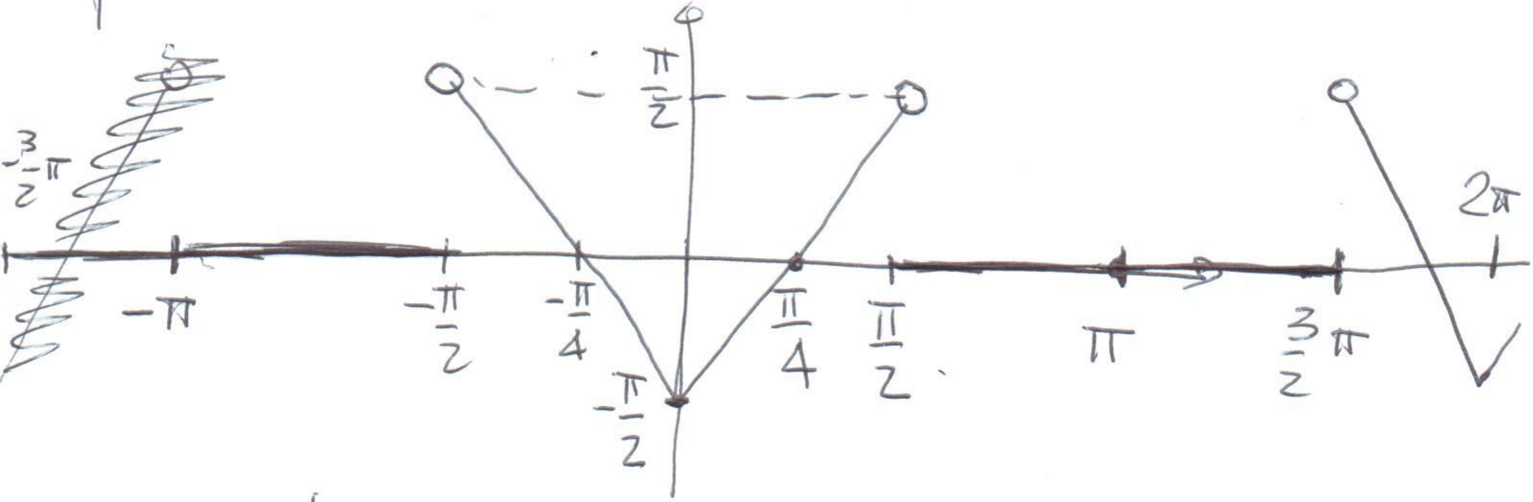
TURNO POMERIDIANO

1) La funzione, 2π-periodica, è ovviamente pari



$$S(x) = \begin{cases} f(x) & \forall x \neq \frac{\pi}{2} + k\pi \\ \frac{\pi}{4} & x = \frac{\pi}{2} + k\pi \end{cases}$$

~~$b_k = 0$~~ CONV. PUNTUALE su $\mathbb{R}$

CONV. UNIFORME in ogni

~~$\left[\frac{\pi}{2} + \varepsilon + 2k\pi, (2k+1)\pi - \frac{\pi}{2} - \varepsilon\right]$~~

$$\left[(2k+1)\pi - \frac{\pi}{2} + \varepsilon ; (2k+1)\pi + \frac{\pi}{2} - \varepsilon\right]$$

e in ogni $\left[2k\pi - \frac{\pi}{2} + \varepsilon ; 2k\pi + \frac{\pi}{2} - \varepsilon\right]$

B_2

$$a_0 = \frac{2}{\pi}\int_0^{\pi/2}\left(2x-\frac{\pi}{2}\right)dx = \frac{2}{\pi}\left[x^2-\frac{\pi}{2}x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{2}{\pi}\left[\frac{\pi^2}{4}-\frac{\pi^2}{4}\right] = 0.$$

$$a_k = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\left(2x-\frac{\pi}{2}\right)\cos(kx)\,dx =$$

$$= \frac{2}{\pi}\left[\left(2x-\frac{\pi}{2}\right)\frac{1}{k}\operatorname{sen}(kx)\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}\frac{2}{k}\operatorname{sen}(kx)\,dx\right]$$

$$= \frac{2}{\pi}\left[\frac{\pi}{2k}\operatorname{sen}\left(k\frac{\pi}{2}\right) + \frac{2}{k^2}\cos(kx)\Big|_0^{\frac{\pi}{2}}\right]$$

$$= \frac{1}{k}\operatorname{sen}\left(k\frac{\pi}{2}\right) + \frac{4}{\pi k^2}\left[\cos\left(k\frac{\pi}{2}\right)-1\right]$$

$$\Rightarrow f(x) \sim \sum_{k=1}^{\infty}\left\{\frac{1}{k}\operatorname{sen}\left(k\frac{\pi}{2}\right) + \frac{4}{\pi k^2}\left[\cos\left(k\frac{\pi}{2}\right)-1\right]\right\}\cos(kx)$$

