SVOLGIMENTO PROVA SCRITTA di ANALISI MAT. 1 del 13/1/2015

$\boxed{\text{COMPITO B}}$ B_1

1) $$f(x) = \frac{\log[(x-2)^2]}{x-2} = \frac{2\log(|x-2|)}{x-2}$$

$$D = \{x \in \mathbb{R} \mid x \neq 2\} = (-\infty, 2) \cup (2, +\infty)$$

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{2\log(|x|)}{x} = 0^{\pm}$$

$y = 0$ AS. ORIZZ. a $\pm\infty$

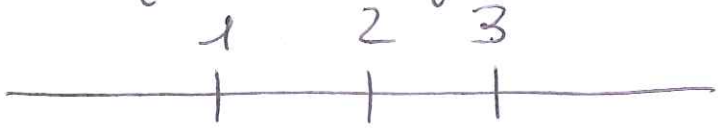
$$\lim_{x \to 2^{\pm}} f(x) = \frac{2\log(0^+)}{0^{\pm}} = \mp\infty$$

$x = 2$ AS. VERT.

Segno di f:

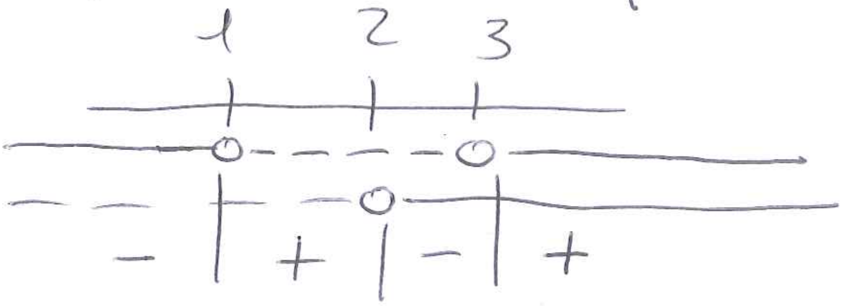
$$\begin{cases} |x-2| > 1 \\ x > 2 \end{cases} \Rightarrow \begin{cases} x-2 > 1 \\ x > 2 \end{cases} \cup \begin{cases} x-2 < -1 \\ x > 2 \end{cases}$$

$$\Rightarrow \begin{cases} x > 3 \\ x > 2 \end{cases} \cup \begin{cases} x < 1 \\ x > 2 \end{cases}$$



Segno di f:

$$\begin{cases} |x-2|>1 \\ x>2 \end{cases} \Rightarrow \begin{cases} 1>x\,;\; x>3 \\ x>2 \end{cases}$$



Intersezioni con assi:

ASSE x:

$$f(x)=0 \iff |x-2|=1 \iff x=1;\; x=3$$

ASSE y:

$$f(0)=-\log 2<0$$

$$f(x)=\begin{cases} \dfrac{2\log(x-2)}{x-2} & \text{se } x>2 \\[2ex] \dfrac{2\log(2-x)}{x-2} & \text{se } x<2 \end{cases}$$

$$f'(x)=\begin{cases} 2\left[\dfrac{1-\log(x-2)}{(x-2)^2}\right] & \text{se } x>2 \\[2ex] 2\left[\dfrac{1-\log(2-x)}{(x-2)^2}\right] & \text{se } x<2 \end{cases}$$

Per $x>2$: $f'(x)>0 \iff \log(x-2)<1$

$\iff x<e+2$

$$f(e+2)=\frac{2}{e}$$

Per $x<2$: $\quad f'(x)>0 \iff \log(2-x)<1$ (B3)

$\iff x>2-e$.

$$f(2-e)=\frac{-2}{e}$$

f decresce in $(-\infty, 2-e)$; cresce in $(2-e, 2)$;

cresce in $(2, e+2)$; decresce in $(e+2, +\infty)$.

$x=2-e$ punto di MIN. REL.

