COMPITO B

B1

1) $f(x) \underset{x\to+\infty}{\sim} \frac{1}{x^2}$, che è integrabile in $[1,+\infty)$.

$$f(x) = \frac{x+1}{x(x+4)^2} = \frac{A}{x} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2}$$

$$= \frac{A(x+4)^2 + Bx(x+4) + Cx}{x(x+4)^2}$$

$$\Rightarrow \begin{cases} A+B=0 \\ 8A+4B+C=1 \\ 16A=1 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{16} \\ B=-\frac{1}{16} \\ C=1-4A=\frac{3}{4} \end{cases}$$

$$\Rightarrow \int_1^{+\infty} \left[\frac{1}{16x} - \frac{1}{16(x+4)} + \frac{3}{4(x+4)^2}\right] dx$$

$$= \frac{1}{16}\log\left|\frac{x}{x+4}\right| - \frac{3}{4(x+4)}\Bigg|_1^{+\infty}$$

$$= \frac{1}{16}\underbrace{\lim_{x\to+\infty}\log\left|\frac{x}{x+4}\right|}_{=0} - \underbrace{\lim_{x\to+\infty}\frac{3}{4(x+4)}}_{=0}$$

B_2

$$-\frac{1}{16}\log\left(\frac{1}{5}\right)+\frac{3}{20}$$

$$\Rightarrow \int_1^{+\infty} f(x)\,dx = \frac{1}{16}\log 5 + \frac{3}{20}.$$

2) $I_{def} = \{x^2-4x\neq 0\} = \mathbb{R}-\{0,4\}$

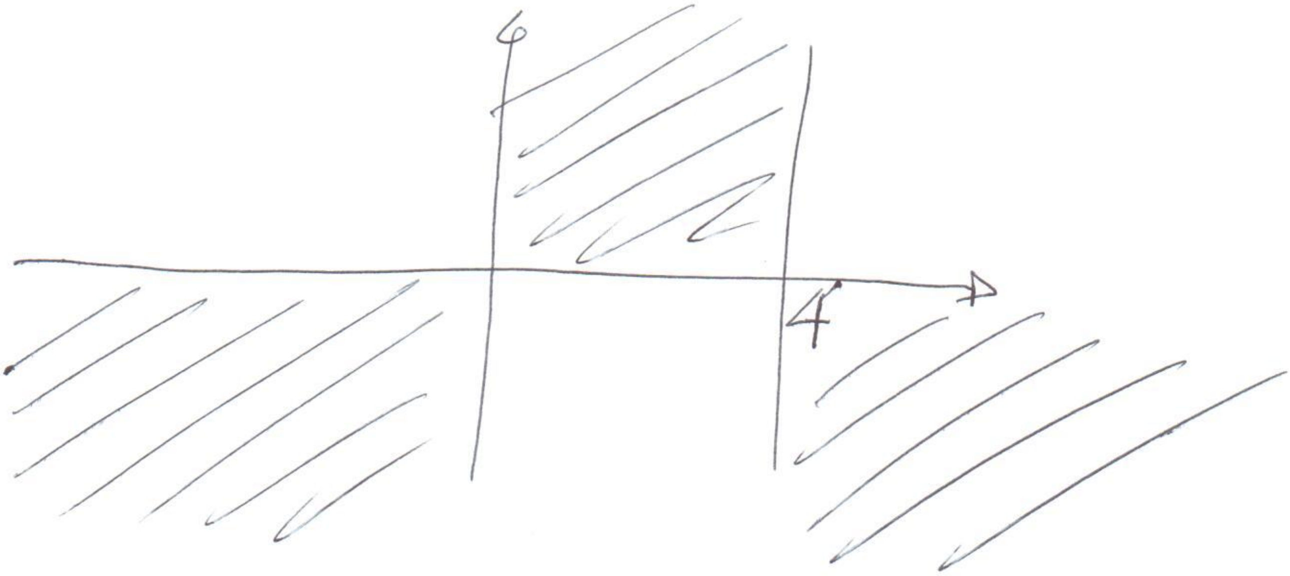
$= (-\infty,0)\cup(0,4)\cup(4,+\infty).$

