A_1

SVOLGIMENTO PROVA SCRITTA di ANALISI 1 del 17/2/2017

COMPITO A

1)
$$a(x) = \cot x \in C^\infty(0,\pi) \ni \frac{\pi}{2}$$
$$b(y) = 1+e^{-y} \in C^\infty(\mathbb{R})$$
$$\Rightarrow \exists I\left(\tfrac{\pi}{2}\right) \subset (0,\pi) \text{ tale che } \exists! \text{ sol. } y \in C^1(I).$$

$$b(y) = 1+e^{-y} \neq 0 \quad \forall y \in \mathbb{R} \Rightarrow \text{NO SOL. SING.}$$

Metodo separazione variabili

$$\int \frac{dy}{1+e^{-y}} = \int \cot x \, dx$$

$$\int \frac{e^y}{1+e^y} dy = \int \frac{\cos x}{\sin x} dx$$

$$\log|1+e^y| = \log|\sin x| + \log C$$

$C>0$ ma $1+e^y>0$; $\sin x > 0$ in $(0,\pi)$

$$\Rightarrow \quad 1+e^y = C \sin x \quad ; \quad C>0.$$

Δ_2

$$y\left(\frac{\pi}{2}\right)=0 \Rightarrow 2=C$$

$$\Rightarrow e^y = 2\operatorname{sen}x - 1$$

$$\Rightarrow y=\log(2\operatorname{sen}x-1)$$

La soluzione è definita per

$$2\operatorname{sen}x-1>0$$

$$\Rightarrow \begin{cases} \operatorname{sen}x > \frac{1}{2} \\ x\in(0,\pi) \end{cases} \Rightarrow x\in\left(\frac{\pi}{6},\frac{7}{6}\pi\right).$$

2) $$\lim_{x\to 0}\frac{e^{x-\cos x}}{\operatorname{sen}x+2e^x}\lim_{x\to 0}\frac{\log(1+\operatorname{sen}^4 x)}{(\cos x+\cosh x-2e^{x^4})}$$

$$=\frac{e^{-1}}{2}\cdot\lim_{x\to 0}\frac{\operatorname{sen}^4 x}{\cancel{x-\frac{x^3}{6}+\frac{x^5}{5!}+x+\frac{x^3}{}} \quad 1-\cancel{\frac{x^2}{2!}}+\frac{x^4}{4!}+1+\cancel{\frac{x^2}{2!}}+\frac{x^4}{4!}-2(1+x^4)+o(x^4)}$$

$$=\frac{1}{2e}\lim_{x\to 0}\frac{\cancel{x^4}}{\cancel{x^4}\left(\frac{2}{\cancel{24}\,12}-2\right)}=\frac{1}{2e}\cdot\left(\frac{1}{\frac{-23}{12}}\right)$$

A 3

$$= \frac{-6}{23e}.$$

3)

$$(z-2)^3\left[2(z-2)^2+1\right]=0$$

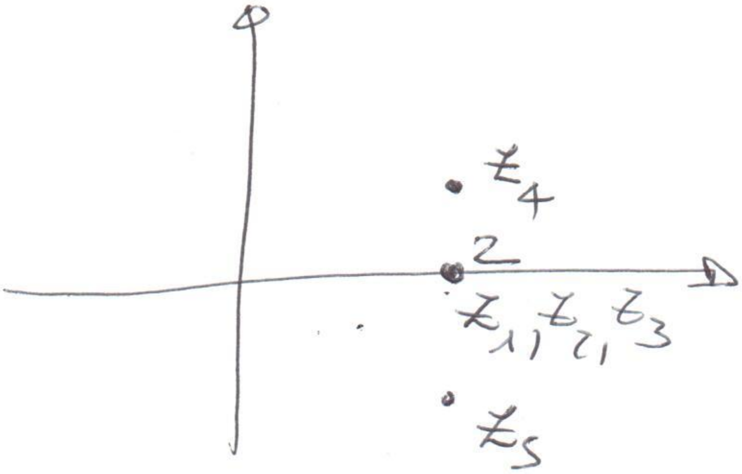
$$\Rightarrow \{z=2 \quad m_a=3\} \cup \left\{(z-2)^2=-\frac{1}{2}\right\}$$

$$(z-2)=\pm\sqrt{-\frac{1}{2}}$$

$$z=2\pm i\sqrt{\frac{1}{2}}$$

$$\Rightarrow z_{1,2,3}=2 \quad ; \quad z_4=2+i\frac{1}{\sqrt{2}} \quad ; \quad z_5=2-i\frac{1}{\sqrt{2}}$$

L'equazione, algebrica di grado 5, ammette 5 soluzioni secondo quanto previsto dal Teorema Fondamentale dell'Algebra.