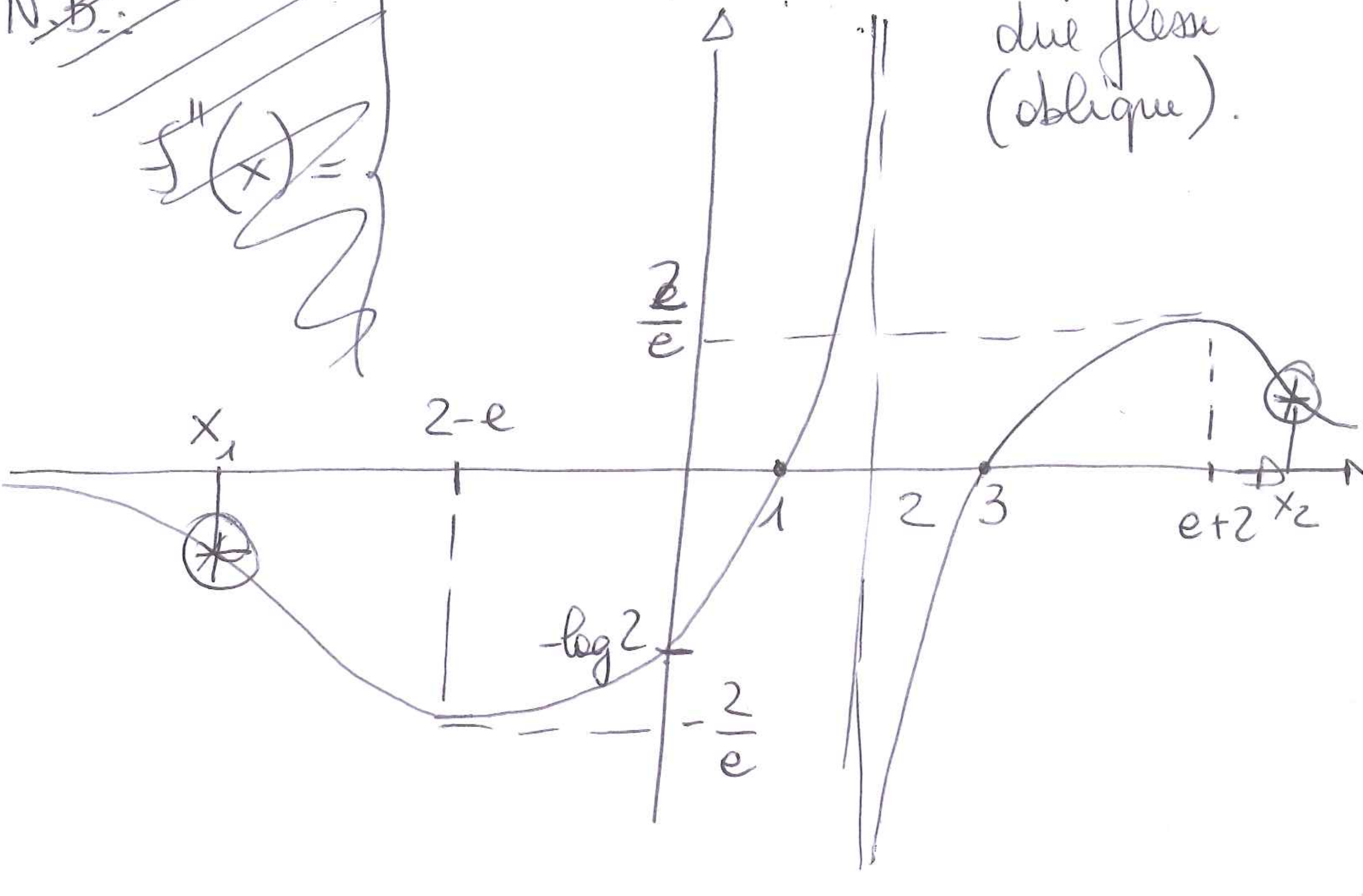
$x=e+2$ punto di MAX. REL.

$\nexists$ MAX. o MIN. ASS.: $\inf f=-\infty$

$\sup f=+\infty$.

N.B.: $\cancel{f''(x)=}$

Si prevedono due flessi (obliqui).



N.B.: Per $x>2$

$$f''(x) = 2\left[\frac{-\frac{1}{\cancel{x-2}}\cancel{(x-2)}^{\cancel{2}} - (1-\log(x-2))\,2\cancel{(x-2)}}{(x-2)^{\cancel{4}\,3}}\right]$$

$$= \frac{2}{(x-2)^3}\left[-3+2\log(x-2)\right] > 0$$

$$\iff 2\log(x-2) > 3 \iff x > 2+e^{3/2}$$

Per $x<2$:

$$f''(x) = 2\left[\frac{-\frac{1}{\cancel{x-2}}\cancel{(x-2)}^{\cancel{2}} - (1-\log(2-x))\,2\cancel{(x-2)}}{(x-2)^{\cancel{4}\,3}}\right]$$

$$= \frac{2}{(x-2)^3}\left[-3+2\log(2-x)\right]$$

Essendo $x-2<0 \implies (x-2)^3<0$

$$f''(x)>0 \iff -3+2\log(2-x)<0$$

$$\iff \log(2-x) < \frac{3}{2} \iff x > 2-e^{3/2}$$

f CONCAVA in $(-\infty, 2-e^{3/2})$; f CONVESSA in $(2-e^{3/2}, 2)$; CONCAVA in $(2, 2+e^{3/2})$; CONVESSA in $(2+e^{3/2}, +\infty)$.

