SVOLGIMENTO PROVA SCRITTA
ANALISI 1 del 9/9/2015

A_1

COMPITO A

1) $(x-iy)^2 + |e^{i(x+iy)}| = e^{-y}$

$x^2 - y^2 - 2ixy + |e^{ix}| \cdot |e^{-y}| = e^{-y}$

$x^2 - y^2 - 2ixy + 1 \cdot e^{-y} = e^{-y}$

$\Rightarrow (x-iy)^2 = 0 \Rightarrow z^2 = 0$

$\Rightarrow z = 0$

2) $n^4 \left[e^{1/n^2} - 1 - \frac{1}{n^2} - \frac{1}{2n^4} \right]$

$= n^4 \left[1 + \frac{1}{n^2} + \frac{1}{2n^4} + \frac{1}{6n^6} + o\left(\frac{1}{n^6}\right) - 1 - \frac{1}{n^2} - \frac{1}{2n^4} \right]$

$\sim \frac{1}{6n^2}$

$\Rightarrow$ la serie converge

A_2

3) $f(x) \underset{x\to+\infty}{\sim} \frac{1}{x^2}$ che è integrabile in $[1, +\infty)$.

$$f(x) = \frac{x+1}{x(x+3)^2} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$= \frac{A(x+3)^2 + Bx(x+3) + Cx}{x(x+3)^2}$$

$$\Rightarrow \begin{cases} A+B=0 \\ 6A+3B+C=1 \\ 9A=1 \end{cases} \Rightarrow \begin{cases} B=-A \\ 9A=1 \\ C=1-3A \end{cases} \Rightarrow \begin{cases} A=\frac{1}{9} \\ B=-\frac{1}{9} \\ C=\frac{2}{3} \end{cases}$$

$$\int_1^{+\infty} \left[\frac{1}{9x} - \frac{1}{9(x+3)} + \frac{2}{3(x+3)^2} \right] dx$$

$$= \frac{1}{9} \log \left| \frac{x}{x+3} \right| - \frac{2}{3(x+3)} \Bigg|_1^{+\infty}$$

$$= \frac{1}{9} \lim_{x\to+\infty} \log \left| \frac{x}{x+3} \right| - \frac{2}{3} \lim_{x\to+\infty} \frac{1}{x+3}$$

$$- \frac{1}{9} \log \left(\frac{1}{4} \right) + \frac{2}{12} =$$

A 3

$= \frac{1}{9}\log 4 + \frac{1}{6}.$

4) $I_{dy} = \{x^2 - 3x \neq 0\} = \mathbb{R} - \{0,3\}$

$= (-\infty, 0) \cup (0,3) \cup (3, +\infty)$

$f(x) > 0 \iff x^2 - 3x > 0$

$\iff x < 0 \; ; \; x > 3$

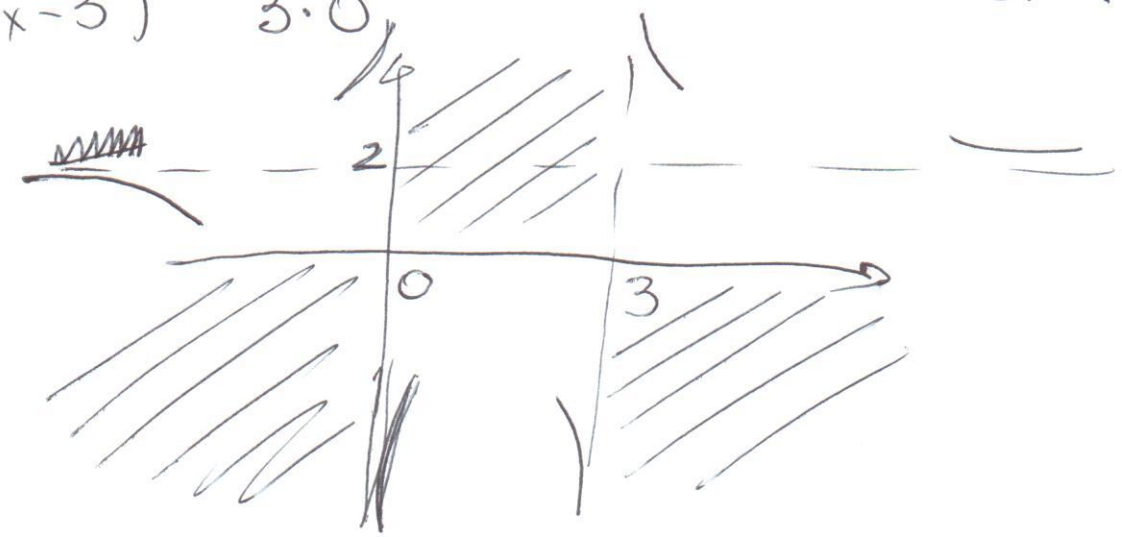
$f(x) < 0 \iff 0 < x < 3$

f NON SI ANNULLA MAI

$\lim_{x \to \pm\infty} f(x) = 2$ $\quad y = 2$ AS. ORIZZ. per $x \to \pm\infty$

$\lim_{x \to 0^{\pm}} \frac{2x^2+3}{x(x-3)} = \frac{3}{0^{\pm} \cdot (-3)} = \mp\infty$ $\quad x = 0$ AS. VERT.

