SVOLGIMENTI PROVA SCRITTA di ANALISI MAT. 1 del 22/9/2017

1) $$f(x) = \frac{\sqrt{(x-1)(4x+1)}}{x}$$

a) $$\begin{cases} 4x^2-3x-1 \geq 0 \\ x \neq 0 \end{cases} \Rightarrow \begin{cases} \left(x+\frac{1}{4}\right)(x-1) \geq 0 \\ x \neq 0 \end{cases}$$

$$\Rightarrow \begin{cases} x \leq -\frac{1}{4} \;;\; x \geq 1 \\ x \neq 0 \end{cases} \Rightarrow D = \left\{x \leq -\frac{1}{4}\right\} \cup \{x \geq 1\}.$$

$f(x) = 0 \iff x = 1 \;;\; x = -\frac{1}{4}$.

$f(x) > 0 \iff x > 0$ cioè $\forall x \geq 1$.

b) $f\left(-\frac{1}{4}\right) = f(1) = 0$ NO AS. VERT.

$$f(x) = \frac{|x|\sqrt{4 - \frac{3}{x} - \frac{1}{x^2}}}{x} \xrightarrow[x \to \pm\infty]{} \pm 2$$

AS. ORIZZONTALI: $y = +2$ per $x \to +\infty$

$y = -2$ per $x \to -\infty$

c) $$f'(x) = \frac{\frac{1}{2\sqrt{4x^2-3x-1}}(8x-3)x - \sqrt{4x^2-3x-1}}{x^2}$$

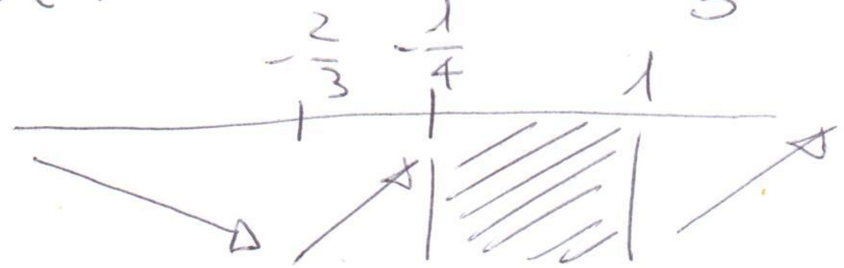
$$= \frac{1}{2x^2\sqrt{4x^2-3x-1}}\left[8x^2-3x-2(4x^2-3x-1)\right]$$

$$f'(x) = \frac{3x+2}{2x^2\sqrt{4x^2-3x-1}}$$

$$\lim_{x \to 1^+} f'(x) = \frac{5}{2 \cdot 0^+} = +\infty$$

$$\lim_{x \to \left(-\frac{1}{4}\right)^-} f'(x) = \frac{\frac{5}{4}}{\frac{2}{16} \cdot 0^+} = +\infty$$