Δ_4

4) $$a_n = \frac{1}{2n^2+(-1)^n} \xrightarrow[n\to\infty]{} 0$$

<u>n pari:</u>

$$a_n = \frac{1}{2n^2+1} \; ; \; a_{n+1} = \frac{1}{2(n+1)^2-1}$$

$$\Rightarrow a_n \geq a_{n+1} \iff \frac{1}{2n^2+1} \geq \frac{1}{2(n+1)^2-1}$$

$$\iff 2(n+1)^2-1 \geq 2n^2+1$$

$$\iff 4n \geq 0 \qquad \forall n \in \mathbb{N}$$

<u>n dispari:</u>

$$a_n = \frac{1}{2n^2-1} \; ; \; a_{n+1} = \frac{1}{2(n+1)^2+1}$$

$$a_n \geq a_{n+1} \iff \frac{1}{2n^2-1} \geq \frac{1}{2(n+1)^2+1}$$

$$\iff 2(n+1)^2+1 \geq 2n^2-1$$

$$\iff 4n+3 \geq -1 \iff n \geq -1$$

$$\forall n \in 2\mathbb{N}+1$$

$\Rightarrow \{a_n\}$ non crescente.

b) poichè $a_n \to 0$ e decrescente

$$\Rightarrow \sum (-1)^n \frac{1}{2n^2+(-1)^n}$$

converge per il criterio di Leibniz.

5)

$$f(x) = \left|\log\left[(x-3)^2\right]\right| \geq 0$$

$$D = \{x \neq 3\}$$

$$\log\left[(x-3)^2\right] \geq 0 \iff (x-3)^2 \geq 1$$

$\cancel{f(x) \neq}$

$$\iff x \leq +2;\ x \geq +4.$$

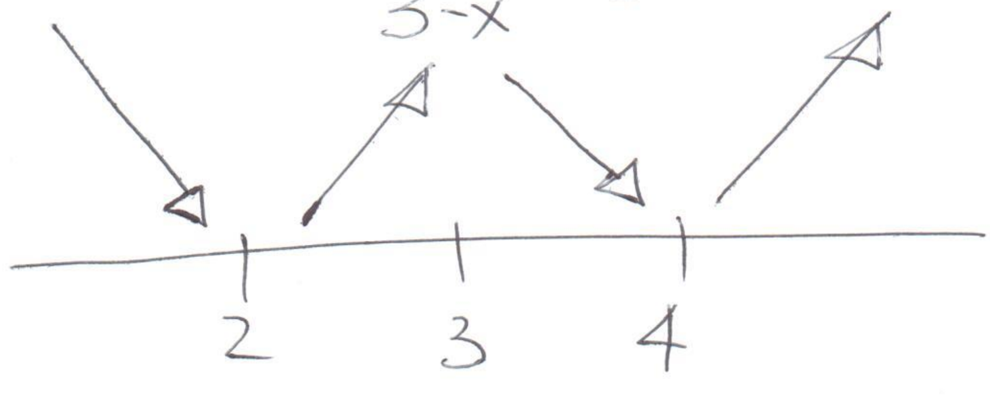
A6

$f(x) = 0 \iff x = 2 \, ; \, x = 4$

$$f(x) = \begin{cases} \log\left[(x-3)^2\right] & x \leq 2 \, ; \, x \geq 4 \\ -\log\left[(x-3)^2\right] & 2 < x < 4 \end{cases} \qquad x \neq 3$$

$$f(x) = \begin{cases} 2\log(x-3) & x \geq 4 \\ -2\log(x-3) & 3 < x < 4 \\ -2\log(3-x) & 2 < x < 3 \\ 2\log(3-x) & x \leq 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{2}{x-3} > 0 & x > 4 \\ -\dfrac{2}{x-3} < 0 & 3 < x < 4 \\ \dfrac{2}{3-x} > 0 & 2 < x < 3 \\ -\dfrac{2}{3-x} < 0 & x < 2 \end{cases}$$



