SVOLGIMENTI PROVA SCRITTA di ANALISI 1 del 13/1/2016

B_1

COMPITO B

1) $z - i = w$

$$w^3 = \frac{1-\sqrt{3}\,i}{\sqrt{3}+i} = \frac{(1-\sqrt{3}\,i)}{(\sqrt{3}+i)} \cdot \frac{(\sqrt{3}-i)}{(\sqrt{3}-i)} =$$

$$= \frac{\cancel{\sqrt{3}} - \cancel{\sqrt{3}} - i - 3i}{4} = -i = \left[\cos\left(-\frac{\pi}{2}\right) + i \operatorname{sen}\left(-\frac{\pi}{2}\right)\right]$$

$$\Rightarrow \begin{cases} w_0 = \cos\left(-\frac{\pi}{6}\right) + i \operatorname{sen}\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - i\,\frac{1}{2} \\ w_1 = \cos\left(\frac{3}{6}\pi\right) + i \operatorname{sen}\left(\frac{3}{6}\pi\right) = i \\ w_2 = \cos\left(\frac{7}{6}\pi\right) + i \operatorname{sen}\left(\frac{7}{6}\pi\right) = \frac{-\sqrt{3}}{2} - i\,\frac{1}{2} \end{cases}$$

$$\begin{cases} z_0 = \cancel{\frac{\sqrt{3}}{2} - i\frac{1}{2}}\; w_0 + i = \frac{\sqrt{3}}{2} + \frac{1}{2}i \\ z_1 = w_1 + i = 2i \\ z_2 = w_2 + i = -\frac{\sqrt{3}}{2} + \frac{1}{2}i \end{cases}$$

B2

2) $$\left(1+\frac{1}{n^2+1}\right)^{\frac{n^2+2}{n-1}} = \left[\left(1+\frac{1}{n^2+1}\right)^{n^2+1}\right]^{\frac{n^2+2}{(n^2+1)(n-1)}}$$

$$\xrightarrow[n\to\infty]{} e^0 = 1$$

Poiché $a_n \not\to 0$, $\Rightarrow$ la serie diverge.

3) $$\frac{\operatorname{arctg}(x^2)+\operatorname{sen}(x^2)-2x^2}{\log(1+x^6)}$$

$$= \frac{\cancel{x^2} - \frac{x^6}{3} + \cancel{x^2} - \frac{x^6}{6} - \cancel{2x^2} + o(x^6)}{x^6+o(x^6)} \underset{x\to 0}{\sim} \frac{-\frac{x^6}{2}}{x^6} \longrightarrow -\frac{1}{2}$$

Poiché $\operatorname{arctg}(x^2)+\operatorname{sen}(x^2)-2x^2 \sim -\frac{x^6}{2}$

allora il suo ordine di infinitesimo è l'esponente 6.

B3

4) $I_{def}: \begin{cases} 2-x^3 \geq 0 \\ \cancel{-1} \leq \sqrt{2-x^3} \leq 1 \end{cases}$

$= \begin{cases} x^3 \leq 2 \\ \cancel{2-x^3 \leq 1} \quad 0 \leq 2-x^3 \leq 1 \end{cases}$

$1 \leq x^3 \leq 2$

$I_{def} = \{ 1 \leq x \leq \sqrt[3]{2} \} = [1, \sqrt[3]{2}]$

$f(1) = \arcsin 1 = \frac{\pi}{2}$

$f(\sqrt[3]{2}) = \arcsin 0 = 0 \leftarrow$ intersezione asse x

NO intersezione asse y.

Segno: $\sqrt{2-x^3} \geq 0 \Rightarrow \arcsin(\sqrt{2-x^3}) \geq 0$

$\forall x \in I_{def}$

$f'(x) = \frac{1}{\sqrt{1-2+x^3}} (-3x^2) = \frac{-3x^2}{\sqrt{x^3-1}} \leq 0$

$\forall x \in I_{def}$

$f'(x) = 0 \iff x = 0$, fuori da I_{def}.

PARTE CORRETTA: (B4)

$$f'(x) = \frac{1}{\sqrt{1-2+x^3}} \cdot \frac{1}{2\sqrt{2-x^3}} \cdot (-3x^2)$$

$$= \frac{-3x^2}{2\sqrt{x^3-1}\,\sqrt{2-x^3}}$$

$f'(x) = 0 \iff x = 0 \notin I_{def}$

$f'(x) < 0 \quad \forall x \in (1, \sqrt[3]{2})$

f sempre decrescente.