$\lim_{x \to 3^{\pm}} \frac{2x^2+3}{x(x-3)} = \frac{21}{3 \cdot 0^{\pm}} = \pm\infty$ $\quad x = 3$ AS. VERT.



A_4

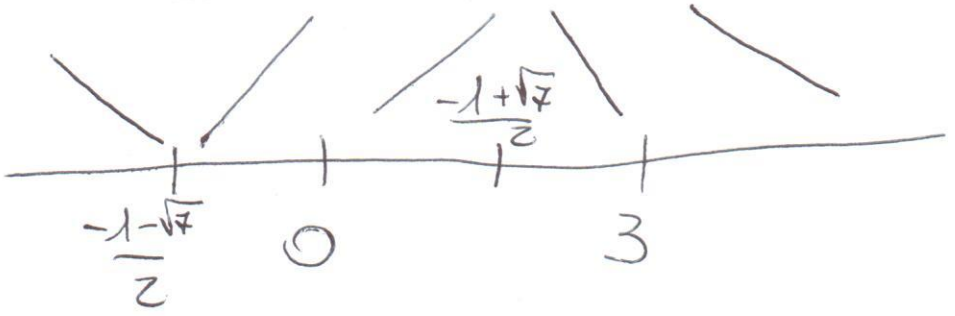
$$f'(x) = \frac{4x(x^2-3x) - (2x^2+3)(2x-3)}{(x^2-3x)^2}$$

$$= \frac{\cancel{4x^3} - 12x^2 - \cancel{4x^3} + 6x^2 - 6x + 9}{(x^2-3x)^2}$$

$$= \frac{-6x^2-6x+9}{(x^2-3x)^2} > 0 \quad \Leftrightarrow \quad 2x^2+2x-3 < 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+6}}{2} = \frac{-1 \pm \sqrt{7}}{2}$$

$$x_1 = \frac{-1-\sqrt{7}}{2} < 0 \quad ; \quad x_2 = \frac{-1+\sqrt{7}}{2} < 3$$



$$f'(x) > 0 \quad \Leftrightarrow \quad x_1 < x < x_2$$

quindi f decresce in $(-\infty, x_1)$
cresce in $(x_1, 0)$
cresce in $(0, x_2)$
decresce in $(x_2, 3)$
decresce in $(3, +\infty)$.

FAC.: dato che $f(x) \xrightarrow[x \to 3^{\pm}]{} \pm\infty$, (A5)

$\nexists$ MAX. e MIN. ASS.

f ammette MIN. REL. in x_1

e MAX. REL in x_2.

5) EQ. OMO. ASSOCIATA: $y'' + 9y = 0$

$\lambda^2 + 9 = 0 \Rightarrow \lambda_{1,2} = \pm 3i$

$\Rightarrow y_0(x) = C_1 \cos(3x) + C_2 \operatorname{sen}(3x)$

EQ. NON OMO: $y_p(x) = (Ax+B)\, e^{3x}$

$y_p'(x) = (A + 3Ax + 3B)\, e^{3x}$

$y_p''(x) = (6A + 9Ax + 9B)\, e^{3x}$

$\Rightarrow 6A + 9Ax + 9B + 9Ax + 9B = 3x$

$$\begin{cases} 18A = 3 \\ 6A + 18B = 0 \end{cases} \qquad \begin{cases} A = \frac{1}{6} \\ B = -\frac{1}{18} \end{cases}$$

$\Rightarrow y(x) = C_1 \cos(3x) + C_2 \operatorname{sen}(3x) + \left(\frac{1}{6}x - \frac{1}{18}\right) e^{3x}$

$$y(0) = C_1 - \frac{1}{18} = 0 \Rightarrow C_1 = \frac{1}{18}$$

A6

$$y'(x) = -3C_1 \operatorname{sen}(3x) + 3C_2 \cos(3x) + \left(\frac{1}{6} + \frac{1}{2}x - \frac{1}{6}\right) e^{3x}$$

$$y'(0) = 3C_2 = 0 \Rightarrow C_2 = 0$$

$$\Rightarrow y(x) = \frac{1}{18}\cos(3x) + \left(\frac{1}{6}x - \frac{1}{18}\right) e^{3x}$$