2) Lungo l'asse x

$$\lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{e^{x^2}-1}{x^2} = 1 \neq f(0,0)$$

f NON è CONTINUA in (0,0).
Quindi NON è DIFFERENZIABILE in tale punto.

$$\frac{df}{d\vec{r}}(0,0) = \lim_{t\to 0^\pm} \frac{e^{t^2(\alpha+\beta)^2}+t^2\alpha\beta-1}{t^2\alpha^2+|t||\beta|}$$

$$= \lim_{t\to 0^\pm} \frac{t^2(\alpha+\beta)^2+t^2\alpha\beta+o(t^2)}{t(t^2\alpha^2+|t||\beta|)}$$

$$= \lim_{t\to 0^\pm} \frac{|t|^{\cancel{2}}\left[(\alpha+\beta)^2+\alpha\beta\right]}{t\cancel{|t|}\left[|t|\alpha^2+|\beta|\right]} = \cancel{\frac{0}{|\beta|} = 0}$$

$\cancel{\forall\beta\neq 0.}$

$$= \pm\frac{\left[\alpha^2+\beta^2+3\alpha\beta\right]}{|\beta|} \qquad \forall\beta\neq 0$$

Essendo $\left(\frac{df}{dr}\right)^+ \neq \left(\frac{df}{dr}\right)^-$, La f non è derivabile per alcuna direzione con $\beta \neq 0$. B4

Ad esempio, per $\beta = 1$

$$\frac{df}{d\vec{\imath}} = \frac{\partial f}{\partial y} = \lim_{k \to 0^\pm} \frac{e^{k^2} - 1}{k|k|} = \lim_{k \to 0^\pm} \frac{k^2}{k|k|} = \pm 1.$$

Per $\beta = 0$:

$$\frac{df}{d\vec{\imath}} = \frac{\partial f}{\partial x} = \lim_{h \to 0^\pm} \frac{e^{h^2} - 1}{h \cdot h^2} = \frac{1}{0^\pm} = \pm\infty$$

$\to 1$

Non esiste alcuna derivata direzionale.

3) In $\mathring{D}$:

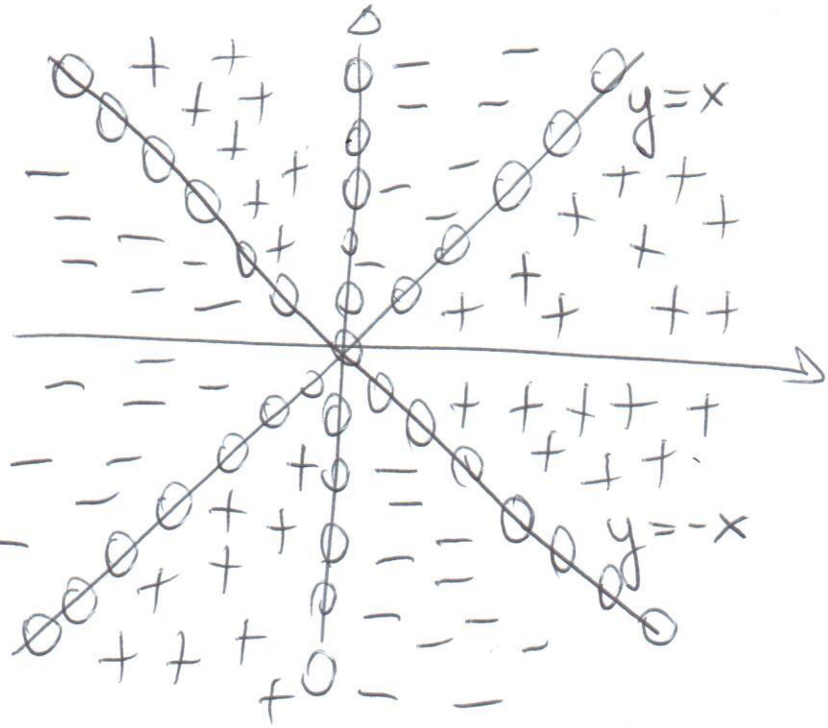
$$\begin{cases} \frac{\partial f}{\partial x} = 3x^2 - y^2 = 0 \\ \frac{\partial f}{\partial y} = -2xy = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \cup \begin{cases} y = 0 \\ x = 0 \end{cases} \Rightarrow$$

Un solo punto stazionario: $(0,0)$.