$$f'(x) = 0 \iff x > -\frac{2}{3}$$



f decresce in $\left(-\infty, -\frac{2}{3}\right)$

cresce in $\left(-\frac{2}{3}, -\frac{1}{4}\right]$

cresce in $[1, +\infty)$

$$f\left(-\frac{2}{3}\right) = -\frac{5}{2}$$

$$f\left(-\frac{1}{4}\right) = f(1) = 0$$

Poiché $f < 0$ per $x < -\frac{1}{4}$

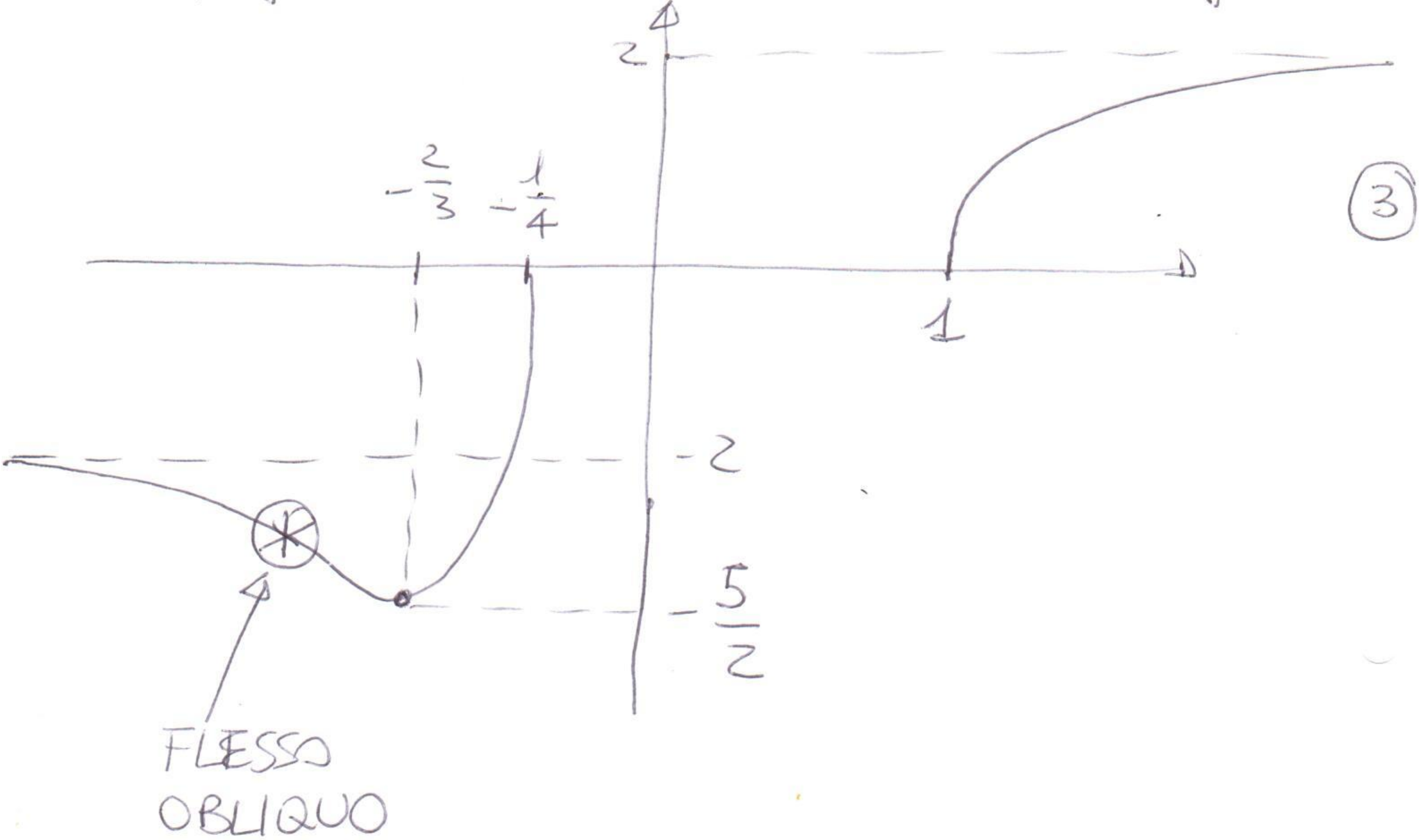
$f > 0$ per $x > 1$

$\Rightarrow$ $x = -\frac{2}{3}$ MIN. REL. e ASS.

$x = -\frac{1}{4}$ MAX. REL.

$x = 1$ MIN. REL.

d) Grafico con numero minimo di flessi:



2) $\log x - 1 \geq 0 \iff x \geq e$. Poiché stiamo integrando in $\left[\frac{1}{e^2}, \frac{1}{e}\right] \Rightarrow$

$|\log x - 1| = 1 - \log x$

$$\Rightarrow \int_{1/e^2}^{1/e} \frac{1}{x\left[\log^2 x - 2\log x + 1\right]}\,dx = \int_{1/e^2}^{1/e} \frac{1}{x(\log x - 1)^2}\,dx$$

$$= \frac{-1}{(\log x - 1)}\Bigg|_{1/e^2}^{1/e} = \frac{-1}{(-1-1)} + \frac{1}{(-2-1)}$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

3) $z^3 = -1 + i$

$$|-1+i| = \sqrt{1+1} = \sqrt{2}$$

$$\Rightarrow \quad z^3 = \sqrt{2}\left[-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\, i\right]$$

$$= \sqrt{2}\left[\cos\left(\frac{3}{4}\pi\right) + i \sin\left(\frac{3}{4}\pi\right)\right]$$

$$\Rightarrow \quad z = \sqrt[6]{2}\left[\cos\left(\frac{\frac{3}{4}\pi + 2k\pi}{3}\right) + i \sin\left(\frac{\frac{3}{4}\pi + 2k\pi}{3}\right)\right]$$

$$\cancel{\sqrt[6]{2}}$$

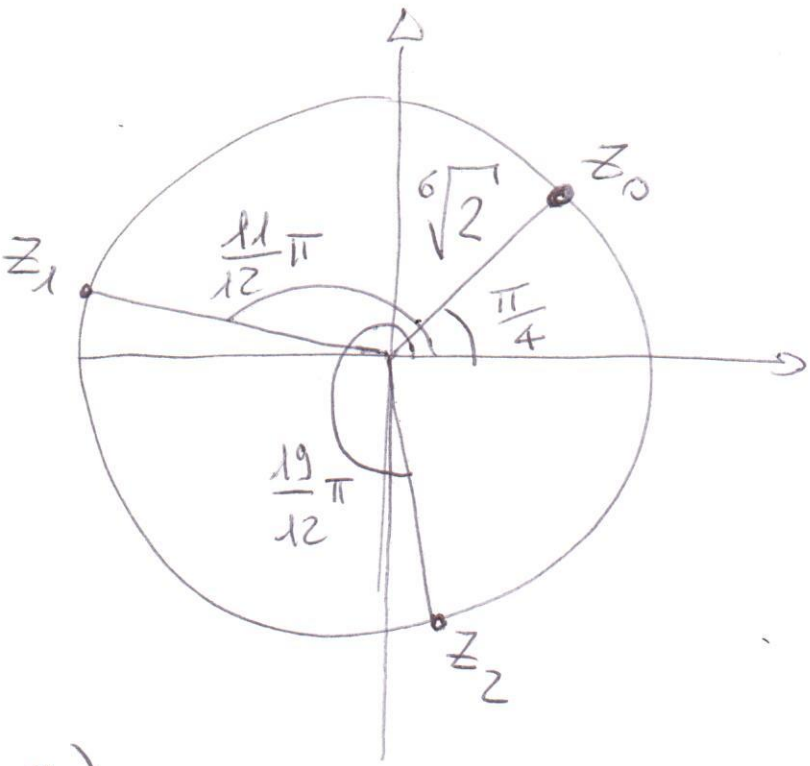
$$z_0 = \sqrt[6]{2}\left[\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right)\right] = \sqrt[6]{2}\left[\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right]$$