$f(x)$ NON SI ANNULLA MAI

$f(x)>0 \iff x(x-4)>0$

$\iff x<0 \;;\; x>4$

$f(x)<0 \iff 0<x<4$



B3

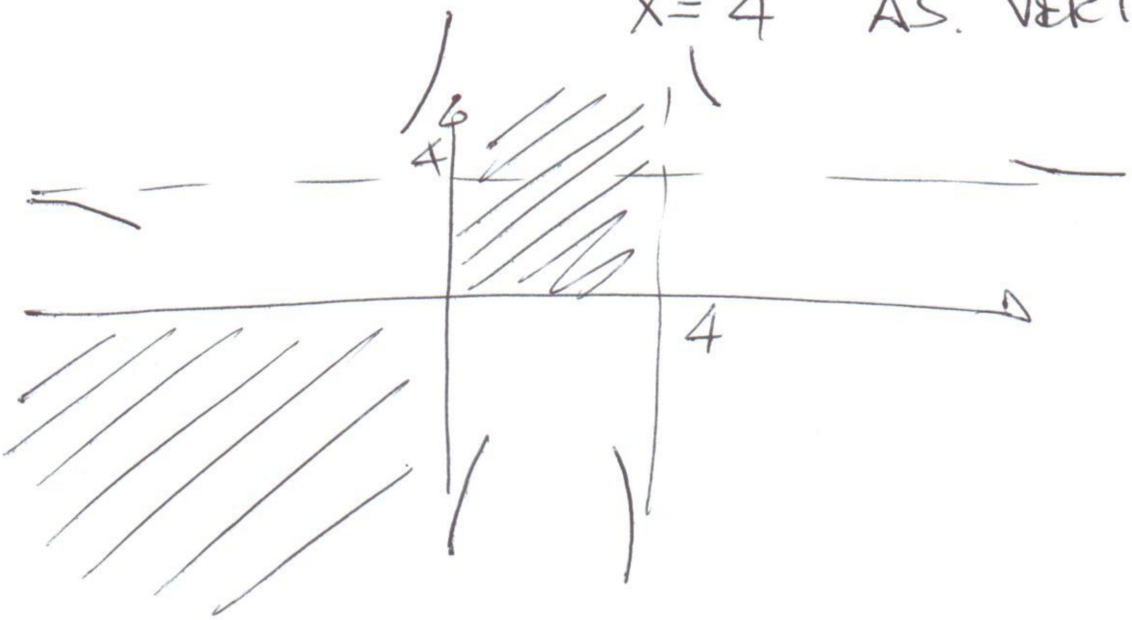
$$\lim_{x\to\pm\infty} f(x) = 3 \qquad y=3 \text{ AS. ORIZZ. per } x\to\pm\infty$$

$$\lim_{x\to 0^\pm} \frac{3x^2+4}{x(x-4)} = \frac{4}{0^\pm\cdot(-4)} = \mp\infty$$

$x=0$ AS. VERT.

$$\lim_{x\to 4^\pm} \frac{3x^2+4}{x(x-4)} = \frac{52}{4\cdot 0^\pm} = \pm\infty$$

$x=4$ AS. VERT.



