COMPITO B

B_1

1) $\bar{z}^2 - z = x^2 - y^2 - 2ixy - x - iy$

$= x^2 - y^2 - x - i(2xy + y)$

$\overline{\bar{z}^2 - z} = x^2 - y^2 - x + i(2xy + y)$

$\mathrm{Im}\,\overline{(\bar{z}^2 - z)} = 2xy + y$

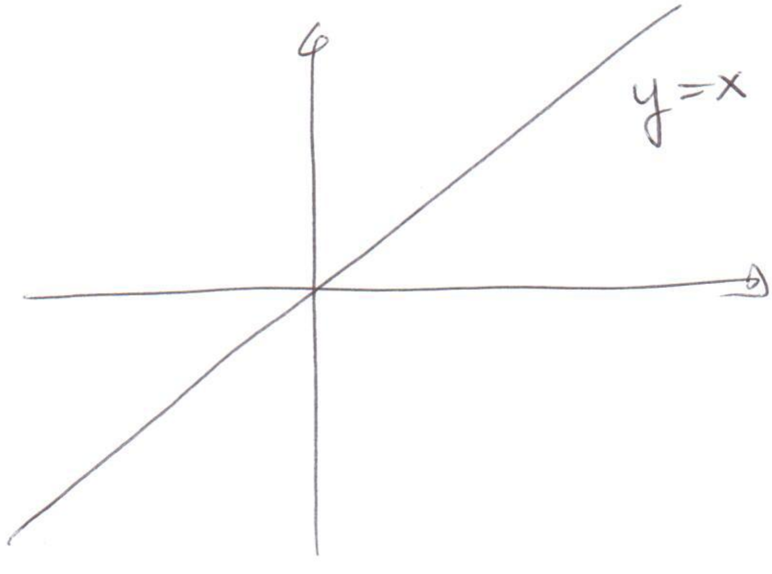
$|z|^2 + i\bar{z} = x^2 + y^2 + i(x - iy)$

$= x^2 + y^2 + y + ix$

$\mathrm{Re}(|z|^2 + i\bar{z}) = x^2 + y^2 + y$

$\Rightarrow x^2 + y^2 + \cancel{y} = 2xy + \cancel{y} \Rightarrow (x-y)^2 = 0$

$\Rightarrow y = x$



B_2

2) Criterio della radice:

$$\sqrt[n]{a_n} = \left(\frac{n+2}{n-1}\right)\left(1-\frac{1}{n}\right)^n \longrightarrow \frac{1}{e} < 1$$

$\Rightarrow$ serie convergente.

3) $D = \{x \neq 1\}$

$f(x) = 0 \iff x = 2$

$f(x) \geq 0 \iff x > 1$

$f(x) \leq 0 \iff x < 1$

$$\lim_{x \to 1^\pm} f(x) = \frac{1}{0^\pm} = \pm\infty$$

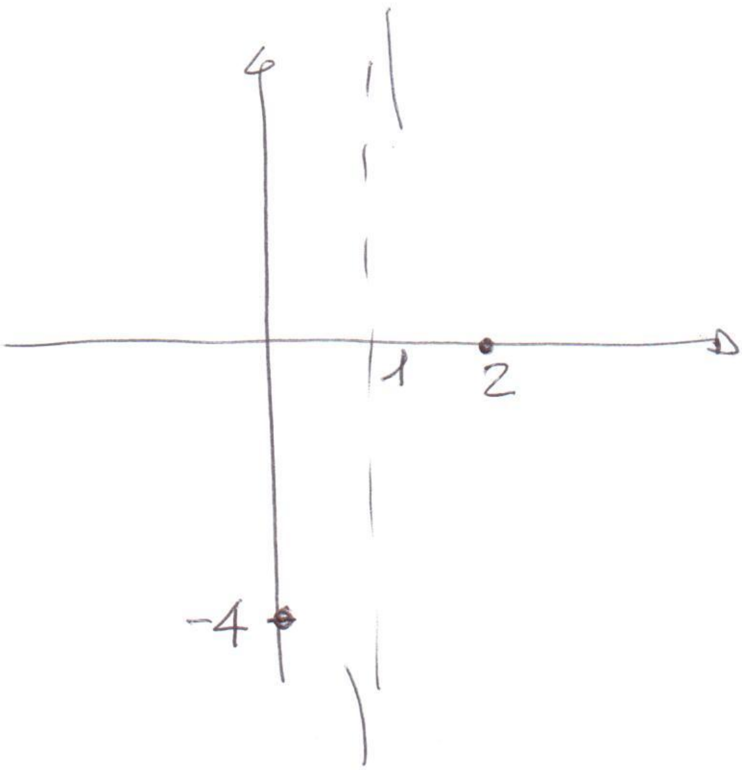
ASINTOTO VERTICALE DX e SX $x = 1$.

