Splines in Isogeometric Analysis

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The powerful framework of Isogeometric Analysis [1] facilitates the exchange of data between tools used for geometric design (CAD) and for numerical analysis. The use of B-splines and NURBS not only for modeling but also for analysis offers numerous advantages over traditional finite element functions. However, surfaces of arbitrary manifold topology still pose some problems due to either trimming and stitching and/or the presence of extraordinary points where other then the regular number of patches meet.

In this presentation, we will explore methods for converting a CAD model to analysis-suitable representations based on Clough-Tocher splines [2] and subdivision surfaces [3, 4], including efficient quadrature rules for these types of splines [5].

This presentation will be largely based on join work with Pieter Barendrecht, Michael Bartoň, Tom Cashman, Neil Dodgson, Malcolm Sabin, and Jingjing Shen [2–5].

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Interpolation with spatial rational Pythagorean–hodograph curves of degree 6

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In the talk a construction of rational Pythagorean–hodograph curves of degree 6 is presented, which is based on a polynomial dual representation of degree 4 derived from a particular quadratic polynomial with quaternion coefficients. It is shown that each such curve depends on twelve free parameters and has a piecewise rational arc–length function. Geometric interpolation of two data points and two tangent directions is considered and it is shown that the nonlinear part of the problem reduces to a quadratic equation that includes two additional shape parameters. Two branches of two–parametric family of G^1 interpolants are given in a closed form and regions of shape parameters are revealed that imply the interpolants to be regular and admissible. It is further shown that one of the shape parameters determines the length of the interpolant and visa versa. Therefore, the interpolation problem is supplemented by prescribing also the length of the interpolant, and it is proven that the solution exists for any nonplanar G^1 data and any prescribed length greater than the difference between the interpolation points. The presented scheme can be used to construct a G^1 Hermite interpolation spline with a prescribed length. The numerical examples are shown that illustrate the derived theoretical results.

Multi-resolution Analysis Using Non Interpolant, Non Translation Invariant, Non Linear Subdivision Schemes

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This paper is devoted to the design of complete multi-resolution frameworks consistent with general subdivision schemes. By general we mean non stationary, non homogeneous or non linear and by consistent we mean stable and numerically efficient. This work follows a first step completed for linear schemes presented in [1] and a second step devoted to some non linear cases ([2]). Here we will describe the properties and the efficiency of the designed multi-resolution frameworks on various aspects.

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Modification of the Circle Average for Improving the Refinement of Point-Normal Pairs in 2D

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Subdivision is a well-known and established method for generating smooth curves/ surfaces from discrete data by repeated refinements. The typical input for such a process is a mesh of vertices. In this talk we discuss the refinement of 2D data consisting of vertices of a polygon and a normal at each vertex. In our previous work, we replaced all the linear averages with circle averages in two subdivision schemes: the Lane-Riesenfeld algorithm and the 4-point scheme, expressed in terms of repeated binary averages. We proved that these modified schemes converge to C^1 curves. Yet the curve of normals to the limit curve is not equal to the curve of the limit of the normals, a property which is expected intuitively by designers. In this talk we propose a modification of the circle average, and show by experimental results that this expected property holds.

Simplex–Splines on the Clough–Tocher split

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We consider a simplex spline basis on the Clough-Tocher split where a triangle is divided into three subtriangles by connecting the vertices to the barycenter. We show that this basis has the following B-spline properties: differentiation formula, stable recurrence relation, knot insertion formula, nonnegative partition of unity, Marsden identity with dual polynomials with linear factors, explicit dual functionals, L_∞ stability, simple conditions for C^1 joins to neighboring triangles, and well conditioned collocation matrices for Lagrange and Hermite interpolation using certain sites.

Analysis of shape preserving representations for circle approximations

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design in other spaces than P_n, the space of polynomials of degree at most n, arises commonly in Computer-Aided Geometric Design. In addition to the rational Bézier model, spaces with exponential polynomials or including trigonometric functions can be used to represent remarkable curves and surfaces. For instance, the circle can be exactly parameterized with respect to the arc length parameter in cycloidal spaces or approximated in suitable subspaces of polynomials. In any case, it is important to ensure the existence of shape preserving representations and to construct proper basis associated to them. We also analyze the behaviour of Greville abscissae in the proposed spaces.