$f(x,y) = x(x^2-y^2)$

B_5



$(0,0)$ punto di sella.

f simmetrica rispetto ad asse x:

$$f(x,-y) = f(x,y)$$

~~è~~ e antisimmetrica rispetto ad asse y:

$$f(-x,y) = -f(x,y).$$

<u>Su ∂D</u>:

Con la restrizione:

$$f\Big|_{\partial D} = \cos^3\vartheta - \sin^2\vartheta\cos\vartheta = 2\cos^3\vartheta - \cos\vartheta$$

$$\frac{d}{d\vartheta}\left(f\big|_{\partial D}\right) = -6\cos^2\vartheta \sin\vartheta + \sin\vartheta$$
$$= \sin\vartheta\left(1 - 6\cos^2\vartheta\right)$$

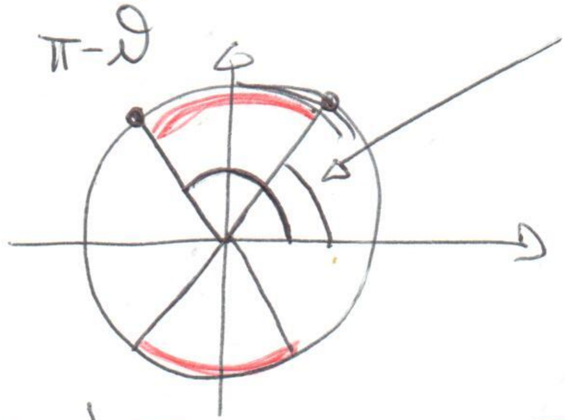
B6

$\sin\vartheta > 0 : \quad \vartheta \in (0, \pi)$

$1 - 6\cos^2\vartheta > 0 \iff \cos^2\vartheta < \frac{1}{6}$

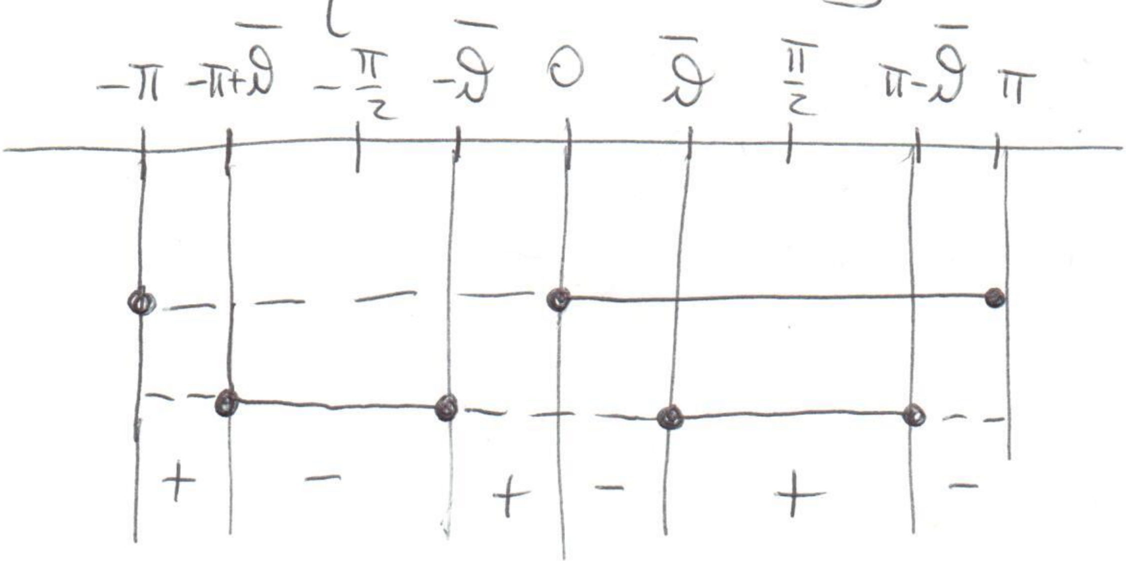
$\iff -\frac{1}{\sqrt{6}} < \cos\vartheta < \frac{1}{\sqrt{6}}$

