SVOLGIMENTI PROVA SCRITTA di ANALISI MAT. 1 - 22/3/2016

A1

COMPITO A

1) Omogenea associata:

$$\alpha^2 + \alpha - 6 = 0 \quad \Rightarrow \quad \alpha_1 = 2\ ;\ \alpha_2 = -3$$

$$\Rightarrow y_0(x) = C_1 e^{2x} + C_2 e^{-3x}$$

Poiché $\alpha = 2$ è radice del polinomio caratteristico, cerco l'integrale particolare nella forma

$$y_p(x) = x\,(ax+b)\,e^{2x}$$

$$\Rightarrow \quad y_p'(x) = (2ax + b + 2ax^2 + 2bx)\,e^{2x}$$

$$y_p''(x) = \left[(2a+2b) + 4ax + 4ax + 4bx + 2b + 4ax^2\right] e^{2x}$$

$$\Rightarrow \begin{bmatrix} \cancel{4ax^2} + (8a+\cancel{4b})x + 2a + 4b + \\ \cancel{2ax^2} + (2a+\cancel{2b})x + b + \\ \cancel{-6ax^2} - \cancel{6bx} \end{bmatrix} e^{2x} = 3x e^{2x}$$

$$\begin{cases} 10a = 3 \\ 2a + 5b = 0 \end{cases} \quad \Rightarrow \quad \begin{cases} a = \dfrac{3}{10} \\ b = -\dfrac{3}{25} \end{cases}$$

$$\Rightarrow y(x) = C_1 e^{2x} + C_2 e^{-3x} + \left(\frac{3}{10}x^2 - \frac{3}{25}x\right) e^{2x}$$

Poiché per $x \to +\infty$ $\;y_p(x) \to +\infty$ allora $\nexists$ soluzioni limitate.

A_2

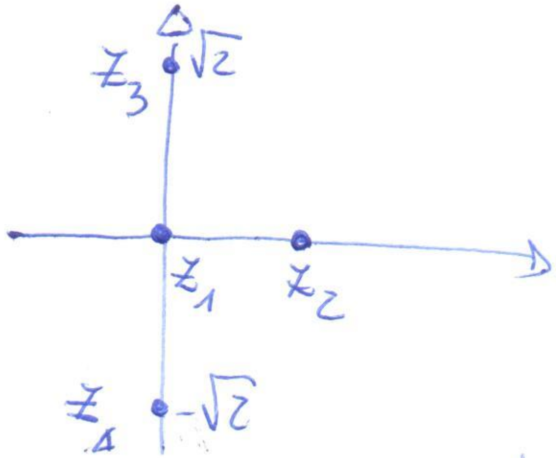
2) $(\bar{z}^2+2)\,z\,[1-\bar{z}]=0$

$\Rightarrow \{\bar{z}^2+2=0\} \cup \{z=0\} \cup \{\bar{z}=1\}$

$\{\bar{z}=\pm\sqrt{-2}\} \cup \{z=0\} \cup \{z=1\}$

$\Rightarrow \{\bar{z}=\pm i\sqrt{2}\} \cup \{z=0\} \cup \{z=1\}$

$\Rightarrow \{z=\pm i\sqrt{2}\} \cup \{z=0\} \cup \{z=1\}$



3) Criterio del rapporto:

$$\frac{a_{n+1}}{a_n} = \frac{[n!]^2}{(2n)!} \cdot \frac{(2n-2)!}{[(n-1)!]^2} = \frac{n^2}{2n\cdot(2n-1)} \longrightarrow \frac{1}{4} < 1$$