Spline quasi-interpolation schemes based on Hermite-Obrechkoff linear one-step methods for ODEs

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The BS linear multistep methods have been introduced in [3-4] to solve general first order boundary value problems, and similarly, the BS2 methods have been proposed in [1] to address second order semi–linear problems. Both BS and BS2 methods are based on B–Splines with non coincident internal knot, it is shown that they have nice convergence and stability features. Moreover, these methods have the peculiar property of admitting an interesting spline extension. Indeed, the numerical solution they produce can be seen as the values at the mesh points of a spline which collocates the differential equation at the given grid. This property allows us to give a dual interpretation of these schemes: both BS and BS2 methods can be also applied in the context of discrete spline quasi–interpolation, requiring first order Hermite [2] or lacunary second order data, respectively.

In this talk we will present an extension of the BS, BS2 methods, which requires high order Hermite data and is based on B-splines with multiple knots. In particular we will focus on a family of one step methods, that lie in the class of Hermite-Obrechkoff methods and are related to the Euler–Maclaurin formulas. Their dual interpretation in the quasi-interpolation context will be also introduced [5,6].

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Regularity of Non-stationary Subdivision schemes: Matrix approach

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Subdivision schemes are a method to generate smooth curves and surfaces from given control data. Convergent subdivision schemes iteratively refine a sequence of points and eventually converge to a continuous limit function. In non-stationary subdivision schemes every refinement step is based on a different uniformly bounded refinement mask, where the anisotropic dilation matrix is the same for all subdivision operators.

In this talk, we show how a finite set of square matrices, derived from the refinement masks, characterizes the Hölder regularity of the limit functions of the underlying non-stationary subdivision scheme as done in [1] for stationary subdivision schemes. We further show how to define the square matrices, proof the existence of a common invariant subspace of the square matrices, and present an algorithm, derived in [2], which may determine the regularity of the limit functions in reasonable time.

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Approximation of Set-Valued Functions with Compact Images in $\mathbb{R}^n$ based on the Metric Linear Combination

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The talk presents results on the approximation of set-valued functions (multifunction), functions mapping the points of a closed real interval to general compact sets in $\mathbb{R}^n$. The research is motivated by the problem of the reconstruction of 3D objects or 2D shapes from their parallel cross-sections. Other possible applications are in Control Theory.

The approach is to adapt approximation methods for real-valued functions to set-valued functions, by replacing operations between numbers by operations between sets. For multifunctions with compact convex images, adaptation based on Minkowski convex combinations of sets yield approximating operators in the Hausdorff metric. Yet, if the images of set-valued functions are not necessarily convex, then the approximations methods may fail. Spline subdivision schemes, based on Minkowski convex combinations, converge to convex-valued multifunctions from any initial sets.

To avoid covexification we adapt approximation methods for set-valued functions with general compact images, using metric linear combinations of sets. This adaptation method is not restricted to positive operators. We regard set-valued analogues of the Bernstein polynomial operator and the Schoenberg spline operator. We give error estimates with respect to Hausdorf distance in terms of the regularity properties of the approximated set-valued function. We also present a new notion of set-valued integral using metric linear combinations of sets. The new integral is termed the metric integral. We use the metric integral to smooth a multifunction by defining its metric Steklov set-valued function. The error is measured by the averaged Hausdorf distance.

Efficient preconditioning strategy for Stokes system in Isogeometric Analysis

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The efficient solution of systems arising from the discretization of a PDE employing high degree and high regularity B-splines has always been a challenging task in Isogeometric analysis. In this talk we focus the attention on efficient and fast methods to solve steady-state Stokes equations. We want to find preconditioning strategies that can reduce computational solving time. According to [1], the main difficulties in achieving this task are the way of approximation of the stiffness block and of the Schur complement block. In order to accomplish this task and inspired by the ideas and good results presented in [2], we develop a strategy that exploits the tensor product hidden behind isogeometric discretization and that is based on the solution of a Sylvester-like equation at each step. We consider as discrete space the one based on Isogeometric Taylor-Hood elements and we build a preconditioning method that is robust with respect to spline degree, i.e. it is p scalable. The greatest difficulty that we have to deal with is the inclusion of the geometry information in the preconditioning matrices. We partially solved this problem by performing a separation of variable based procedure that allows to incorporate some data regarding the geometry in our strategy. By presenting a variety of examples, we highlight the good properties and the open questions as well as possible ways to continue this work.

