COMPITO B

$B_1$

1)

$$a_n = \frac{\left[\sqrt{n^2-n} - n + \frac{1}{2}\right]}{\left[\sqrt{n^2+n} - n - \frac{1}{2}\right]} \cdot \frac{\left[\sqrt{n^2-n} + n - \frac{1}{2}\right]}{\left[\sqrt{n^2+n} + n + \frac{1}{2}\right]} \cdot \frac{\left[\sqrt{n^2+n} + n + \frac{1}{2}\right]}{\left[\sqrt{n^2-n} + n - \frac{1}{2}\right]}$$

$$= \frac{(n^2-n) - \left(n - \frac{1}{2}\right)^2}{(n^2+n) - \left(n + \frac{1}{2}\right)^2} \cdot \left[\frac{\sqrt{n^2+n} + n + \frac{1}{2}}{\sqrt{n^2-n} + n - \frac{1}{2}}\right]$$

$$= \frac{\cancel{-\frac{1}{4}}}{\cancel{-\frac{1}{4}}} \left[\frac{\sqrt{n^2+n} + n + \frac{1}{2}}{\sqrt{n^2-n} + n - \frac{1}{2}}\right]$$

$$\underset{n\to+\infty}{\sim} \frac{2n}{2n} \xrightarrow[n\to+\infty]{} 1$$

Pertanto $\sum a_n$ diverge

2) $f(x) = \log\left[(1-x)^2\right] = 2\log(|1-x|)$ (B2)

$D = \{x \neq 1\}$

$f$ continua e derivabile in $D$, perchè composizione di funzioni derivabili. ~~...~~

$f(x) = 0 \iff |1-x| = 1$

$\iff x = 0 \; ; \; x = 2$

$f(x) > 0 \iff |1-x| > 1$

$\iff x < 0 \; ; \; x > 2$

$\left( f(0) = 0 \right)$.

ASINTOTO VERTICALE:

$\lim_{x \to 1} f(x) = 2\log(0^+) = -\infty$

AS. VERT. $x = 1$.

$\lim_{x \to \pm\infty} f(x) = +\infty$ B3

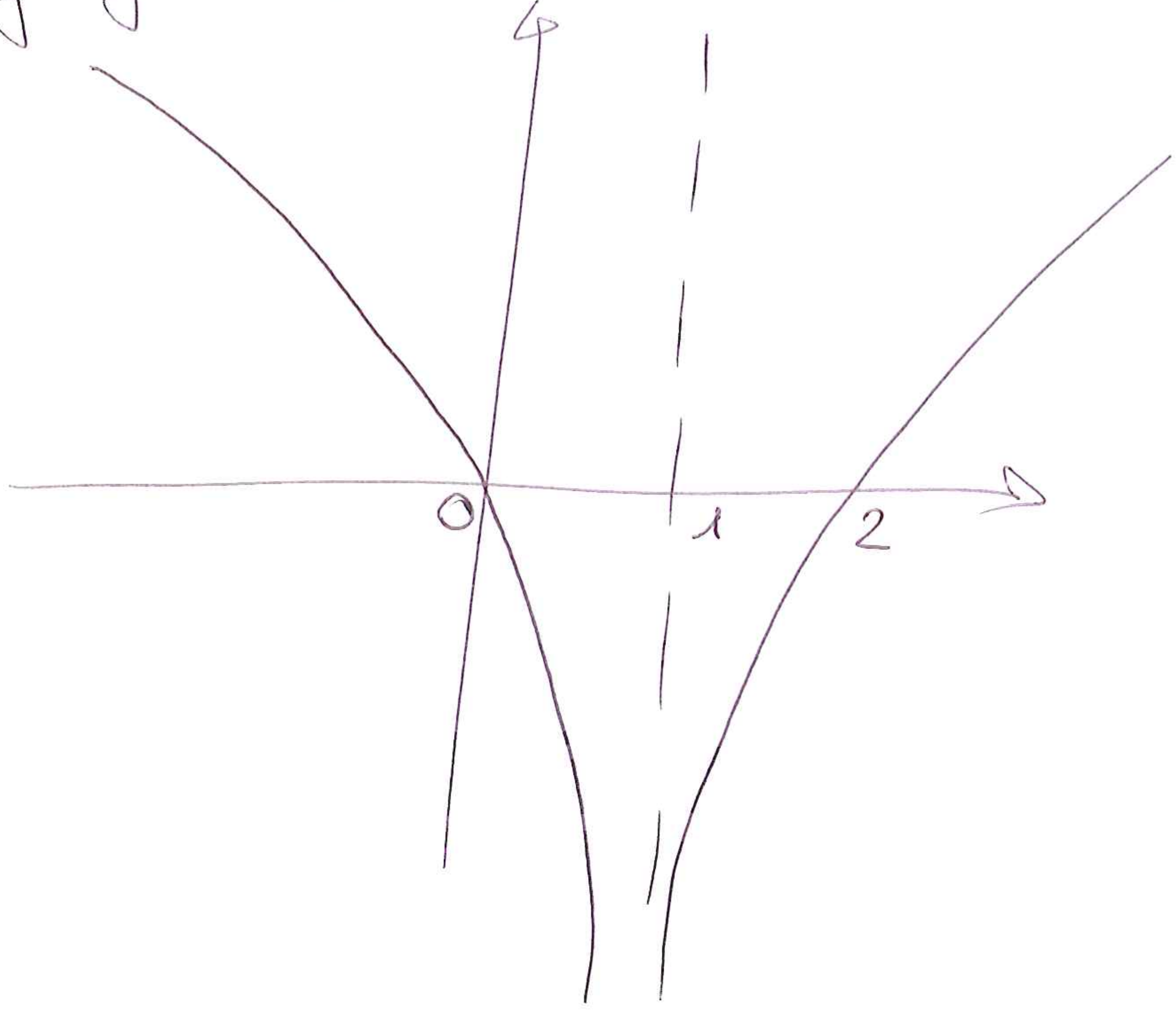
∄ AS. OBLIQUO perché il logaritmo è sublineare.

