FROM HERMITE SUBDIVISION SCHEMES TO LAGRANGE SUBDIVISION SCHEMES

J. L. Merrien, S. Dubuc, T. Sauer
INSA, Renne, France
University of Montreal, Canada
Universität of Gießen, Germany

Abstract

Recently in [1], for dimension 1, a transformation of a *Hermite Subdivision Scheme* into a *Lagrange Subdivision Scheme* was proposed provided the Hermite scheme satisfies a spectral property. This property is equivalent to the sum rule given in [2].

In this lecture, we propose a generalization for the multidimension case then we prove that if the Lagrange subdivision operator is C^0–convergent, then Hermite subdivision scheme is C^d*–convergent*.

Let $s_k := \binom{s+k-1}{s-1}$ be the dimension of the space of homogeneous polynomials of degree k and $r_k := \binom{k+s}{s}$ the dimension of the finite dimensional vector space of all polynomials of total degree at most k so that $r_k = s_0 + \cdots + s_k$. We recall that for dimension s with d derivatives, the Hermite operator $H_{\boldsymbol{A}}$ operates on $\ell^{r_d}(\mathbb{Z}^s)$ and is defined by

$$\mathcal{D}^{n+1} H_{\boldsymbol{A}} \boldsymbol{c}(\alpha) = \sum_{\beta \in \mathbb{Z}} \boldsymbol{A}(\alpha - 2\beta) \mathcal{D}^n \boldsymbol{c}(\beta), \qquad \boldsymbol{c} \in \ell^{r_d}\left(\mathbb{Z}^s\right), \tag{1}$$

where $\mathcal{D}$ is the diagonal matrix with diagonal entries $(1, \underbrace{1/2, \ldots, 1/2}_{s_1 = s \text{ times}}, \ldots, \underbrace{1/2^d, \ldots, 1/2^d}_{s_d \text{ times}})$.

Now if $\boldsymbol{c}_n = (H_{\boldsymbol{A}})^n \boldsymbol{c}_0$, we decompose the vector $\boldsymbol{c}_n(\beta) = (\underbrace{\boldsymbol{c}_n^{(0)}(\beta)}_{s_0=1}, \underbrace{\boldsymbol{c}_n^{(1)}(\beta)}_{s_1}, \ldots, \underbrace{\boldsymbol{c}_n^{(d)}(\beta)}_{s_d})^T$ and the first component $\boldsymbol{c}_n^{(0)}(\beta)$ can be read as the value of a function f_n at $\beta/2^n$, the s_1 following ones, $\boldsymbol{c}_n^{(1)}(\beta)$, are for the first derivatives $D^1 f_n(\beta/2^n)$ and so one up to the last s_d ones, $\boldsymbol{c}_n^{(d)}(\beta)$ which are $D^d f_n(\beta/2^n)$ where $D^j := \left[D^\alpha = \frac{\partial^j}{\partial x^\alpha}\right]_{|\alpha|=j}$.

References

[1] S. Dubuc and J.-L. Merrien, Hermite Subdivision Schemes and Taylor Polynomials, *Const App.*29 2009, 219-245,

[2] B. Han, T. Yu and Y. Xue, Noninterpolatory Hermite subdivision schemes, *Math. Comp.* **74** (2005), 1345-1367.