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A Collocation Method in Refinable Spaces for the Solution of Fractional Dynamical Systems

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For dynamical system we mean a particle, or set of particles, whose state changes in the time. The dynamics of the system is described by a set of differential equations.When the involved derivatives are of non integer order, we get a fractional dynamical system [2].
In this work we consider a fractional dynamical system of the following type:

$$\begin{cases} D^\gamma X(t) = A\,X(t) \quad 0 < \gamma < 1,\, t \geq t_0 \\ X(t_0) = X_0 \end{cases}$$

where D^γ denotes the Caputo fractional derivative [1]. We numerically solve the system by a collocation method that uses approximating spaces belonging to multiresolution analysis generated by a class of fractional refinable functions [3]. Some numerical results on several test problems are shown.

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Multivariate Haar functions and one problem of the automata theory

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Multivariate Haar functions on $\mathbf{R}^d$ can be constructed with an arbitrary dilation integer expansive $d \times d$ matrix M and arbitrary set of $m = |\det M|$ "digits" from the corresponding quotient set. They are generated by refinable functions that are indicator functions of special self-similar sets. In contrast to the univariate case, multivariate Haar functions can have various Hölder exponents in L_2, their computation may be a hard problem especially for anisotropic matrices M.

A formula for the Hölder exponent in case of general dilation matrices was presented in [1]. We show that the Hölder exponent can be expressed by the boundary Minkowski dimension of the self-similar set. Moreover, the same value has an interpretation in terms of the problem of synchronizing automata. A finite automata is determined by a directed multigraph with N vertices (states) and with all edges (transfers) coloured with m colours so that each vertex has precisely one outgoing edge of each colour. The automata is synchronizing if there exists a finite sequence of colours such that all paths following that sequence terminate at the same vertex independently of the starting vertex. The problem of synchronizing automata has been studied in great detail (see [2] for a survey). It turns out that each Haar function can be naturally associated with a finite automata and the Hölder exponent is related to the length of the synchronizing sequence. We introduce a concept of synchronizing rate and show that it is actually equal to the Hölder exponent of the corresponding Haar function. Applying this result we prove that the Hölder exponent can be found within finite time by a combinatorial algorithm.

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Variational Polynomial Interpolation

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Given $n+1$ data points $(x_i, y_i)_{i=0:n}$ meeting $i \neq j \implies x_i \neq x_j$, we want to do true polynomial interpolation of these points without getting the Runge oscillations, or at least with much smaller oscillations.

The idea is quite simple. Instead of using degree n polynomial, we use a degree $n+k$ polynomial, for some integer number k. So we have k extra degrees of freedom. Now we use these extra degrees of freedom for minimizing the oscillations of the so-obtained polynomial. To do so the first idea is to minimize $\sum(p''(x))^2\,dx$ under the interpolation constraint. Using Lagrangean coefficients, this leads to an order $2n+k+2$ linear system.

The surprising point here is that this is very efficient to cut down the oscillations. For example, 11 equidistant data points on the Runge function f gives for degree 10 polynomial p_{10} a distance $\|f - p_{10}\|_\infty$ close to 2, as for a degree 12 polynomial, the distance $\|f - p_{12}\|_\infty$ is less than .35, and less than .07 for a degree 14 polynomial.

But we still have more : it is a numerical (and a "moral") evidence that the interpolating polynomial p_{n+k} tends to the interpolating natural cubic spline when k tends to infinity. We presently have no theoretical proof for that, but this seems quite "sure".

We can do even more ! Let us remember that the function interpolating some data while minimizing $\sum(g'(x))^2\,dx$ over all functions whose first derivative is square integrable, is the piecewise linear function interpolating the data, which obviously has no oscillation. So instead of minimizing $\sum(p''(x))^2\,dx$ let us minimize $\sum(p'(x))^2\,dx$. The resulting polynomial has better result in terms of the uniform norm of $f - p$. And the experimental results show a convergence towards the piecewise linear interpolating function of the data.

We still have some work to do, for proving the convergence, for getting a convergence rate, condition number bounds... but we already can say that this way of doing polynomial interpolation cuts down in a drastic way the Runge oscillations.