A7

$$\lim_{x \to 3^{\pm}} f(x) = +\infty$$

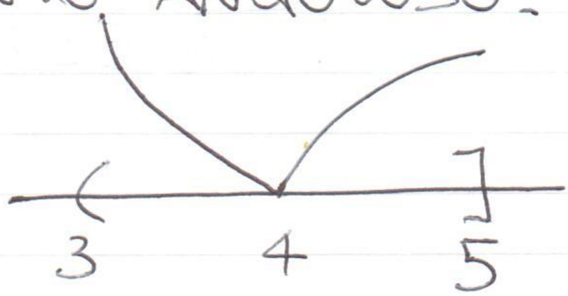
$$\lim_{x \to 2^+} f'(x) = 2 > 0 \; ; \; \lim_{x \to 2^-} f'(x) = -2 < 0$$

PUNTO ANGOLOSO

$$\lim_{x \to 4^+} f'(x) = 2 > 0 \; ; \; \lim_{x \to 4^-} f'(x) = -2 < 0$$

PUNTO ANGOLOSO.

In $(3,5]$



$x = 4$ MIN. ASS.

$f(5) = \log 4$ MAX. REL.

$\lim_{x \to 3^+} f(x) = +\infty$ $\nexists$ MAX. ASS.

A_8

Più rapidamente, osservando che

$$f(x) = \left| \log\left[(x-3)^2\right] \right|$$

e ponendo $t = x-3$, si ha il grafico di

$$\tilde{f}(t) = \left|\log(t^2)\right| = 2\left|\log|t|\right|$$

funzione pari.

grafico di $g(t) = \log|t|$:

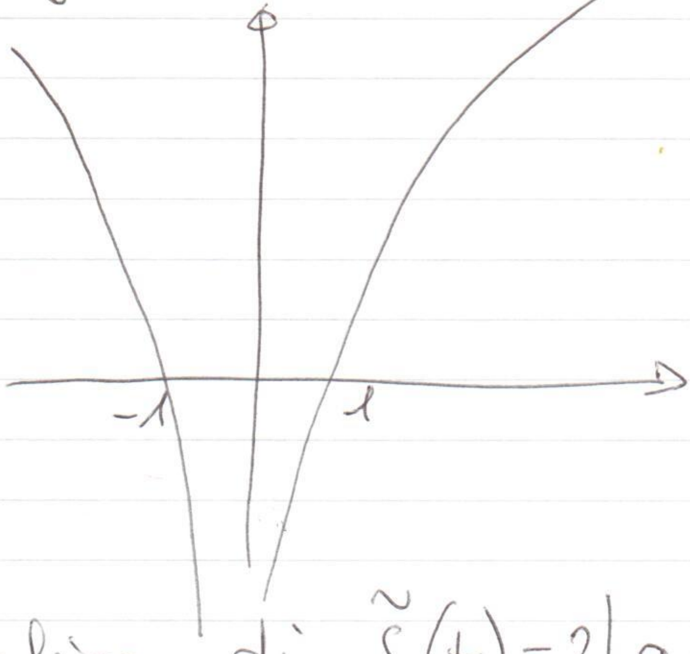
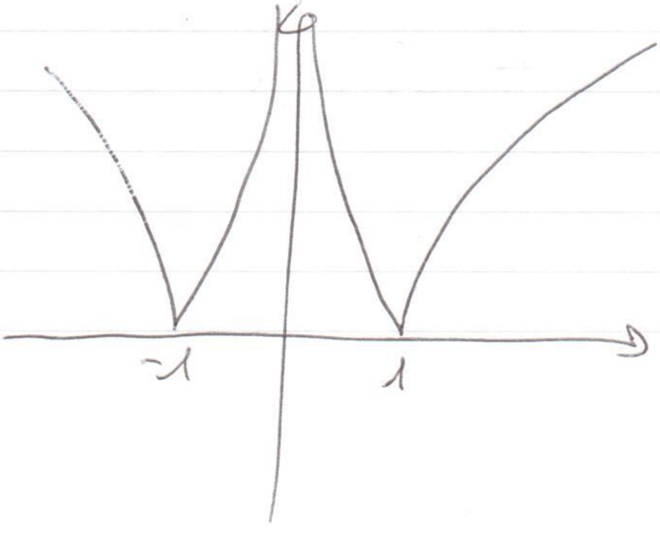


grafico di $\tilde{f}(t) = 2|g(t)| = 2\left|\log|t|\right|$



Traslando di 3, otteniamo il grafico di f(x): (A9)