SVOLGIMENTO PROVA SCRITTA di ANALISI 1 del 11/1/2018

B1

$$\boxed{\text{COMPITO B}}$$

1) $z \neq -\frac{1}{2} + \frac{i\sqrt{3}}{2}$

$$2z^2 + 2(1-i\sqrt{3})z - 1 - i\sqrt{3} = 0$$

$$\Rightarrow z_{1,2} = \frac{(i\sqrt{3}-1) \pm \sqrt{(i\sqrt{3}-1)^2 + 2 + 2i\sqrt{3}}}{2}$$

$$= \frac{(i\sqrt{3}-1) \pm \sqrt{-3+1-2i\sqrt{3}+2+2i\sqrt{3}}}{2} = \frac{i\sqrt{3}-1}{2}$$

$$= -\frac{1}{2} + i\frac{\sqrt{3}}{2} \quad \text{molteplicità } 2$$

Ma $z \neq -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ $\Rightarrow$ NESSUNA SOLUZIONE.

2) convergenza assoluta

$$|a_n| = \frac{|\sin n^3|}{(n+2)^3} \le \frac{1}{(n+2)^3} \sim \frac{1}{n^3}$$

CONFRONTO

CONFRONTO ASINTOTICO

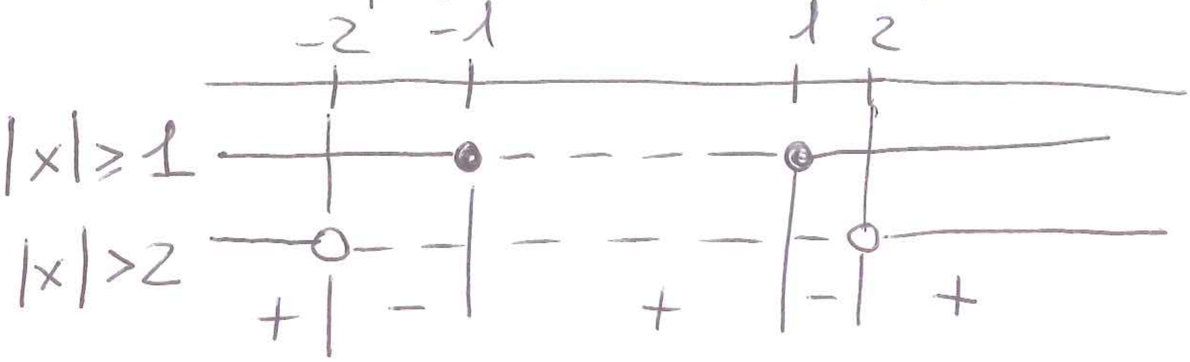
B_2

$\sum \frac{1}{n^3}$ converge $\Rightarrow \sum |a_n|$ converge

$\Rightarrow \sum a_n$ converge.

3) La funzione è definita e <u>non negativa</u>

per $\frac{1-|x|}{2-|x|} \geq 0 \iff \frac{|x|-1}{|x|-2} \geq 0$



$\Rightarrow D = (-\infty, -2) \cup (-1, 1) \cup (2, +\infty)$

f è pari. Lo studiamo solo per $x \geq 0$

$\cancel{f(0)=}$

$$f(x) = \sqrt{\frac{x-1}{x-2}} \quad \text{per } x \geq 0$$

$f(0) = \sqrt{\frac{1}{2}}$.

$f(x) = 0 \iff x = \pm 1$

$$\lim_{x\to 2^+} f(x) = \sqrt{\frac{1}{0^+}} = +\infty = \lim_{x\to -2^-} f(x)$$

$\Rightarrow$ AS. VERTICALI : $x = \pm 2$

$$\lim_{x\to +\infty} f(x) = 1 = \lim_{x\to -\infty} f(x)$$

$\Rightarrow$ AS. ORIZZONTALE per $x \to \pm\infty$: $y = 1$.

B3

$x = 0$ è punto di raccordo.

Per $D \cap \{x > 0\}$:

$$f'(x) = \frac{1}{2\sqrt{\frac{x-1}{x-2}}}\left[\frac{x-2-(x-1)}{(x-2)^2}\right] =$$

$$\frac{1}{2}\sqrt{\frac{x-2}{x-1}}\left[\frac{-1}{(x-2)^2}\right] < 0$$

$\forall x \in D \cap \{x > 0\}$; $x \neq 1$

$$\lim_{x\to 1^-} f'(0) = \frac{1}{2}\sqrt{\frac{-1}{0^-}}\left(\frac{-1}{1}\right) = \frac{-1}{0^+} = -\infty$$

$x = \pm 1$ punti di non derivabilità.

$$\lim_{x \to 0^+} f'(x) = \frac{1}{2\sqrt{2}}\left(\frac{-1}{4}\right) = \frac{-1}{8\sqrt{2}}$$

Per simmetria:

$$f'(x) < 0 \quad \forall x \in D \cap \{x < 0\} \; ; \; x \neq -1$$

$$\lim_{x \to 0^-} f'(x) = \frac{1}{8\sqrt{2}}$$

Pertanto: f cresce in $(-\infty, -2)$

cresce in $(-1, 0)$

decresce in $(0, 1)$

decresce in $(2, +\infty)$

In $x = 0$ punto angoloso di MAX. RELATIVO. ~~In $x = \pm 1$ punto di~~

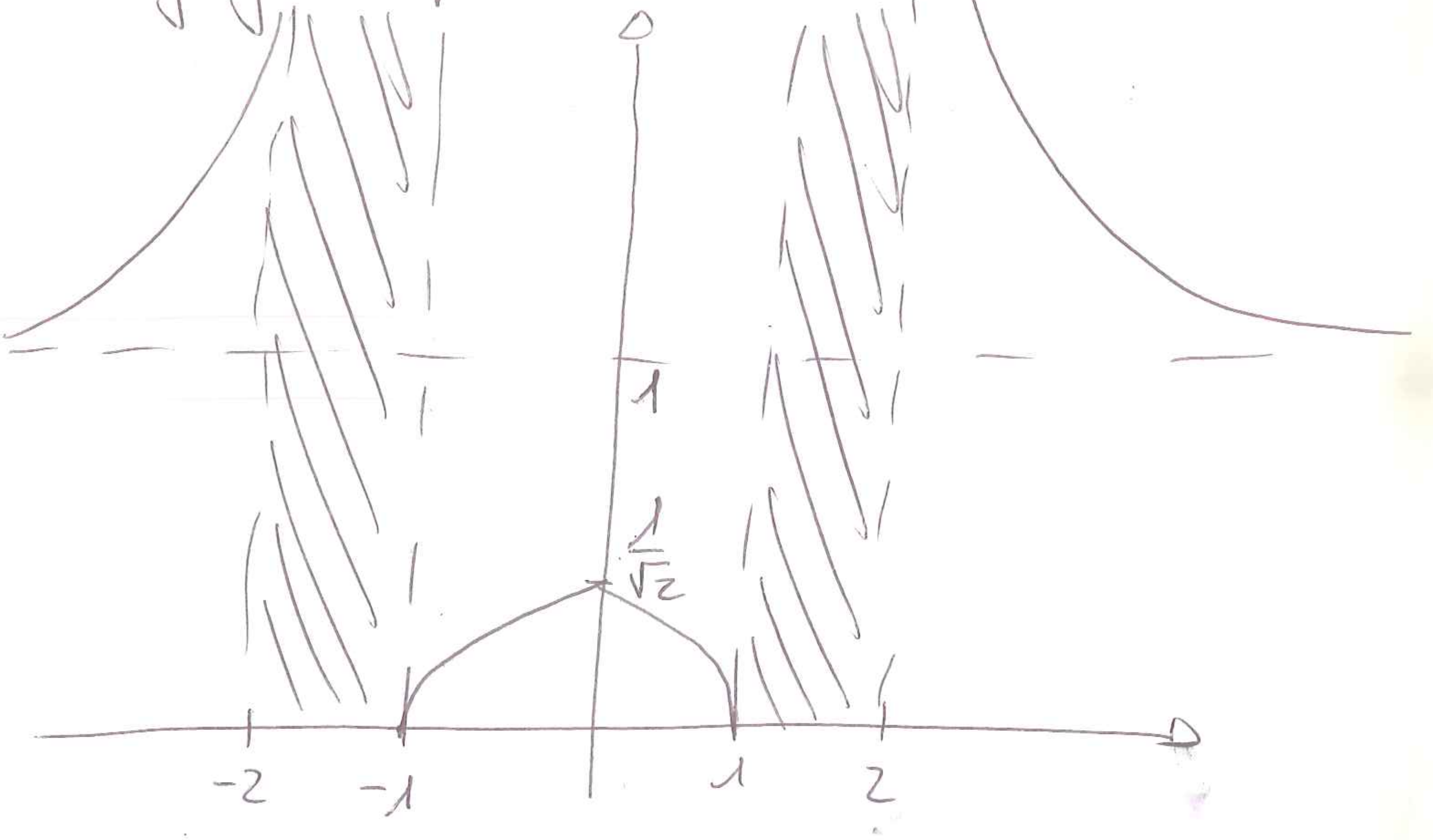
Poichè $\lim_{x \to 2^+} f(x) = +\infty \Rightarrow \nexists$ MAX. ASS.

Poichè $f(x) \geq 0$ e $f(\pm 1) = 0$

$\Rightarrow x = \pm 1$ punti di MIN. ASS.

Il grafico qualitativo è:

R5



4) $y''(x) - tg\,x\; y'(x) = 0$

Definita in $\left(-\frac{\pi}{2} + k\pi \,;\, +\frac{\pi}{2} + k\pi\right)$.

Poiché $x_0 = 0 \Rightarrow$ studiamo l'equazione in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$-tg\,x \in C^0\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \exists!$ sol. $y \in C^2\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y'(x) = z(x)$

$\Rightarrow z(x) = C_1 e^{\int tg\,x\, dx} = C_1 e^{-\ln|\cos x|}$

Ma in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\cos x > 0$

$\boxed{B_6}$

$$\Rightarrow \quad z(x) = y'(x) = C_1 e^{-\ln(\cos x)}$$
$$= \frac{C_1}{\cos x}$$

$$y'(0) = 0 = \frac{C_1}{\cos 0} = C_1 \quad \Rightarrow \quad y'(x) = 0$$

$$\Rightarrow y(x) = C_2$$

$$y(0) = 1 \quad \Rightarrow \quad C_2 = 1 \quad \Rightarrow \boxed{y(x) \equiv 1}$$

5)

$$\lim_{x \to 0} \frac{\left(-2x^2 - \frac{(-2x^2)^2}{2} + o(x^4)\right) + \left(1 + \frac{(2x)^2}{2} + \frac{(2x)^4}{4!} + o(x^4)\right) - 1}{\left(1 + \frac{x^4}{2} + o(x^4)\right) - \left(1 - \frac{x^4}{2} + o(x^4)\right)}$$

$$= \lim_{x \to 0} \frac{\cancel{-2x^2} - \frac{4x^4}{2} + \cancel{1} + \cancel{2x^2} + \frac{16}{24} x^4 \cancel{-1} + o(x^4)}{x^4 + o(x^4)}$$

$$= \lim_{x \to 0} \frac{\left(-2 + \frac{2}{3}\right)\cancel{x^4}}{\cancel{x^4}} = -\frac{4}{3}.$$