Refinable Hermite Vector Splines

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Refinable vector splines are the basic building blocks of spline multi-wavelets. We consider here specifically the Hermite vector spline of arbitrary length and with support [-1,1], and derive explicit formulations for both the spline and its refinement matrix sequence.

Moments of Sets with Refinable Boundary

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We present a method for determining moments (volume, center of mass, inertia tensor, etc.) for objects which are bounded by refinable functions, such as 2d sets bounded by subdivision curves and 3d sets bounded by subdivision surfaces. The approach is based on the solution of an eigenequation resulting from the interrelation between a certain multilinear form and its refined counterparts. Precomputed data for various subdivision schemes (Doo-Sabin, Catmull-Clark, Loop, $\sqrt{3}$, ...) are available for download at [1].

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Old Problems and New Challenges in Subdivision

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The idea of subdivision for the generation of curves and surfaces is over forty years old. Meanwhile, subdivision has become the de facto standard in computer animation, and also the analysis of the underlying mathematical principles has reached a state of maturity. Still, some old problems are still waiting to be solved, such as the construction of high quality surface schemes. At the same time, new challenges arise as subdivision is attracting new interest in engineering applications, such as isogeometric analysis and additive manufacturing.

The two parts of the talk will explore the territory from the mathematical and application viewpoints, respectively, to trigger an extended discussion among participants on the topic.

Isogeometric methods for high order problems: spectral analysis

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When discretizing a linear PDE by a linear numerical method, the computation of the numerical solution reduces to solving a linear system. The size of this system grows when we refine the discretization mesh. Refinement usually leads to a sequence of discretization matrices with an asymptotic spectral distribution. Roughly speaking, this means that there exists a function, the so-called symbol of the given sequence, such that the eigenvalues of the considered matrices behave like a sampling of the symbol over an equispaced grid on its domain.

Isogeometric analysis (IgA) is nowadays a well-established paradigm for the analysis of problems governed by partial differential equations [1]. The main idea is to use the functions adopted in CAD systems not only to describe the domain geometry, but also to represent the numerical solution of the differential problem, within an isoparametric framework. Galerkin and collocation formulations has been intensively employed in this context.

In this talk we focus on the numerical solution of high order differential problems by the IgA approach based on B-splines. In particular, we prove that any sequence of corresponding matrices possess an asymptotic spectral distribution. Our results extend those obtained in [2, 3, 4] for second order elliptic problems. The provided spectral information can be exploited in the design of fast solvers for the related linear systems.

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Applications of Anisotropic Multiple Multiresolution Analyses

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The concept of multiple multiresolution analysis (MMRA) generalizes the standard wavelet analysis and synthesis technique, in the sense that at each level the filterbank makes use of several scaling matrices and pairs of low-pass/high-pass filters chosen from finite given sets. This approach allows, in the 2D case, for a better detection of directional singularities in images, in particular when sets of anisotropic scaling matrices are employed.

In this talk we present many examples illustrating the effectiveness of several multiple multiresolution analyses based on appropriate choices of scaling matrices and corresponding filters [1,2] when applied to some image processing problems, where compression and detection capabilities of the overall system are specifically required.

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Efficient assembly in IgA-FEM and IgA-BEM The use of weighted quadrature and modified moments

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In this poster we present recent results on the assembly of the linear system arising in the Galerkin Isogeometric Finite Element Method (IgA-FEM) [1] and Isogeometric Symmetric Galerkin Boundary Element Method (IgA-SGBEM) [2].
Key ingredients are the application of weighted quadrature and calculation via recurrence relations of modified-moments. These can be found because in the Isogeometric paradigm B-splines are used to represent both the boundary geometry and the approximate solution of the problem at hand.
The main interest are the cases where the degree of the approximation is raised, so that the computational cost in assembling become challenging. We discuss the application of such novel paradigm in the construction: these modifications demand for a change of paradigm in the existing codes.
Finally we present some tests highlighting the saving achieved.