$\nexists f'$ per $x = 1$; $x = \sqrt[3]{2}$

$$\lim_{x \to \sqrt[3]{2}^-} f'(x) = \frac{-3\sqrt[3]{4}}{2 \cdot 1 \cdot \sqrt{0^+}} = -\infty$$

$$\lim_{x \to 1^+} f'(x) = \frac{-3}{2\sqrt{0^+} \cdot 1} = -\infty$$

B_{4bis}

$$f''(x) = \frac{-3x}{4(2-x^3)^{3/2}(x^3-1)^{3/2}}\left[2x^6+3x^3-8\right]$$

(si confrontino pagg. A_2/A_{2bis} del compito A)

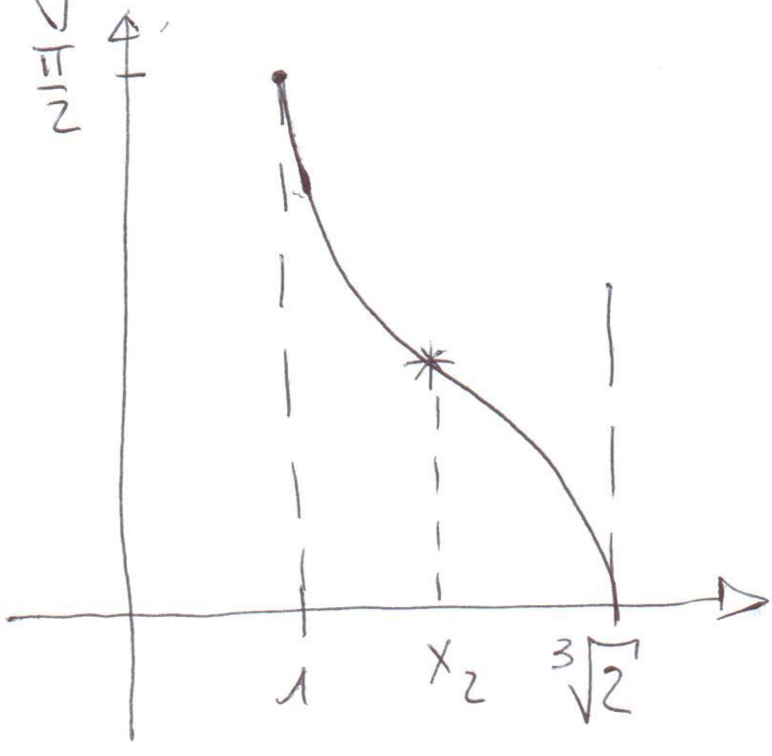
$f''(x) < 0$ per $x > x_2 = \dfrac{-3+\sqrt{73}}{4}$

$\Rightarrow$ f convessa in $[1, x_2)$

concava in $(x_2, \sqrt[3]{2}]$.

