COMPITO A

A₁

1) $y''(x) - \cot g\, x \cdot y'(x) = 0$

$a(x) = -\cot g\, x \in C^0(k\pi, k\pi+\pi)$

Poiché $\frac{\pi}{2} \in (0,\pi) \Rightarrow$ studiamo l'equazione in $(0,\pi)$.

$\exists!$ sol. $y \in C^2(0,\pi)$.

$$y'(x) = z(x)$$

$$z'(x) - \cot g\, x\, z(x) = 0$$

$$z(x) = C_1 e^{\int \cot g\, x\, dx} = C_1 e^{\log(|\sin x|)}$$

Poiché $\sin x > 0$ in $(0,\pi) \Rightarrow$

$$z(x) = C_1 e^{\log(\sin x)} = C_1 \sin x$$

$$z\left(\frac{\pi}{2}\right) = y'\left(\frac{\pi}{2}\right) = 0 = C_1 \sin\left(\frac{\pi}{2}\right) = C_1$$

$$\Rightarrow z(x) = y'(x) \equiv 0$$

$$\Rightarrow y(x) \equiv C_2$$

$$y\left(\frac{\pi}{2}\right) = 1 \Rightarrow \boxed{y(x) \equiv 1}$$

A_2

2)

$$\lim_{x\to 0} \frac{x\left[x - \cancel{\frac{x^3}{6}} + \frac{x^5}{5!} + x + \cancel{\frac{x^3}{6}} + \frac{x^5}{5!} + o(x^5)\right] + 2\left[\cancel{1} - \frac{x^2}{2} + \cancel{\frac{x^4}{4!}} - \frac{x^6}{6!} - \cancel{1} - \frac{x^2}{2} - \cancel{\frac{x^4}{4!}} - \frac{x^6}{6!} + o(x^6)\right]}{x^6}$$

$$= \lim_{x\to 0} \frac{x\left[2x + \frac{2}{120}x^5 + o(x^5)\right] + 2\left[-x^2 - \frac{2}{720}x^6 + o(x^6)\right]}{x^6}$$

$$= \lim_{x\to 0} \frac{\cancel{2x^2} + \frac{1}{60}x^6 \cancel{- 2x^2} - \frac{1}{180}x^6 + o(x^6)}{x^6}$$

$$= \lim_{x\to 0} \frac{\frac{2}{180}x^6}{x^6} = \frac{1}{90}$$

3) $z \neq \frac{1}{2} - \frac{i}{2}\sqrt{3}$

$$z_{1,2} = \frac{(1-i\sqrt{3}) + \sqrt{(1-i\sqrt{3})^2 + 2 + 2i\sqrt{3}}}{2}$$

$$z_{1,2} = \frac{1 - i\sqrt{3} + \sqrt{1 - 3 - 2i\sqrt{3} + 2 + 2i\sqrt{3}}}{2} = \frac{1 - i\sqrt{3}}{2}$$

molteplicità $= 2$

Ma $z \neq \frac{1}{2} - i\frac{\sqrt{3}}{2}$ A_3

$\Rightarrow$ NESSUNA SOLUZIONE.

4) CONVERGENZA ASSOLUTA

$$\sum |a_n| = \sum \frac{|\operatorname{arctg}(n^3)|}{(n-1)^3}$$

$$|a_n| = \frac{|\operatorname{arctg}(n^3)|}{(n-1)^3} \leq \frac{\pi}{2(n-1)^3} \sim \frac{\pi}{2n^3}$$

CONFRONTO

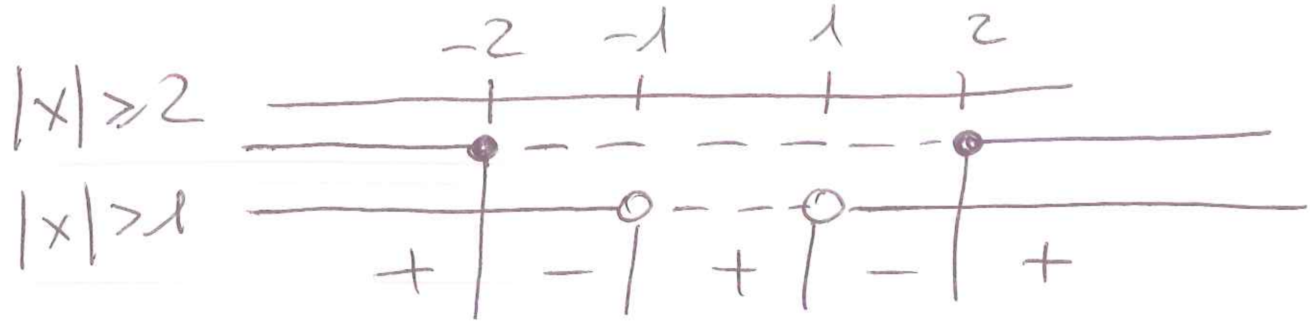
CONFRONTO ASINTOTICO

$$\sum \frac{\pi}{2n^3} = \frac{\pi}{2} \sum \frac{1}{n^3} \text{ converge}$$

$\Rightarrow \sum |a_n|$ converge $\Rightarrow \sum a_n$ converge.

5) La funzione è definita (e non negativa) per $\dfrac{2-|x|}{1-|x|} \geq 0 \iff \dfrac{|x|-2}{|x|-1} \geq 0$

A_4



$$D = (-\infty, -2] \cup (-1, 1) \cup [2, +\infty)$$

f PARI. La studiamo per $x \geq 0$:

$$f(0) = \sqrt{2} \qquad f(x) = \sqrt{\frac{x-2}{x-1}}$$

$$\lim_{x \to 1^-} f(x) = \sqrt{\frac{-1}{0^-}} = +\infty = \lim_{x \to -1^+} f(x)$$

$x = \pm 1$ AS. VERTICALI

$$\lim_{x \to \pm\infty} f(x) = 1 \implies y = 1$$ AS. ORIZZONTALE per $x \to \pm\infty$

$$f(x) = 0 \iff x = \pm 2.$$

Studiamo la derivata per $x \geq 0$ A_5

$$f'(x) = \frac{1}{2\sqrt{\frac{x-2}{x-1}}}\left[\frac{x-1-(x-2)}{(x-1)^2}\right]$$

$$= \frac{1}{2}\sqrt{\frac{x-1}{x-2}}\left[\frac{1}{(x-1)^2}\right] > 0$$

$$\forall x \in D \cap \{x>0\} \quad ; \quad x \neq 2$$

f NON è derivabile in $x = \pm 2$.

$$\lim_{x \to 2^+} f'(x) = \frac{1}{2}\sqrt{\frac{1}{0^+}} = +\infty$$

$$\lim_{x \to 0^+} f'(x) = \frac{1}{2\sqrt{2}} \implies \text{per simmetria}$$

a) $$\lim_{x \to 0^-} f'(x) = \frac{-1}{2\sqrt{2}}$$

In $x=0$ PUNTO ANGOLOSO.

b) $f'(x) < 0 \quad \forall x \in D \cap \{x<0\} \; ; \; x \neq -2$

f decresce in $(-\infty, -2]$

decresce in $(-1, 0)$

cresce in $(0, 1)$

cresce in $[2, +\infty)$

A 6

$x = 0$ punto di MIN. REL.

$x = \pm 2$ punti di MIN. ASS.

∄ punti di MAX. REL. o ASS.

Il grafico qualitativo è