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C^1 isogeometric spaces on multipatch domains

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One key feature of isogeometric analysis is that it allows smooth shape functions. Indeed, when isogeometric spaces are constructed from p-degree splines (and extensions, such as NURBS), they enjoy up to C^{p-1} continuity within each patch. However, global continuity beyond C^0 on so-called multi-patch geometries poses some significant difficulties. We consider planar multi-patch domains that have a parametrization which is only continuous at the patch interface. On such domains we study the h-refinement of C^1 -continuous isogeometric spaces. These spaces in general do not have optimal approximation properties. The reason is that the C^1-continuity condition easily over-constrains the solution which is, in the worst cases, fully locked to linears at the patch interface. However, recent studies have given numerical evidence that optimal convergence occurs for bilinear two-patch geometries and cubic (or higher degree) C^1 splines. This is the starting point of our study. We introduce the class of analysis-suitable G^1 geometry parametrizations, which includes piecewise bilinear parametrizations. We then analyze the structure of C^1 isogeometric spaces over analysis-suitable G^1 parametrizations and, by theoretical results and numerical testing, discuss their approximation properties. We also consider examples of geometry parametrizations that are not analysis-suitable, showing that in this case optimal convergence of C^1 isogeometric spaces is prevented.

Edge detection methods based on RBF interpolation

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Edge detection is a widely used tool in signal/image processing with the aim of identifying abrupt changes or discontinuities in a signal/digital image. In [2] and [3] an iterative adaptive method based on RBF interpolation with suitable scale parameters has been proposed for the detection of jump discontinuities in 1D and 2D problems. Generalizing the 1D method using variably scaled kernels [1] we obtain a new iterative method that compares favorably with the existing one since performs a smaller number of iterations and detects jumps more accurately.
We also present a new non-iterative technique based on RBF interpolation, that detects jumps/edges by identifying the local maxima of the absolute values of the expansion coefficients. To illustrate the effectiveness and efficiency of this last method, numerical examples in 1D and 2D are provided.

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Spline Fitting with Normal Boundary Conditions

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Motivated by the reconstruction of turbine blades for aircraft engines, we study a spline fitting technique that realizes the simultaneous approximation of point and normal data. More precisely, we optimize an objective function that penalizes weighted residuals of points and of unit normals of the resulting surface x_h

$$\|x_h - f\|_{L^2}^2 + \gamma \|N_{x_h} - N_f\|_{L^2}^2 \to \ \min, \tag{1}$$

where f is the surface we wish to approximate and N denotes the unit normal vector of the respective surface. In applications, the normals control the behaviour of the spline surface especially along the domain boundary. If the given normal data are taken from neighboring spline patches, the approximation of these vectors will enforce approximate geometrical smoothness of the resulting composite spline surface.

We ensure the existence of a solution x_h to this problem for every mesh parameter h and show that one can guarantee optimal convergence rates (cf. [2]) for the sequence of solutions $\{x_h\}_h$ by suitably choosing the weight γ of the normal vector residuals. We also explore an iterative method for choosing the weight that takes the different error thresholds into account, as well as generalization to other norm-like functions, cf. [1]. Finally, we apply our method to industrial data sets of turbine blades.

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Subdivision Curves in the Complex Plane

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In 2010, Tsianos and Goldman [3] extended de Casteljau's subdivision algorithm for planar Bézier curves from the real interval to the complex domain. This method permits the construction of many well known fractal shapes such as the C-curve, the Koch curve, and the Sierpinski gasket. The Bézier representation in the complex domain allows the user to change the shape of a fractal in an interactive manner by moving the control points or adjusting the control parameter.

On the other hand, the study of de Casteljau's algorithm led to an algebraic generalization through Matrix Subdivision Schemes (MSS) at the end of the eighties (see e.g. [1, 2]). And it is worth to know that some curves obtained by this latter process are typically fractals. We propose here to study the connection between the two approaches, through the extension of the MSS to the Complex Plane.

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Polar Forms – A Physical Perspective

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Bézier curves and surfaces are fundamental for geometric modeling and isogeometric analysis, and polar forms (or blossoms) play a central role in their theory. Bézier constructions are closely related to toric varieties [1, 2]. Making use of the fact that these are also symplectic manifolds [3] i.e. phase spaces of integrable mechanical systems, we establish a connection with theoretical physics and in particular with the quantum mechanics of spin. Remarkably, the theory of polar forms immediately follows from the basic properties of spin systems. We outline connections with group representations and the numerous possible avenues for future research.