$\bar{\vartheta} = \arccos\left(\frac{1}{\sqrt{6}}\right)$

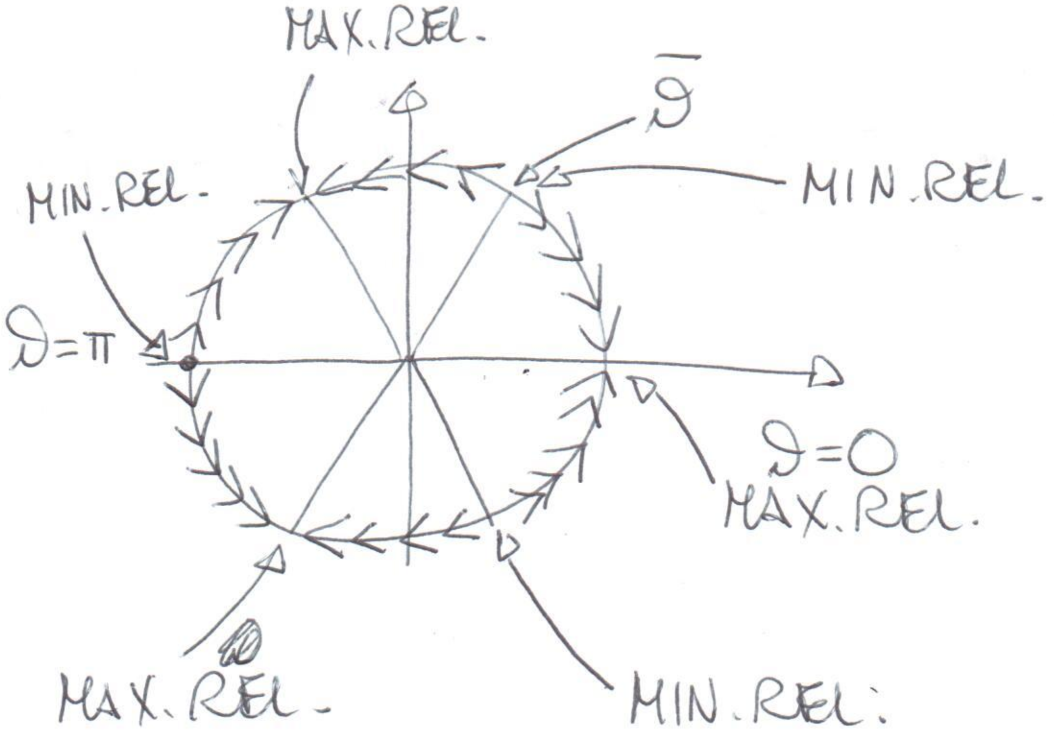


$\iff \cancel{\pi - \bar{\vartheta} < \vartheta}\ \left\{\bar{\vartheta} < \vartheta < \pi - \bar{\vartheta}\right\}$

$\cup \left\{-\pi + \bar{\vartheta} < \vartheta < -\bar{\vartheta}\right\}$



Mi sembra di



B_7

risultati concordi con le simmetrie della f.

$$f\big|_{\partial D}(0) = 1 \qquad f\big|_{\partial D}(\pi) = -1$$

$$f\big|_{\partial D}\left(\arccos\left(\frac{1}{\sqrt{6}}\right)\right) = 2\left(\frac{1}{\sqrt{6}}\right)^3 - \frac{1}{\sqrt{6}} = \frac{1}{3}\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}} = -\frac{2}{3\sqrt{6}}$$

$$= f\big|_{\partial D}\left(-\arccos\left(\frac{1}{\sqrt{6}}\right)\right)$$

$$= -f\big|_{\partial D}\left(\pi - \arccos\left(\frac{1}{\sqrt{6}}\right)\right) = -f\big|_{\partial D}\left(-\pi + \arccos\left(\frac{1}{\sqrt{6}}\right)\right)$$

