SVOLGIMENTI PROVA SCRITTA di ANALISI 1 del 13/1/2016

A_1

COMPITO A

1) $$\frac{-x^3+e^{x^3}-\cos(x^3)}{\sin(x^6)} = \frac{\cancel{-x^3}\cancel{+1}\cancel{+x^3}+\frac{x^6}{2}\cancel{-1}+\frac{x^6}{2}+o(x^6)}{x^6+o(x^6)}$$

$$= \frac{x^6+o(x^6)}{x^6+o(x^6)} \xrightarrow[x\to 0]{} 1$$

Poichè $-x^3+e^{x^3}-\cos(x^3) \underset{x\to 0}{\sim} x^6$, allora il suo ordine di infinitesimo è 6.

2) $$I_{df}: \begin{cases} x^3-1 \geq 0 \\ \cancel{-1}\leq \sqrt{x^3-1} \leq 1 \end{cases} \Rightarrow 0 \leq x^3-1 \leq 1$$

$$1 \leq x^3 \leq 2 \Rightarrow I_{df} = \{1 \leq x \leq \sqrt[3]{2}\}$$

$$= [1, \sqrt[3]{2}].$$

NO intersezione con asse y.

$f(x) = 0 \iff \sqrt{x^3-1} = 0 \iff x = 1$

Segno: $f(x) \geq 0 \iff \arcsin\left(\sqrt{x^3-1}\right) \geq 0$ A_2

$\iff \sqrt{x^3-1} \geq 0 \quad \forall x \in I_{def}$

$f\left(\sqrt[3]{2}\right) = \arcsin(1) = \frac{\pi}{2}.$

PARTE CORRETTA

$$f'(x) = \frac{1}{\sqrt{1-x^3+1}} \cdot \frac{1}{2\sqrt{x^3-1}} \cdot 3x^2 = \frac{3x^2}{2\sqrt{2-x^3}\sqrt{x^3-1}}$$

$f'(x) = 0 \iff x = 0 \notin I_{def}$

$\boxed{\nexists f' \text{ per } x = 1, \; x = \sqrt[3]{2}}$

$f'(x) > 0 \iff x \neq 0 \implies \forall x \in \left(1, \sqrt[3]{2}\right)$

f sempre crescente

$$\lim_{x \to \sqrt[3]{2}^-} f'(x) = \frac{3\sqrt[3]{4}}{2\sqrt{0^+}\cdot 1} = +\infty$$

$$\lim_{x \to 1^+} f'(x) = \frac{3}{2\cdot 1 \cdot \sqrt{0^+}} = +\infty$$

f NON derivabile in $x = 1$; $x = \sqrt[3]{2}$.

$$f''(x) = \frac{3}{2}\left[\frac{2x\cdot\sqrt{x^3-1}\cdot\sqrt{2-x^3} - x^2 \frac{1}{2\sqrt{2-x^3}\cdot\sqrt{x^3-1}}\left[-6x^5+9x^2\right]}{(2-x^3)(x^3-1)}\right]$$

$$f''(x) = \frac{3}{2}\left[\frac{4x(-x^6+3x^3-2)+x^2(6x^5-9x^2)}{2(2-x^3)^{3/2}(x^3-1)^{3/2}}\right]$$

A 2bis

$$= \frac{3}{4}\,\frac{x}{(2-x^3)^{3/2}(x^3-1)^{3/2}}\left[-4x^6+12x^3-8+6x^6-9x^3\right]$$

$$= \frac{3x}{4(2-x^3)^{3/2}(x^3-1)^{3/2}}\left[2x^6+3x^3-8\right]$$

$$x^3 = t \implies 2t^2+3t-8=0$$

$$t_{1,2} = \frac{-3\pm\sqrt{9+64}}{4}$$

$$t_1 = \frac{-3-\sqrt{73}}{4} < 0 \implies (x_1)^3 < 0 \quad \notin I_{def}$$

$t_2 = \dfrac{-3+\sqrt{73}}{4}$; si osservi che $1 < \dfrac{5}{4} < \dfrac{-3+\sqrt{73}}{4} < \dfrac{3}{2} < 2$

$$\implies 1 < x_2 < \sqrt[3]{2} \quad \in I_{def}$$

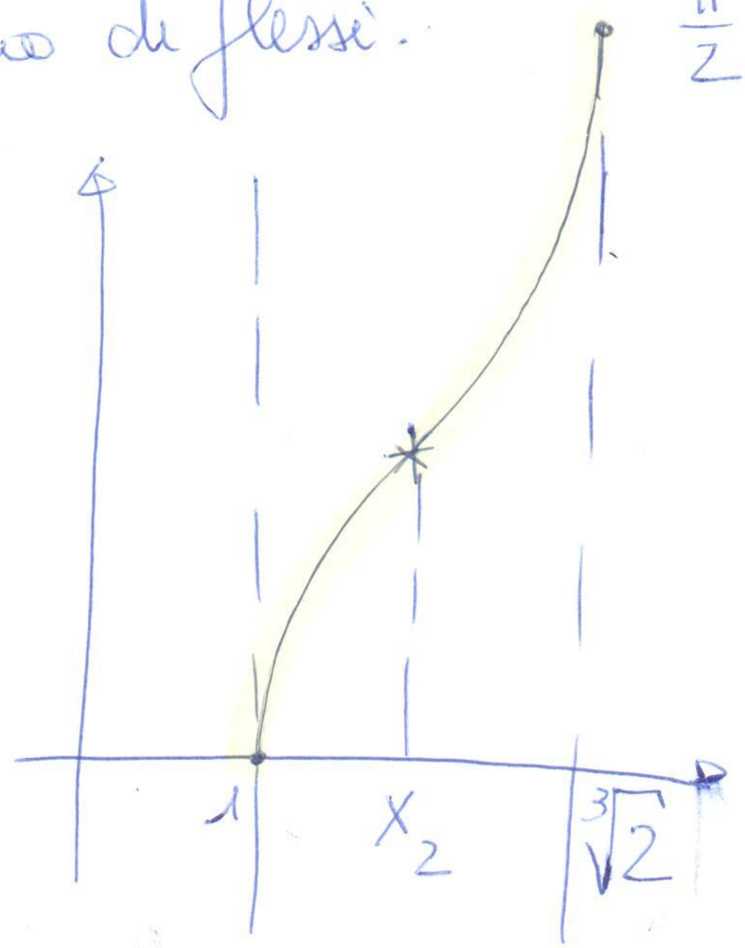
$f'' > 0$ per $t < t_1$ NON INTERESSANTE
e per $t > t_2$