$\Rightarrow$ convergente

$$\Rightarrow \quad a_n = \frac{[(n-1)!]^2}{(2n-2)!} \longrightarrow 0.$$

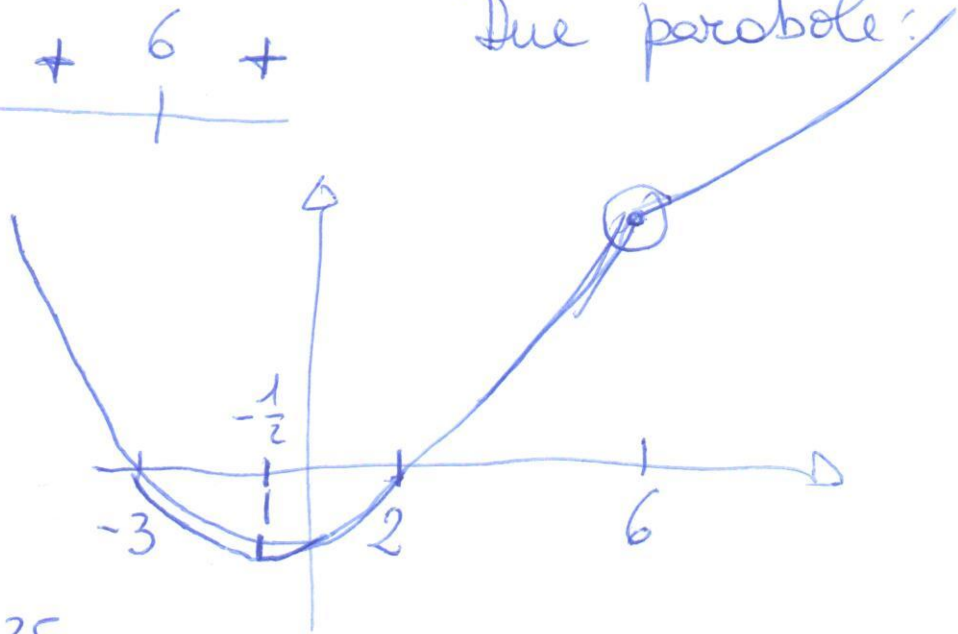
4)

$$f(x)=\begin{cases} x^2-x+6 & \text{se } x\geq 6 \\ x^2+x-6 & \text{se } x<6 \end{cases}$$

$$=\begin{cases} \cancel{(x-3)}\, \underbrace{x^2-x+6}_{>0} & \text{se } x\geq 6 \\ (x+3)(x-2) & \text{se } x<6 \end{cases} \qquad (\Delta<0)$$

$+ \quad -3 \quad - \quad 2 \quad + \quad 6 \quad +$

Due parabole:



$f(-\frac{1}{2})=\frac{-25}{4}$ MIN. ASS.

$\lim_{x\to\pm\infty} f(x)=+\infty$ NO MAX. REL. o ASS.

$$f'(x)=\begin{cases} 2x-1 & x>6 \\ 2x+1 & x<6 \end{cases}$$

$\lim_{x\to 6^+} f'(x)=11 \neq \lim_{x\to 6^-} f'(x)=13$

$x=6$ PUNTO ANGOLOSO

5)

$$f(x) = \frac{x}{\sqrt{x^2-4}} = \frac{x}{\sqrt{x+2}\cdot\sqrt{x-2}} \underset{x\to 2^+}{\sim} \frac{1}{\sqrt{x-2}}$$

A 4

integrabile in un intorno di $x_0 = 2$.

$$\int_2^4 \frac{x}{\sqrt{x^2-4}}\,dx = \sqrt{x^2-4}\,\Big|_2^4 = \sqrt{12} = 2\sqrt{3}.$$