MAX. ASS. in $\vartheta = 0$, cioè in $(1,0)$:

$$f(1,0) = 1$$

MIN. ASS. in $\vartheta = \pi$, cioè in $(-1,0)$:

$$f(-1,0) = -1.$$

<u>In alternativa</u>, con i moltiplicatori di Lagrange:

$$\mathcal{L}(x,y,\lambda) = x^3 - y^2 x + \lambda(x^2+y^2-1)$$

$$\begin{cases} \mathcal{L}_x = 3x^2 - y^2 + 2\lambda x = 0 \\ \mathcal{L}_y = -2xy + 2\lambda y = 2y(\lambda - x) = 0 \\ x^2+y^2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} y = 0 \\ 3x^2 + 2\lambda x = 0 \\ x = \pm 1 \end{cases} \cup \begin{cases} \lambda = x \\ 5x^2 = y^2 \\ x^2+y^2 = 1 \end{cases}$$

B8

$$\begin{cases} y = 0 \\ x = \pm 1 \\ \lambda = \mp \frac{3}{2} \end{cases} \cup \begin{cases} \lambda = x \\ y = \pm\sqrt{5}\, x \\ 6x^2 = 1 \end{cases}$$

B_9

$$\begin{cases} y = 0 \\ x = \pm 1 \\ \lambda = \mp \frac{3}{2} \end{cases} \cup \begin{cases} x = \pm \frac{1}{\sqrt{6}} \\ y = \pm \sqrt{\frac{5}{6}} \\ \lambda = \pm \frac{1}{\sqrt{6}} \end{cases}$$

(2 punti) (4 punti)

MAX. ASS. MIN. ASS.

$$f(1,0) = 1 \, ; \qquad f(-1,0) = -1$$

$$f\left(+\frac{1}{\sqrt{6}}, \pm\sqrt{\frac{5}{6}}\right) = \left(\frac{1}{\sqrt{6}}\right)^3 - \frac{5}{6}\left(\frac{1}{\sqrt{6}}\right)$$

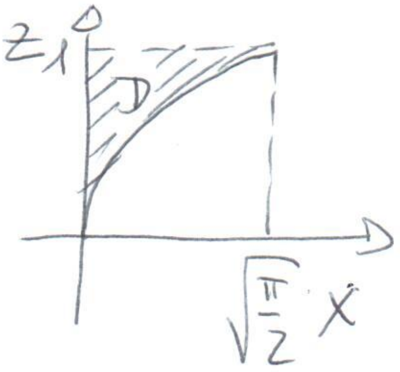
$$= \left(\frac{1}{6} - \frac{5}{6}\right)\frac{1}{\sqrt{6}} = -\frac{2}{3}\cdot\frac{1}{\sqrt{6}}$$

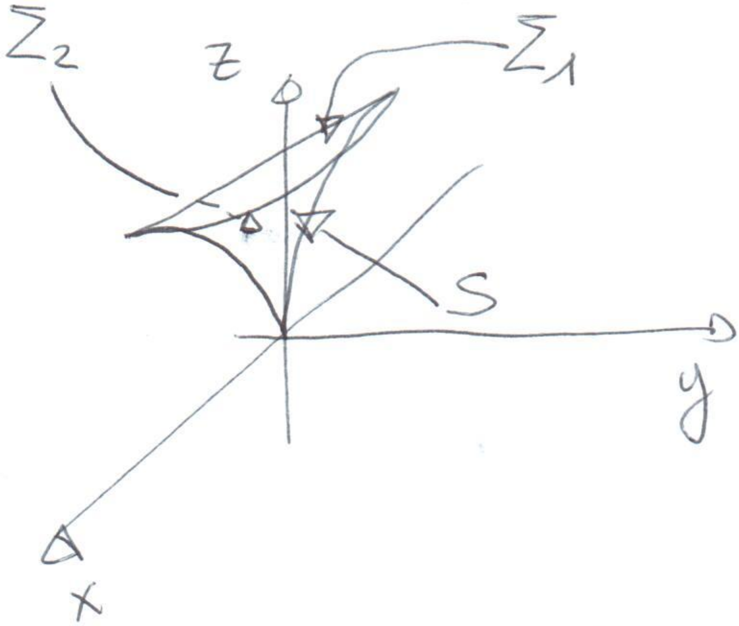
$$f\left(-\frac{1}{\sqrt{6}}, \pm\sqrt{\frac{5}{6}}\right) = \left(-\frac{1}{\sqrt{6}}\right)^3 + \frac{5}{6}\cdot\frac{1}{\sqrt{6}}$$

$$= \left(-\frac{1}{6} + \frac{5}{6}\right)\frac{1}{\sqrt{6}} = \frac{2}{3}\cdot\frac{1}{\sqrt{6}}$$