$f(0) = -4$

$$f(x) = \frac{x^2-4x+4}{x-1}$$

$$= \frac{x^2-x-3x+3+1}{x-1}$$

$$= x-3+\frac{1}{x-1}$$



B_3

$\Rightarrow$ AS. OBLIQUO per $x \to \pm\infty$

$$y = x - 3.$$

$$f'(x) = 1 - \frac{1}{(x-1)^2} = \frac{(x-1)^2 - 1}{(x-1)^2} = 0$$

$$\Leftrightarrow (x-1)^2 = 1 \quad \Leftrightarrow x = 2;\ x = 0$$

$$f'(x) > 0 \Leftrightarrow (x-1)^2 > 1. \quad \Leftrightarrow x < 0;\ x > 2$$

f cresce in $(-\infty, 0)$; decresce in $(0, 1)$; decresce in $(1, 2)$; cresce in $(2, +\infty)$

$x_1 = 0$ punto di MAX. REL.

$x_2 = 2$ punto di MIN. REL.

Poichè $\lim_{x \to \pm\infty} f(x) = \pm\infty$, $\nexists$ MAX-MIN ASSOLUTI

$$f''(x) = \frac{2}{(x-1)^3} > 0 \quad \Leftrightarrow x > 1$$

f concava in $(-\infty, 1)$; convessa in $(1, +\infty)$.