COMPITO B

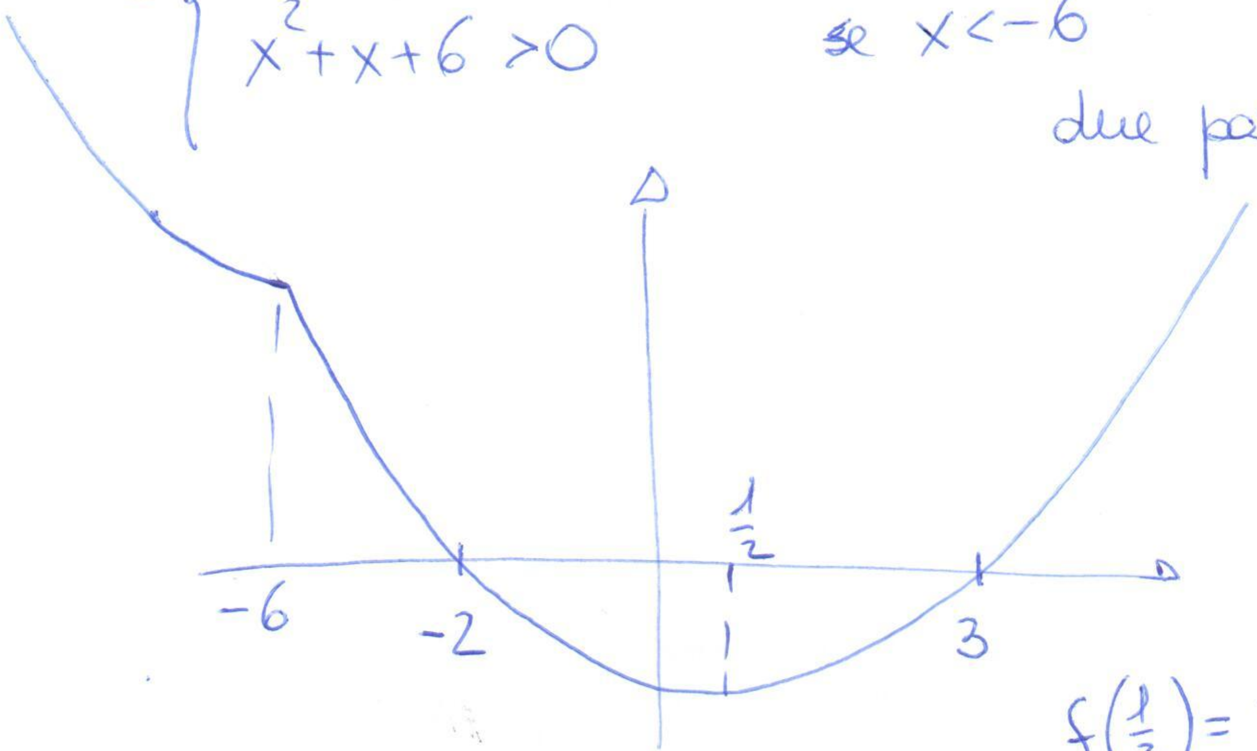
B1

1) $$f(x) = \begin{cases} x^2 - x - 6 & \text{se } x \geq -6 \\ x^2 + x + 6 > 0 & \text{se } x < -6 \end{cases}$$

$\Delta < 0$

$$= \begin{cases} (x-3)(x+2) & \text{se } x \geq -6 \\ x^2 + x + 6 > 0 & \text{se } x < -6 \end{cases}$$

due parabole



$f\left(\frac{1}{2}\right) = \frac{-25}{4} < 0$

$f(-6) = 36$

$$f'(x) = \begin{cases} 2x - 1 & \text{se } x > -6 \\ 2x + 1 & \text{se } x < -6 \end{cases}$$

$$\lim_{x \to -6^+} f'(x) = -13 \neq \lim_{x \to -6^-} f'(x) = -11$$

$x = -6$ PUNTO ANGOLOSO

$x = \frac{1}{2}$ punto di MIN. ASS. B2

$\nexists$ punti di MAX. REL. o ASS.

2) $\frac{x}{\sqrt{x^2-9}} = \frac{x}{\sqrt{x+3}\sqrt{x-3}} \underset{x \to 3}{\sim} \frac{\sqrt{3}}{\sqrt{2}\sqrt{x-3}}$

integrabile in un intorno di $x_0 = 3$

$$\int_3^6 \frac{x}{\sqrt{x^2-9}}\,dx = \sqrt{x^2-9}\Big|_3^6 = \sqrt{27} = 3\sqrt{3}.$$

3) omogeneo associato:

