SVOLGIMENTI PROVA SCRITTA di ANALISI 1 del 16/6/2017.

COMPITO A

A1

1) $D = \mathbb{R}$

$f(x) = 0 \iff x_1 = 0;\ x_2 = -1.$

$f(x) > 0 \iff x(x+1) > 0 \iff x < -1;\ x > 0.$

$$f'(x) = \frac{1}{3(x^2+x)^{2/3}}(2x+1) = 0 \iff x = -\frac{1}{2}$$

$$f'(x) > 0 \iff x > -\frac{1}{2}.$$

f decresce in $\left(-\infty, -\frac{1}{2}\right)$, cresce in $\left(-\frac{1}{2}, +\infty\right)$.

Essendo continua, $x = -\frac{1}{2}$ è punto di MIN. REL. e ASS. $f\left(-\frac{1}{2}\right) = \sqrt[3]{\frac{1}{4} - \frac{1}{2}} = -\sqrt[3]{\frac{1}{4}}$.

$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} x^{\frac{2}{3}} = +\infty$ $\quad \nexists$ MAX. REL. o ASS.

$$f(\mathbb{R}) = \left[-\sqrt[3]{\frac{1}{4}}, +\infty\right)$$

N.B.: f' NON è definita in x_1 e x_2.

$$\left.\begin{aligned} &\lim_{x \to 0^{\pm}} f'(x) = \frac{1}{3 \cdot 0^+} = +\infty \\ &\lim_{x \to -1^{\pm}} f'(x) = \frac{-1}{3 \cdot 0^+} = -\infty \end{aligned}\right\} \text{PUNTI A TANGENTE VERTICALE}$$

$$f''(x) = \frac{1}{3}\left[\frac{2(x^2+x)^{2/3} - (2x+1)\frac{2}{3}(x^2+x)^{-1/3}(2x+1)}{(x^2+x)^{4/3}}\right]$$

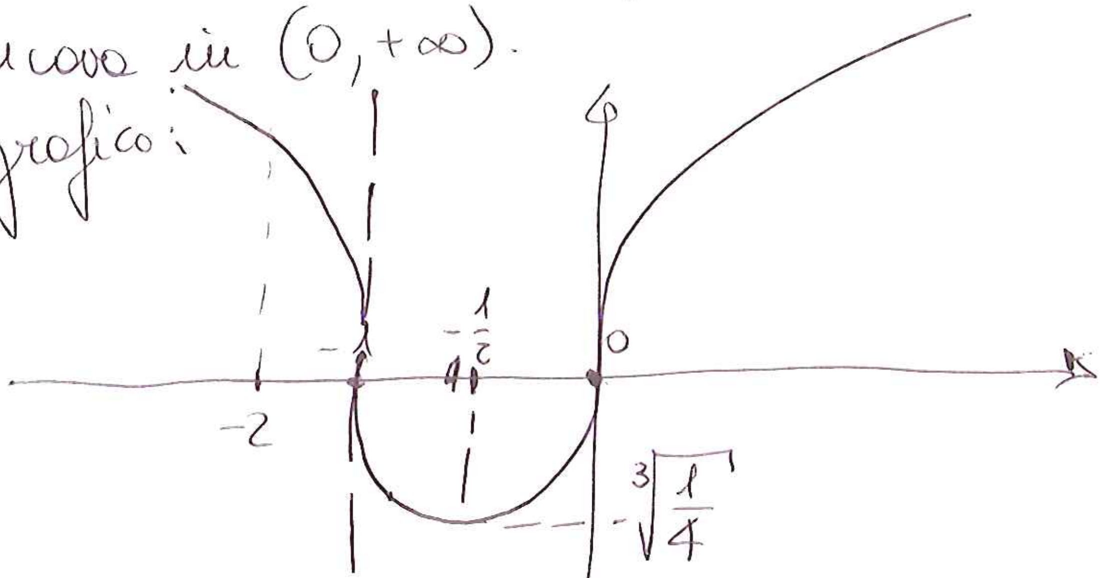
$$= \frac{2}{3}\left[\frac{3(x^2+x)-(2x+1)^2}{(x^2+x)^{5/3}}\right]$$

$$= \frac{2}{3}\left[\frac{3x^2+3x-4x^2-4x-1}{(x^2+x)^{5/3}}\right] = \frac{-2}{3(x^2+x)^{5/3}}(x^2+x+1)$$

$f''(x)>0 \iff x^2+x<0 \iff x_2<x<x_1$

f concava in $(-\infty,-1)$; convessa in $(-1,0)$; concava in $(0,+\infty)$.