$$z_1 = \sqrt[6]{2}\left[\cos\left(\frac{\frac{3}{4}\pi + 2\pi}{3}\right) + i \sin\left(\frac{\frac{3}{4}\pi + 2\pi}{3}\right)\right]$$

$$= \sqrt[6]{2}\left[\cos\left(\frac{11}{12}\pi\right) + i \sin\left(\frac{11}{12}\pi\right)\right]$$

$$z_2 = \sqrt[6]{2}\left[\cos\left(\frac{\frac{3}{4}\pi + 4\pi}{3}\right) + i \sin\left(\frac{\frac{3}{4}\pi + 4\pi}{3}\right)\right]$$

$$= \sqrt[6]{2}\left[\cos\left(\frac{19\pi}{12}\right) + i \sin\left(\frac{19}{12}\pi\right)\right]$$



4) a) OMOGENEA ASSOCIATA:

$$\alpha^2 - \alpha - 2 = 0$$

$$(\alpha - 2)(\alpha + 1) = 0$$

$$y_0(x) = C_1 e^{2x} + C_2 e^{-x}$$

NON OMOGENEA:

Poiché $\alpha = -1$ è radice del polinomio caratteristico, allora

$$y_P(x) = A x e^{-x}$$

$$y_P'(x) = A(1-x)e^{-x}; \quad y_P'' = A(x-2)e^{-x}$$

$$A\left[(x-2)-(1-x)-2x\right]\cancel{e^{-x}}=3\cancel{e^{-x}}$$ ⑥

$$A(\cancel{x}\cancel{+x}\cancel{-2x}-2-1)=3$$

$$\Rightarrow A=\cancel{\tfrac{1}{3}}-1$$

$$y(x)=C_1e^{2x}+C_2e^{-x}-\cancel{\tfrac{1}{3}}xe^{-x}$$

b) $\lim\limits_{x\to+\infty} \cancel{C_1}e^{2x}=+\infty$

allora devo porre $C_1=0$

$\lim\limits_{x\to+\infty} C_2e^{-x}-\cancel{\tfrac{1}{3}}xe^{-x}=0 \quad \forall C_1\in\mathbb{R}$

$\Rightarrow$ sol. limitate: $y(x)=C_2e^{-x}-\cancel{\tfrac{1}{3}}xe^{-x}$

c) $\lim\limits_{x\to-\infty} -\cancel{\tfrac{1}{3}}xe^{-x}=+\infty$

$\Rightarrow \nexists$ sol. limitate su tutto $\mathbb{R}$.

5) a) $\log\left(1+\frac{1}{n}\right) = \frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3} + o\left(\frac{1}{n^3}\right)$

$$\Rightarrow \log^2\left(1+\frac{1}{n}\right) = \left[\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3} + o\left(\frac{1}{n^3}\right)\right]^2$$

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$$= \frac{1}{n^2} + \frac{1}{4n^4} + \cancel{\frac{1}{9n^6}} - \frac{1}{n^3} + \frac{2}{3n^4} + o\left(\frac{1}{n^4}\right)$$

$$\sin\left(\frac{1}{n^2}\right) = \frac{1}{n^2} - \frac{1}{6n^6} + o\left(\frac{1}{n^6}\right)$$

$$\Rightarrow a_n = \cancel{\frac{1}{n^2}} - \cancel{\frac{1}{n^3}} + \left(\frac{1}{4} + \frac{2}{3}\right)\frac{1}{n^4} - \cancel{\frac{1}{n^2}} + \cancel{\frac{1}{n^3}} + o\left(\frac{1}{n^4}\right)$$

$$= \frac{11}{12}\cdot\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)$$

$$\Rightarrow \lim_{n\to\infty} n^4 \cdot a_n = \frac{11}{12}$$

b) $\forall \alpha > 4 \quad \lim_{n\to\infty} n^\alpha a_n = +\infty$

$\forall \alpha < 4 \quad \lim_{n\to\infty} n^\alpha a_n = 0$

c) Poichè $a_n \sim \frac{11}{12} \cdot \frac{1}{n^4}$ 8

e $\frac{11}{12} \sum \frac{1}{n^4}$. converge, allora

$\sum a_n$ converge.