MONOTONIA:

$f'(x) = \frac{-2}{1-x} = \frac{2}{x-1} > 0 \iff x > 1$

$f$ decresce in $(-\infty, 1)$;
cresce in $(1, +\infty)$.

Grafico (non richiesto):



3) $f > 0 \iff \sqrt{x+2} - \sqrt{x}$ (sempre vero). B4

f è comunque definita $\forall x > 0$.

$$f(x) = \frac{4}{x^3\left(\sqrt{x+2}-\sqrt{x}\right)} \left[\frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}\right]$$

$$= \frac{4\left[\sqrt{x+2}+\sqrt{x}\right]}{x^3\left[\cancel{x}+2-\cancel{x}\right]} = \frac{2\left[\sqrt{x+2}+\sqrt{x}\right]}{x^3}$$

$$\underset{x\to+\infty}{\sim} \frac{4\sqrt{x}}{x^3} = \frac{4}{x^{5/2}}$$ che è integrabile a $+\infty$.

$\cancel{\int f(x)\,dx = 2}$

$$\int_2^4 \frac{4}{\sqrt{x+2}-\sqrt{x}}\,dx = \int_2^4 2\left[\sqrt{x+2}+\sqrt{x}\right]dx$$

$$= 2\left[\frac{2}{3}(x+2)^{\frac{3}{2}} + \frac{2}{3}x^{\frac{3}{2}}\right]_2^4 =$$

B5

$$\cancel{= \frac{4}{3}\left[8+\sqrt{8}-\sqrt{8}\right] = \frac{32}{3}}$$

$$\frac{4}{3}\left[(6)^{\frac{3}{2}}-(2)^{\frac{3}{2}}\right]$$

4) omogenea associata:

$$y''-2y'=0 \quad \Rightarrow \alpha^2-2\alpha=0$$

$$\Rightarrow \alpha(\alpha-2)=0$$

$$\Rightarrow y_0(x)=C_1e^{2x}+C_2$$

non omogenea: principio di sovrapposizione. Poiché $\lambda=0$ è radice del polinomio caratteristico, allora

$$y_p(x)=Ae^x+x(ax+b).$$

$$y_p'(x)=Ae^x+2ax+b$$

$$y_p''(x)=Ae^x+2a$$

$$\Rightarrow Ae^x+2a-2Ae^x-4ax-2b$$

$$=e^x+x$$

$$\Rightarrow \begin{cases} -A=1 \\ -4a=1 \\ 2a-2b=0 \end{cases} \Rightarrow \begin{cases} A=-1 \\ a=-1/4 \\ b=-1/4 \end{cases}$$

$$\Rightarrow y(x) = C_1 e^{2x} + C_2 - e^{x} - \frac{1}{4}(x^2 + x).$$

Poichè $\lim_{x\to+\infty} e^{x} = +\infty$, tutte le soluzioni sono illimitate.

B6

5)

$$e^{-i} = 5e^{x-iy}$$

$$\Rightarrow \begin{cases} 1 = 5e^{x} \\ 1 = y + 2k\pi \end{cases} \Rightarrow \begin{cases} x = -\log 5 \\ y = 1 + 2k\pi \ ; \ k \in \mathbb{Z} \end{cases}$$