Grafico:



Nell'intervallo $[-2,0]$ la f decresce in $[-2,-\frac{1}{2})$ e cresce in $(-\frac{1}{2},0]$

$\Rightarrow$ $x=-2$ punto di MAX. REL. $\quad f(-2)=\sqrt[3]{2}$

$x=0$ punto di MAX. REL. $\quad f(0)=0$

$\Rightarrow$ MAX. ASS. $\sqrt[3]{2}$ in $x=-2$.

$x=-\frac{1}{2}$ è punto di MIN. ASS.

2) $f(x) = \dfrac{x}{4+x^4} \underset{x\to+\infty}{\sim} \dfrac{1}{x^3}$ che è integrabile a $+\infty$.

A3

$\Rightarrow f$ è integrabile in $[0,+\infty)$.

$$\int_0^{+\infty} \frac{x}{4+(x^2)^2}\,dx = \frac{1}{2}\int_0^{+\infty} \frac{dt}{4+t^2} = \frac{1}{4}\operatorname{arctg}\left(\frac{t}{2}\right)\Big|_0^{+\infty}$$

$$\boxed{\begin{array}{l} t = x^2;\ dt = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}dt \\ t(0)=0;\ t(+\infty) = +\infty \end{array}}$$

$$= \frac{\pi}{8}.$$

3) $$\left(1+\frac{1}{i^3}\right)^8 = \left(1-\frac{1}{i}\right)^8 = (1+i)^8$$

$$= \left[\sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]\right]^8 = 16\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]^8$$

$$= 16\left[\cos(2\pi) + i\sin(2\pi)\right] = 16e^{i2\pi} = 16.$$

4) Equazione definita su tutto $\mathbb{R}$

$$y(x) = e^{-\int \frac{x}{x^2+1}dx}\left[\int e^{\int \frac{x}{x^2+1}dx} \frac{1}{\sqrt{x^2+1}}\,dx + C\right]$$

$$= e^{-\frac{1}{2}\ln(x^2+1)}\left[\int e^{\frac{1}{2}\ln(x^2+1)} \frac{1}{\sqrt{x^2+1}}\,dx + C\right]$$

$$= \frac{1}{\sqrt{x^2+1}}\left[\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}\,dx + C\right] = \frac{C+x}{\sqrt{x^2+1}}.$$

Poiché $\lim_{x \to -\infty} y(x) = \lim_{x\to-\infty} \frac{x}{\sqrt{x^2}} = \lim_{x\to-\infty} \frac{x}{|x|}$

$= -1,$

nessuna soluzione particolare soddisfa la condizione al limite richiesta. (A4)

5)

$$a_n = \frac{1}{n}\left[\cancel{\frac{1}{\sqrt{n+1}}} - \frac{1}{2(n+1)} + o\left(\frac{1}{n+1}\right)\right.$$
$$\left. - \cancel{\frac{1}{\sqrt{n+1}}} - \frac{1}{6(n+1)^{3/2}} + o\left(\frac{1}{(n+1)^{3/2}}\right)\right] \operatorname{arctg} n$$

$$= \frac{1}{n}\left[-\frac{1}{2(n+1)} + o\left(\frac{1}{n+1}\right)\right] \cdot \operatorname{arctg} n$$

$$\sim \frac{-\pi}{4n(n+1)} \sim \frac{-\pi}{4n^2}$$

La serie converge perché converge la serie

$$\sum \frac{-\pi}{4n^2} = -\frac{\pi}{4} \sum \frac{1}{n^2}.$$