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Hölder Regularity of Univariate Semi-regular Subdivision via Tight Wavelet Frames

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Univariate tight wavelet frames are special function families $\{\psi_j\}_{j\in J} \subset L^2(\mathbb{R})$ (J is a countable index set) that are used for decomposition and analysis of functions in $L^2(\mathbb{R})$. We show that, if the frame $\{\psi_j\}_{j\in J} \subset C^q(\mathbb{R})$, $q \in \mathbb{N}$, has q vanishing moments, i.e.

$$(x^k, \psi_j)_{L_2} = 0 \quad \text{for all} \quad k \in \{0, \dots, q-1\},$$

then the decay rate of the frame coefficients $(\cdot, \psi_j)_{L^2}$, $j \in J$, characterizes the Hölder regularity of semi-regular subdivision. Semi-regular subdivision schemes are such that finitely many of their basic limit functions are not integer shifts of the other ones. The first step towards characterizations of Hölder regularity, in the semi-regular setting, is our construction of the appropriate tight wavelet frame based on Dubuc-Deslauriers $2n$-point semi-regular interpolatory subdivision, $n \in \mathbb{N}$. This construction is done using a so-called modified Unitary Extension Principle. We illustrate our results with several numerical examples.

Space of C^2-smooth isogeometric functions on planar multi-patch geometries

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In this talk, the space of C^2-smooth isogeometric functions on bilinearly parameterized multi-patch domains $\Omega \subset \mathbb{R}^2$, where the graph of each isogeometric function is a multi-patch spline surface of bidegree (d, d), $d \in \{5, 6\}$, is considered. The space is fully characterized by the equivalence of the C^2-smoothness of an isogeometric function and the G^2-smoothness of its graph surface.

The dimension of this C^2-smooth isogeometric space is investigated. The study is based on the decomposition of the space into the direct sum of three subspaces, which are called patch space, edge space and vertex space. Whereas the computation of the dimension of the first two subspaces is based on the two-patch domain study, the computation of the dimension of the vertex space for all possible configurations of bilinear multi-patch domains is a non-trivial task.

C^2-smooth geometrically continuous isogeometric functions are required for solving 6-th order PDEs over multi-patch domains by means of isogeometric analyis. In this talk, we consider examples of solving the triharmonic equation on different bilinear multi-patch domains using our constructed C^2-smooth geometrically continuous isogeometric functions. Furthermore, numerical results obtained by performing L^2 approximation on different multi-patch domains indicate optimal approximation order of the considered isogeometric spline spaces.

Comparing triangular Bézier surfaces for coincidence

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It is known that Bézier curves and surfaces may have multiple representations by different control polygons. The polygons may have different number of control points and may even be disjoint. Up to our knowledge, Pekerman et al. [1] were the first to address the problem of testing two parametric polynomial curves for coincidence. An alternative geometric approach was proposed in [2]. In the same paper the problem of testing tensor product Bézier surfaces (TPBS) for coincidence was considered. It was shown that the approach in [2] developed for curves could be extended and applied to TPBS under certain conditions. The comparison of TPBS for coincidence was later fully resolved by using a new technique proposed in [3].

Here we consider the problem of testing triangular Bézier surfaces (TBS) for coincidence. It turns out that none of the existing approaches can be applied directly and, as we know, no solution to this problem is available. We propose an algorithm based on a reduction of the problem for testing TBS to the corresponding problem for TPBS.

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Near-optimal Tension Parameters in Convexity Preserving Interpolation by Generalized Cubic Splines

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Polynomial splines currently are the main tool of interpolation, especially if we deal with a large number of points. Cubic splines of smoothness class C^2 are the most popular among them. There are a large number of interpolation problems where it is important that the solution satisfies to certain geometric properties like nonnegativity, monotonicity or convexity. Unfortunately, classical splines and in particular cubic splines ignore such properties, do not reproduce the shape of given data and may provide solutions to unwanted bends or waves. There are many constructions of splines (hyperbolic, exponentional, rational splines, splines with additional nodes, and so on) based on the same idea consisting in generalization of a construction of conventional cubic spline by introduction of so-called tension or design parameters which allow to ensure sufficient tension of links of the spline in critical areas, i.e. to control the shape of the spline. All of these splines are called the generalized cubic splines. A suitable choice of parameters should provide a small deviation from a classical spline and at the same time to suppress unwanted waves.