2 FLESSI in $x_1 = 2-e^{3/2}$; $x_2 = 2+e^{3/2}$.

B_5

2) $\Re e(\bar{z}) = \Re e(z) = x$

$z^2 = x^2 - y^2 + 2ixy \Rightarrow \Im m(z^2) = 2xy$

$\Rightarrow e^{x + i \frac{2\cancel{x}y}{\cancel{x}}} = 2\left[+\frac{\sqrt{3}}{2} - \frac{i}{2}\right]$

$(x \neq 0)$

$$e^x \cdot e^{2yi} = 2 \cdot e^{-i\frac{\pi}{6}}$$

Per la periodicità dell'esponenziale immaginario, si ha

$$\begin{cases} e^x = 2 \\ 2y = \frac{-\pi}{6} + 2k\pi \end{cases} ; \; k \in \mathbb{Z}$$

$$\begin{cases} x = \log 2 \\ y = \frac{-\pi}{12} + k\pi \end{cases} ; \; k \in \mathbb{Z}$$

3) La serie è di segno qualsiasi, a causa della presenza di $\cos(n)$.
Studiamo la convergenza assoluta.

B6

$$\left| n^2 \cos(n) \left[\log\left(1+\frac{1}{n^2}\right) - \frac{1}{n^2} \right] \right|$$

criterio del confronto

$$\leq n^2 \left| \log\left(1+\frac{1}{n^2}\right) - \frac{1}{n^2} \right|$$

confronto asintotico

$$\sim n^2 \left| -\frac{1}{2n^4} \right| = \frac{1}{2n^2}$$

Poichè $\sum \frac{1}{2n^2}$ converge, allora converge assolutamente, e quindi semplicemente, la serie in esame.

4) Per $x \to +\infty$

$$e^{1/x} - 1 \sim \frac{1}{x} \qquad e^{1/x} \to e^{\frac{1}{\infty}} = e^0 = 1$$

$$\cancel{\sin\left(\frac{1}{x} - 1\right) \sim \frac{1}{x}} \Rightarrow \sin(e^{1/x} - 1) \sim \frac{1}{x}$$

$$\Rightarrow f(x) \underset{x \to +\infty}{\sim} \frac{1}{x^3}$$ che è integrabile a $+\infty$.

Operiamo la sostituzione

$$e^{1/x} - 1 = t \Rightarrow dt = -\frac{1}{x^2} e^{\frac{1}{x}} dx$$

$$t(1) = \cancel{\sqrt{e} - 1}; \ e - 1; \quad t(+\infty) = 0$$

$$\Rightarrow \int_1^{+\infty} f(x)\,dx = -\int_{e-1}^{0} \sin t\,dt = \cos t \Big|_{e-1}^{0}$$

$$= 1 - \cos(e-1)$$

5) OMO: $\alpha^2 + 2\alpha = 0 \Rightarrow \alpha(\alpha+2) = 0$

$$\Rightarrow y_0(x) = C_1 + C_2 e^{-2x}$$

Poichè $\alpha = 0$ è radice del polinomio caratteristico con $m_a = 1$, allora

$$w(x) = Ax \quad ; \; w'(x) = A \; ; \; w''(x) = 0$$

$$\Rightarrow \quad 2A = \frac{2}{5} \quad \Rightarrow A = \frac{1}{5}$$

$$\Rightarrow y_{NO}(x) = C_1 + C_2 e^{-2x} + \frac{1}{5}x$$

$$y(0) = \underline{C_1 + C_2 = 1}$$

$$y'(x) = -2C_2 e^{-2x} + \frac{1}{5}$$

$$y'(0) = -2C_2 + \frac{1}{5} = 1$$

$$\Rightarrow C_2 = -\frac{2}{5} \; ; \; C_1 = 1 - C_2 = \frac{7}{5}$$

$$\Rightarrow \quad y(x) = \frac{7}{5} - \frac{2}{5} e^{-2x} + \frac{1}{5}x.$$