4)

B 10





Teorema della Divergenza:

$$\iiint_E \operatorname{div}\vec{F}\,dx\,dy\,dz = \int_{\partial E} \vec{F}\cdot\vec{n}_e\,dS =$$

$$= \int_{S\cup\Sigma_1\cup\Sigma_2} \vec{F}\cdot\vec{n}_e\,dS$$

B 11

Su Σ_1: $\vec{n}_e = (0, 0, 1)$

$\vec{F} = (x^2, xy, 1)$

$\Rightarrow \vec{n}_e \cdot \vec{F} = 1$.

Su Σ_2: $\vec{n}_e = (0, -1, 0)$

$\vec{F} = (x^2, 0, z)$ $\Rightarrow \vec{F} \cdot \vec{n}_e = 0$

$$\Rightarrow \int_S \vec{F} \cdot \vec{n}_e \, dS = \iiint_E (2x + x + 1) \, dx\,dy\,dz - \int_{\Sigma_1} dS$$

$$= 3 \iiint_E x \, dx\,dy\,dz + \mathrm{vol}\, E - \text{Area}\, \Sigma_1$$

$= 0$ per la simmetria del dominio e quindi del baricentro

B_{12}

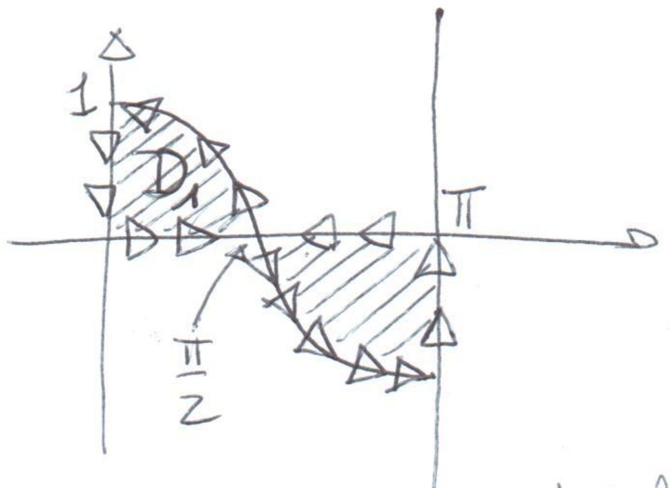
$$= \pi \iint_D x\,dx\,dz - \frac{\pi}{2}\cdot\frac{\pi}{2}$$

$$= \pi \int_0^{\sqrt{\frac{\pi}{2}}} x\,dx \left[1 - \sin(x^2)\right] dx - \frac{\pi^2}{4}$$

$$= \pi \left[\frac{x^2}{2} + \frac{1}{2}\cos(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}} - \frac{\pi^2}{4}$$

$$= \pi\left[\frac{\pi}{4} - \frac{1}{2}\right] - \frac{\pi^2}{4} = -\frac{\pi}{2}.$$

5) Il dominio D è



L'area di D è ovviamente il doppio dell'area di D_1, la cui frontiera è formata dalle curve

$$\gamma_1^+ : \begin{cases} y = 0 \\ x = t \quad ; t \in \left[0, \frac{\pi}{2}\right] \end{cases} \quad ; \quad \gamma_2^- = \begin{cases} x = t \; ; t \in \left[0, \frac{\pi}{2}\right] \\ y = \cos(t) \end{cases}$$

$$\gamma_3^- : \begin{cases} x = 0 & dx = 0 \\ y = t & ; \; t \in [0,1]. \end{cases}$$

B_{13}

$$\text{Area } D = 2 \text{Area } D_1$$

$$= -2 \int_{\partial D_1} y \, dx = -2 \int_{\gamma_2^-} y \, dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \cos t \, dt = 2 [\sin t]_0^{\frac{\pi}{2}} = 2.$$

Infatti:

$$\text{Area } D = 2 \text{Area } D_1 = 2 \int_0^{\frac{\pi}{2}} \cos x \, dx = 2.$$