$$f'(x) = \frac{6x(x^2-4x)-(3x^2+4)(2x-4)}{(x^2-4x)^2}$$

$$= \frac{\cancel{6x^3}-24x^2-\cancel{6x^3}+12x^2-8x+16}{(x^2-4x)^2}$$

$$= \frac{-12x^2-8x+16}{(x^2-4x)} > 0$$

B_4

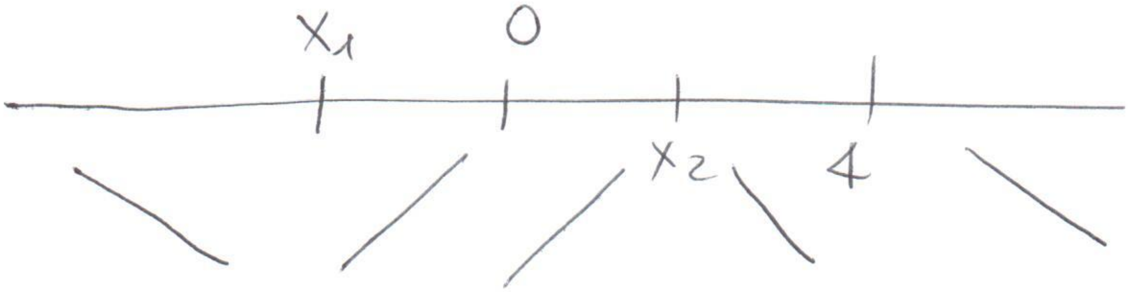
$$\iff 3x^2+2x-4<0$$

$$x_{1,2} = \frac{-1\pm\sqrt{1+12}}{3} = \frac{-1\pm\sqrt{13}}{3}$$

$\cancel{f'(x)>0 \iff}$

$$x_1 = \frac{-1-\sqrt{13}}{3} < 0 \quad ; \quad x_2 = \frac{-1+\sqrt{13}}{3} < 4$$

$$f'(x)>0 \iff x_1 < x < x_2$$



f decresce in $(-\infty, x_1)$

cresce in $(x_1, 0)$

cresce in $(0, x_2)$

decresce in $(x_2, 4)$

decresce in $(4, +\infty)$.

B_5

FAC.: poiché $\lim\limits_{x \to 4^{\pm}} f(x) = \pm\infty$,

$\nexists$ MAX. e MIN. ASSOLUTI

x_1 punto di MIN. REL.

x_2 punto di MAX. REL.

3) OMO. ASSOCIATA: $\lambda^2 + 4 = 0$

$$\Rightarrow \lambda = \pm 2i$$

$$\Rightarrow y_0(x) = C_1 \cos(2x) + C_2 \sin(2x)$$

NON. OMO.:

$$y_p(x) = (Ax + B)e^{-2x}$$

$$y_p'(x) = (A - 2Ax - 2B)e^{-2x}$$

$$y_p''(x) = (-4A + 4Ax + 4B)e^{-2x}$$

$$\Rightarrow -4A + 4Ax + 4B + 4Ax + 4B = 2x$$

$$\Rightarrow \begin{cases} -4A + 8B = 0 \\ 8A = 2 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{4} \\ B = \frac{1}{8} \end{cases}$$

$$y(x) = C_1 \cos(2x) + C_2 \operatorname{sen}(2x) + \left(\frac{1}{4}x + \frac{1}{8}\right)e^{-2x}$$

$$y(0) = C_1 + \frac{1}{8} = 0 \quad \Rightarrow C_1 = -\frac{1}{8}$$

B6

$$y'(x) = -2C_1 \operatorname{sen}(2x) + 2C_2 \cos(2x)$$
$$+ \left(\cancel{\frac{1}{4}} - \frac{1}{2}x - \cancel{\frac{1}{4}}\right)e^{-2x}$$

$$y'(0) = 2C_2 = 0 \quad \Rightarrow \quad C_2 = 0$$

$$y(x) = -\frac{1}{8}\cos(2x) + \left(\frac{1}{4}x + \frac{1}{8}\right)e^{-2x}$$

4) $$z^2 + \left|e^{i(x-iy)}\right| = e^y$$

$$z^2 + \left|e^{ix+y}\right| = e^y$$

$$z^2 + \left|e^{ix}\right| \cdot \left|e^y\right| = e^y$$

$$z^2 + \cancel{e^y} = \cancel{e^y}$$

$$z^2 = 0 \quad \Rightarrow \quad z = 0$$

5) $n^3\left[\sin\left(\frac{1}{n}\right)-\frac{1}{n}+\frac{1}{6n^3}\right]$

B7

$$=n^3\left[\cancel{\frac{1}{n}}-\cancel{\frac{1}{6n^3}}+\frac{1}{5!\,n^5}+o\left(\frac{1}{n^5}\right)-\cancel{\frac{1}{n}}+\cancel{\frac{1}{6n^3}}\right]$$

$$\sim\frac{1}{5!\,n^2}$$

quindi la serie converge.