flesso discendente in x_2

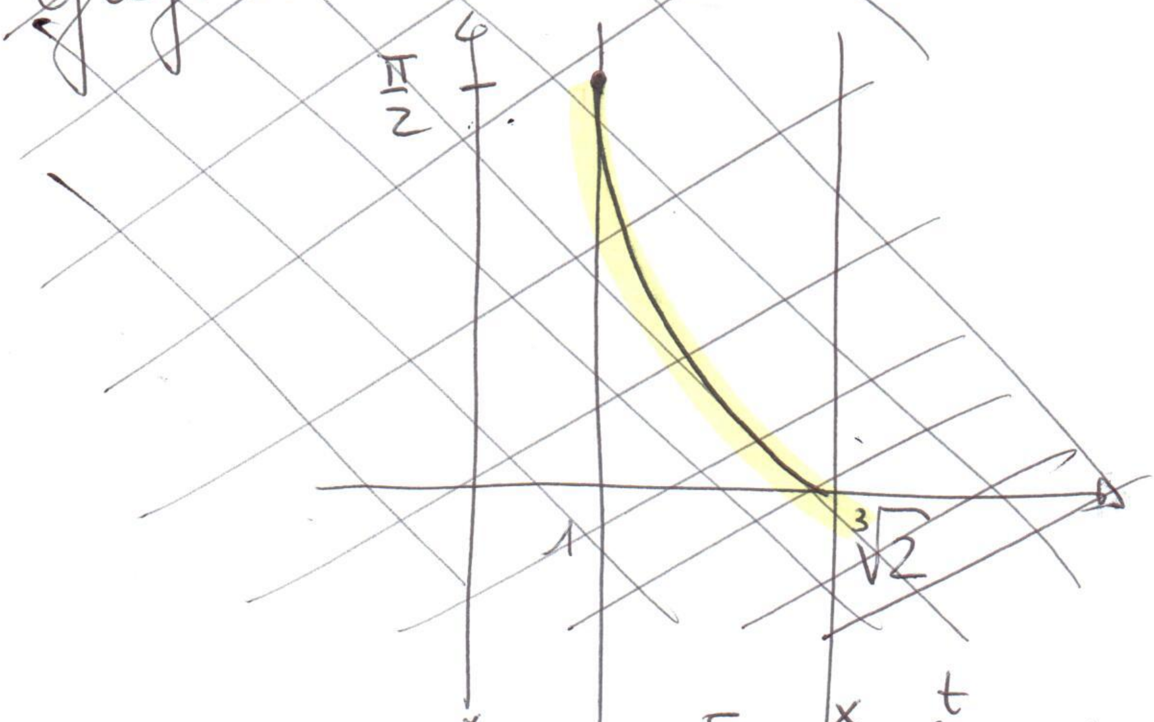
Il grafico corrispondente a quello svolto sotto ipotesi di numero minimo di flessi.



f sempre convesse

B5

Grafico:



5) $$y(x) = e^{-\int_0^x \cos^2 t\,dt}\left[\int_0^x e^{\int_0^t \cos^2 s\,ds}\cdot\left[\cos(2t)+1\right]dt + 2\right]$$

$$= e^{-\int_0^x \left[\frac{\cos 2t+1}{2}\right]dt}\cdot\left[\int_0^x e^{\int_0^t \left[\frac{\cos 2s+1}{2}\right]ds}\left[\cos(2t)+1\right]dt + 2\right]$$

$$= e^{-\left[\frac{\sin 2t}{4}+\frac{1}{2}t\right]_0^x}\cdot\left[\int_0^x e^{\left[\frac{\sin 2s}{4}+\frac{1}{2}s\right]_0^t}\left[\cos(2t)+1\right]dt + 2\right]$$

B6

$$= e^{-\frac{1}{4}\operatorname{sen}(2x)-\frac{1}{2}x} \cdot \left[\int_0^x e^{\left(\frac{\operatorname{sen}(2t)}{4}+\frac{1}{2}t\right)} \left(\cos(2t)+1\right) dt + 2\right]$$

$$\frac{\operatorname{sen}(2t)}{4}+\frac{1}{2}t = u$$

$$\Rightarrow du = \left[\frac{\cos(2t)}{2}+\frac{1}{2}\right] dt$$

$$\Rightarrow \left(\cos(2t)+1\right) dt = 2\,du$$

$$u(0)=0 \qquad u(x) = \frac{\operatorname{sen}(2x)}{4}+\frac{1}{2}x$$

$$= e^{-\frac{1}{4}\operatorname{sen}(2x)-\frac{1}{2}x} \cdot \left[\int_0^{u(x)} e^{u} \cdot 2\,du + 2\right]$$

$$= e^{-\frac{1}{4}\operatorname{sen}(2x)-\frac{1}{2}x} \left[2e^{u}\Big|_0^{u(x)} + 2\right]$$

$$= e^{-\frac{1}{4}\operatorname{sen}(2x)-\frac{1}{2}x} \left[2e^{\frac{1}{4}\operatorname{sen}(2x)+\frac{1}{2}x} \cancel{-2+2}\right]$$

$$= 2$$

$\Rightarrow \quad y(x) \equiv 2$

Infatti; $\quad \cos(2x) + 1 = 2\cos^2(x)$

$\Rightarrow$ l'equazione diventa

$$y' + \cos^2(x)\, y = 2\cos^2 x$$

La funzione $y \equiv 2$, effettivamente, soddisfa l'equazione e il problema di Cauchy.