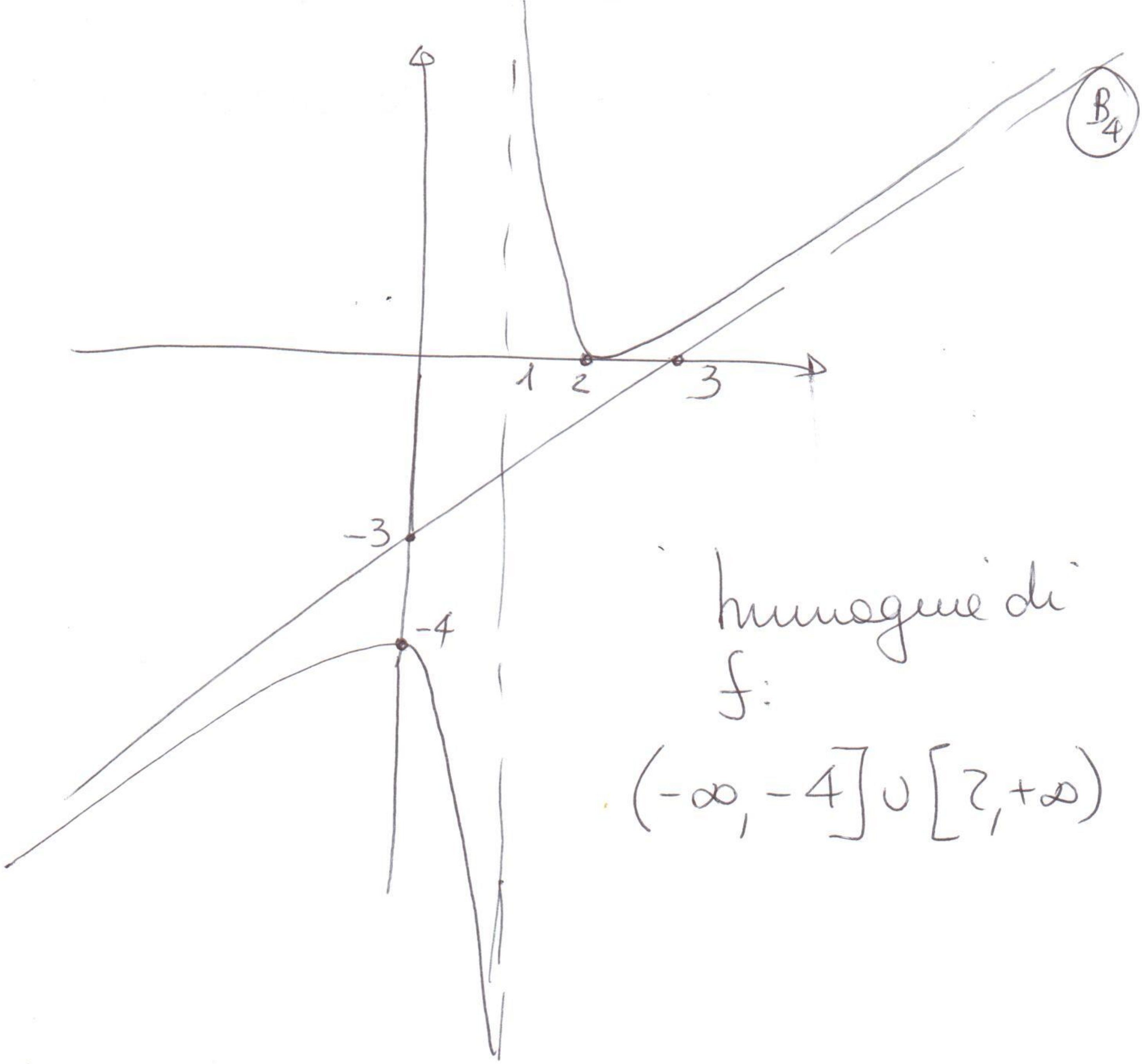


Immagine di f:

$(-\infty, -4] \cup [2, +\infty)$

4) $f \in C^0\left((-\infty, 0) \cup (0, +\infty)\right)$

Poichè $(-\log 3) < 0 \Rightarrow$ l'equazione è definita in $(-\infty, 0)$.

$\exists !$ soluzione $y \in C^1\left((-\infty, 0)\right)$

soluzione GLOBALE.

B5

$$\cancel{y(x) = e^{-x}}$$

$$y(x) = e^{-\int_{-\log 3}^{x} 1\,dt} \left[\int_{-\log 3}^{x} e^{\int_{-\log 3}^{t} 1\,ds} \frac{1}{e^{2t}-1}\,dt + 3\log 2 \right]$$

$$= e^{-x-\log 3} \left[\int_{-\log 3}^{x} e^{t+\log 3} \frac{1}{e^{2t}-1}\,dt + 3\log 2 \right]$$

$$= \frac{e^{-x}}{\cancel{3}} \left[\cancel{3} \int_{-\log 3}^{x} \frac{e^{t}}{e^{2t}-1}\,dt + \cancel{3}\log 2 \right]$$

$$\cancel{= e^{-x}} \qquad e^{t} = u \qquad du = e^{t}\,dt$$

$$u(-\log 3) = \frac{1}{3} \ ; \ u(x) = e^{x}$$

$$= e^{-x} \left[\int_{1/3}^{e^{x}} \frac{du}{u^2-1} + \log 2 \right]$$

$$= e^{-x} \left[\frac{1}{2} \int_{1/3}^{e^{x}} \left[\frac{-1}{u+1} + \frac{1}{u-1} \right] du + \log 2 \right]$$

$$= e^{-x}\left[\frac{1}{2}\log\left(\left|\frac{u-1}{u+1}\right|\right)\Bigg|_{1/3}^{e^x} + \log 2\right]$$

$$= e^{-x}\left[\frac{1}{2}\log\left(\left|\frac{e^x-1}{e^x+1}\right|\cdot\left|\frac{\frac{4}{3}}{-\frac{2}{3}}\right|\right) + \log 2\right]$$

$$= e^{-x}\left[\frac{1}{2}\log\left(2\left(\frac{1-e^x}{e^x+1}\right)\right) + \log 2\right]$$

si ricordi che $x<0$.

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5) $$\sqrt{1+x^2+x^3}-1 \sim \frac{x^2}{2}$$

$$\operatorname{arctg}\left(\sqrt{1+x^2+x^3}-1\right) \sim \frac{x^2}{2}$$

$$\operatorname{sen}(x) \sim x$$

$$\Rightarrow \quad f(x) \sim \frac{x}{\left(\frac{x^2}{2}\right)} = \frac{2}{x}$$

In $\left(0,\frac{1}{2}\right]$ $1+x^2+x^3>1>0$

$\Rightarrow$ RADICE SEMPRE DEFINITA.
DENOMINATORE MAI NULLO.

NON INTEGRABILE

in $\left(0,\frac{1}{2}\right)$.