Scalar Subdivision Schemes and Box Splines

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Abstract

We study convergent scalar d-variate subdivision schemes satisfying sum rules of order $k \in \mathbb{N}$, with dilation matrix $2I$. Using the results of Möller and Sauer in [3], stated for general expanding dilation matrices, we characterize the structure of the mask symbols of such schemes by showing that they must be linear combinations of shifted box spline generators of a certain polynomial ideal $\mathcal{I}^k$. The directions of the corresponding box splines are $\theta \in \{0,1\}^d \setminus \{(0,\ldots,0)\}$. The ideal $\mathcal{I}^k$, as shown in [3], is determined by the given order of the sum rules or, equivalently, by the order of the Strang-Fix conditions.

Our results open a way to a systematic study of subdivision schemes. For example, in the bivariate case, if the mask symbol of any convergent subdivision scheme is in $\mathcal{I}^k$, then the mask is an affine combination of smoothed versions of three-directional box splines. Many special cases, including affine combinations of convergent schemes, can be looked at this way; see, e.g., [2] and the references given therein.

As in the univariate case, this characterization seems to be the proper way of matching the smoothness, as determined in [1], of the box spline building blocks with the order of polynomial reproduction of the corresponding scheme. Due to the interaction of the building blocks, the convergence and smoothness, however, are usually destroyed, if several convergent schemes are combined in this way. We address this problem and give a sufficient condition for convergence of such combinations.

We illustrate our result with several examples.

References

[1] C. de Boor and K. Höllig, B-Splines from parallelepipeds, *J. Anal. Math.* **42** (1983), 99–115.

[2] C. Conti, Stationary and non stationary affine combination of subdivision masks, *Math. Comput. Simulation*, to appear.

[3] H. M. Möller and T. Sauer, Multivariate refinement functions of high approximation order via quotient ideals of Laurent polynomials, *Adv. Comput. Math.* **20** (2004), 205–228.