The problem of a suitable choice of design parameters of the generalized splines is very important. Initially there have been proposed heuristic ways of a choice of operating parameters. Later there appeared the iterative schemes of automatic choice of parameters. But the result of Miroshnichenko [1] about non-negative solution of tridiagonal system of equations with diagonal dominance became the most effective tool for these purposes.

We apply similar results to the problem of convex interpolation by generalized cubic splines [2]. We offer a common scheme of a choice of the tension parameters, close to optimum, for ensuring the convexity of interpolation spline. The described scheme leads to easily realizable algorithms of a choice of parameters for the most popular in practice kinds of generalized splines. Note an important fact, that the generalized spline will be conventional cubic spline if sufficient conditions of its convexity [1] are satisfied.

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Directional Multiresolution in Two and More Variables

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It is well known that wavelets are a useful tool in signal processing and they allow a fast implementation thanks to their relation with multiresolution analysis, subdivision schemes and filterbanks. Despite their success, they are not suited to handle anisotropic signals and singularities along some directions or curves because they use isotropic scaling matrices.

Several alternatives to wavelets are known. Among them, shearlets allow for the definition of a directional multiple multiresolution analysis, where perfect reconstruction of the filterbank can be easily ensured by choosing an appropriate multiple subdivision scheme (see e.g. [3]). In this case we have anisotropic scaling matrices that are product of a parabolic matrix and the so-called shear matrix. A drawback of shearlets is their relative large determinant that leads to a substantial complexity.

In the two dimensional case an anisotropic directional multiple multiresolution, based on a particular choice of scaling matrices with minimum determinant, is presented in [1].

In this talk we show a system of anisotropic directional scaling matrices in $\mathbb{Z}^d$, $d \geq 2$, that keeps the crucial properties of shearlets but with lower determinant (the case $d = 2$ is studied in [2]). In particular we will prove that they satisfy the slope resolution property in any dimension d. For the two dimensional case we provide a concrete example of multiple subdivision scheme and some numerical results of image decomposition and reconstruction.

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Computing Straight Skeletons for Arc Polygons

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We generalize the offsetting process that defines straight skeletons of polygons [1] to *arc* polygons, i.e., to planar shapes with piecewise circular boundaries. The offsets are obtained by shrinking or expanding the circular arcs on the boundary and tracing the paths of the vertices. These paths define the associated skeletons and the associated decomposition into patches. While the skeleton forms a tree, the patches of our decomposition have a radial monotonicity property. Analyzing the events that occur during the offsetting process is nontrivial; for example one has to ensure that the offsetting object stays an arc polygon. This leads us to an event-driven algorithm for offset and skeleton computation. Several examples (both manually created ones and approximations of planar free-form shapes by arc spline curves) will be presented to analyze the performance of our algorithm.

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Adaptive progressive interpolation subdivision surfaces based on local feature

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Progressive interpolation subdivision surfaces generated by approximating schemes was presented recently, it can handle control meshes of any size and any topology while generating smooth subdivision surfaces that faithfully resemble the shape of the initial meshes. When the number of original vertices becomes huge, the convergence speed becomes slow and computation complexity becomes big. In order to solve this kind of problem, one adaptive progressive interpolation subdivision scheme is presented in this paper. The vertices of control mesh are classified into two classes, active vertices and fixed ones. When precision is given, two classes vertices are changed dynamically according to result of each iteration. Only the active vertices are adjusted, thus the class of active vertices keep running down while the fixed ones keep rising, which saves computation greatly. Furthermore, weights are assigned to these vertices to accelerate convergence speed. Theoretical analysis and numerical examples are also given to illustrate the correctness and effectiveness of the method.

Non Polynomial Divided Differences and B-splines

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One of the recent works on the splines develops a general, unified theory of splines for a wide collection of spline spaces, including trigonometric splines, hyperbolic splines, and special Müntz spaces of splines by invoking a novel variant of the homogeneous polar form (see [1]). Our main goal is to represent non-polynomial B-splines using divided differences. We define divided differences for the space $\pi_n(\gamma_1, \gamma_2)$ spanned by $\left\{\gamma_1^{n-k}\gamma_2^k\right\}_{k=0}^n$ where γ_1 and γ_2 are two linearly independent functions. We also derive some properties and identities of divided differences. Defining a novel variant of truncated power function, we express non polynomial B-splines explicitly in terms of divided differences of truncated power function.

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