$\implies f$ concava in $\left[1,\ x_2 = \sqrt[3]{\dfrac{-3+\sqrt{73}}{4}}\right)$

convessa in $\left(x_2, \sqrt[3]{2}\right]$

In x_2 flesso ascendente. $\left(A_3\right)$

Lo studio di $f''(x)$ corrisponde allo studio in ipotesi di numero minimo di flessi.



3) A_4

$$y(x) = e^{\int_0^x \sin^2 t\, dt} \cdot \left[\int_0^x e^{-\int_0^t \sin^2 s\, ds} \cdot (\cos(2t) - 1)\, dt + 2 \right]$$

$$= e^{\int_0^x \left[\frac{1}{2} - \frac{\cos(2t)}{2}\right] dt} \cdot \left[\int_0^x e^{-\int_0^t \left[\frac{1}{2} - \frac{\cos(2s)}{2}\right] ds} \cdot \left[\cos(2t) - 1\right] dt + 2 \right]$$

$$= e^{\left[\frac{1}{2}t - \frac{\sin(2t)}{4}\right]_0^x} \cdot \left[\int_0^x e^{-\left[\frac{1}{2}s - \frac{\sin(2s)}{4}\right]_0^t} \cdot \left[\cos(2t) - 1\right] dt + 2 \right]$$

$$= e^{\left[\frac{1}{2}x - \frac{\sin(2x)}{4}\right]} \left[\int_0^x e^{-\frac{1}{2}t + \frac{\sin(2t)}{4}} \cdot \left[\cos(2t) - 1\right] dt + 2 \right]$$

$$u = -\frac{1}{2}t + \frac{\sin(2t)}{4} \Rightarrow du = -\frac{1}{2} + \frac{1}{2}\cos(2t)$$

$$\Rightarrow \cos(2t) - 1 = 2du$$

$$u(0) = 0 \quad ; \quad u(x) = -\frac{1}{2}x + \frac{\sin(2x)}{4}$$

A5

$$= e^{\left[\frac{1}{2}x - \frac{\operatorname{sen}(2x)}{4}\right]} \cdot$$

$$\left[\int_0^{u(x)} 2e^{u}\,du + 2\right] = e^{\left[\frac{1}{2}x - \frac{\operatorname{sen}(2x)}{4}\right]}\left[2e^{u}\Big|_0^{u(x)} + 2\right]$$

$$= e^{\left[\frac{1}{2}x - \frac{\operatorname{sen}(2x)}{4}\right]}\left[2e^{-\frac{1}{2}x + \frac{\operatorname{sen}(2x)}{4}} \cancel{-2} \cancel{+2}\right]$$

$$= 2$$

Quindi la soluzione è $y(x) \equiv 2$.

~~Infatti, si può riscrivere l'equaz~~

Infatti, poiché $\cos(2x) - 1 = -2\operatorname{sen}^2 x$

possiamo riscrivere l'equazione nella forma

$$y' - \operatorname{sen}^2(x)\, y = -2\operatorname{sen}^2 x$$

che è soddisfatta da $y \equiv 2$, la quale soddisfa anche il problema di Cauchy.

A_6

4) $z+i=w \qquad z=w-i$

$$w^3=\left(\frac{1+\sqrt{3}\,i}{\sqrt{3}-i}\right)\cdot\frac{(\sqrt{3}+i)}{(\sqrt{3}+i)}=\frac{\cancel{\sqrt{3}}-\cancel{\sqrt{3}}+3i+i}{3+1}$$

$$=i=\cos\left(\frac{\pi}{2}\right)+i\operatorname{sen}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow\begin{cases} w_0=\cos\left(\frac{\pi}{6}\right)+i\operatorname{sen}\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}+i\frac{1}{2} \\ w_1=\cos\left(\frac{5}{6}\pi\right)+i\operatorname{sen}\left(\frac{5}{6}\pi\right)=-\frac{\sqrt{3}}{2}+i\frac{1}{2} \\ w_2=\cos\left(\frac{9}{6}\pi\right)+i\operatorname{sen}\left(\frac{9}{6}\pi\right)=-i \end{cases}$$

$$\Rightarrow\begin{cases} z_0=w_0-i=\frac{\sqrt{3}}{2}-\frac{i}{2} \\ z_1=w_1-i=-\frac{\sqrt{3}}{2}-\frac{i}{2} \\ z_2=w_2-i=-2i \end{cases}$$

5) $$a_n=\left[\left(1-\frac{1}{n^2+4}\right)^{-(n^2+4)}\right]^{-\frac{(n^2+1)}{(n-1)(n^2+4)}}\xrightarrow[n\to\infty]{}e^0=1$$

Poiché $a_n \not\to 0 \Rightarrow$ la serie diverge.