$\alpha^2 - \alpha - 6 = 0 \quad \Rightarrow (\alpha-3)(\alpha+2) = 0$

$\Rightarrow y_0(x) = c_1 e^{3x} + c_2 e^{-2x}$

Poiché $\alpha = 3$ è radice del polinomio caratteristico, $\Rightarrow \quad y_p(x) = x(ax+b)e^{3x}$

$\Rightarrow y_p'(x) = [2ax + b + 3(ax^2+bx)]e^{3x}$

$y_p''(x) = [2a + 6ax + 3b + 6ax + 3b + 9ax^2 + 9bx]e^{3x}$

$$= \left[2a + 6b + (12a + 9b)x + 9ax^2\right] e^{3x}$$ B_3

$$\Rightarrow \left[\begin{array}{llll} 2a + 6b & + (12a + \cancel{9b})x & + \cancel{9ax^2} \\ -b & -2ax \cancel{-3bx} & \cancel{-3ax^2} \\ & \cancel{-6bx} & \cancel{-6ax^2} \end{array}\right] \cancel{e^{3x}} = 2x\cancel{e^{3x}}$$

$$\begin{cases} 2a + 5b = 0 \\ 10a = 2 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{5} \\ b = \cancel{-\frac{1}{5}} -\frac{2}{5}a = -\frac{2}{25} \end{cases}$$

$$\Rightarrow y(x) = C_1 e^{3x} + C_2 e^{-2x} + \frac{1}{5}\left(x^2 - \frac{2}{5}x\right) e^{3x}$$

Poiché $\lim_{x \to +\infty} y_p(x) = +\infty$

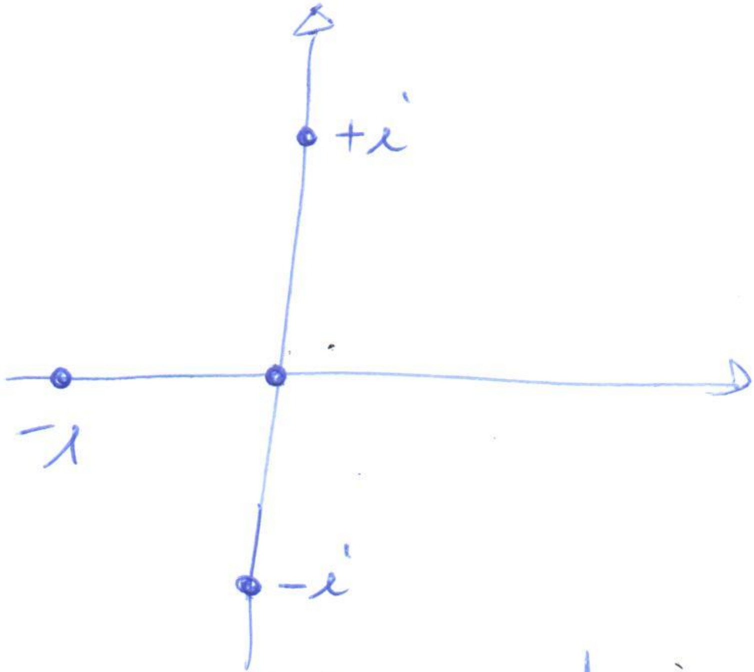
NON ESISTONO SOL. LIMITATE.

4) $(z^2+1)\bar{z}(1+z) = 0$

$\{z^2 = -1\} \cup \{\bar{z} = 0\} \cup \{z = -1\}$

$\{z = \pm i\} \cup \{z = 0\} \cup \{z = -1\}$

$\textcircled{B_4}$



5) Criterio del rapporto:

$$\frac{a_{n+1}}{a_n} = \frac{[(n-1)!]^2}{(2n-2)!} \cdot \frac{(2n-4)!}{[(n-2)!]^2} =$$

$$= \frac{(n-1)(n-1)}{(2n-2)(2n-3)} \xrightarrow[n\to\infty]{} \frac{1}{4} < 1$$

$$\Rightarrow \text{CONV.} \Rightarrow \lim_{n\to+\infty} \frac{[(n-2)!]^2}{(2n-4)!} = 0.$$