C1

$\boxed{\text{COMPITO C}}$

1)
$$f(x) = \begin{cases} \log\left[4(x^2-x)\right] = \log 4 + \log(x^2-x) & \text{se } x<0;\ x>1 \\ \log\left[4(x-x^2)\right] = \log 4 + \log(x-x^2) & \text{se } 0<x<1 \end{cases}$$

$$\lim_{x\to 0^{\pm}} \log\left(|4x^2-4x|\right) = \lim_{x\to 1^{\pm}} \log\left(|4x^2-4x|\right) = -\infty$$

ASINTOTI VERTICALI, DX e SX, $x=0$ e $x=1$.

I_{def}: $4x^2-4x \neq 0 \Rightarrow x\neq 0;\ x\neq 1$

$$I_{def} = (-\infty, 0) \cup (0,1) \cup (1,+\infty)$$

$f(x) \geq 0 \iff |4x^2-4x| \geq 1$

$\underline{x<0;\ x>1:}$ $\quad 4x^2-4x-1 \geq 0$

$$x_{1,2} = \frac{2\pm\sqrt{4+4}}{4} = \frac{1\pm\sqrt{2}}{2}$$

$\Rightarrow x \leq x_1 = \frac{1-\sqrt{2}}{2} < 0$; $\quad x \geq x_2 = \frac{1+\sqrt{2}}{2} > 0$

$f(x) > 0$ in $(-\infty, x_1)$ e in $(x_2, +\infty)$

$f(x) < 0$ in $(x_1, 0)$ e in $(0, x_2)$

$f(x) = 0$ per $x = x_1$; $x = x_2$.

$0<x<1$: $\quad 4x-4x^2 \geq 1$

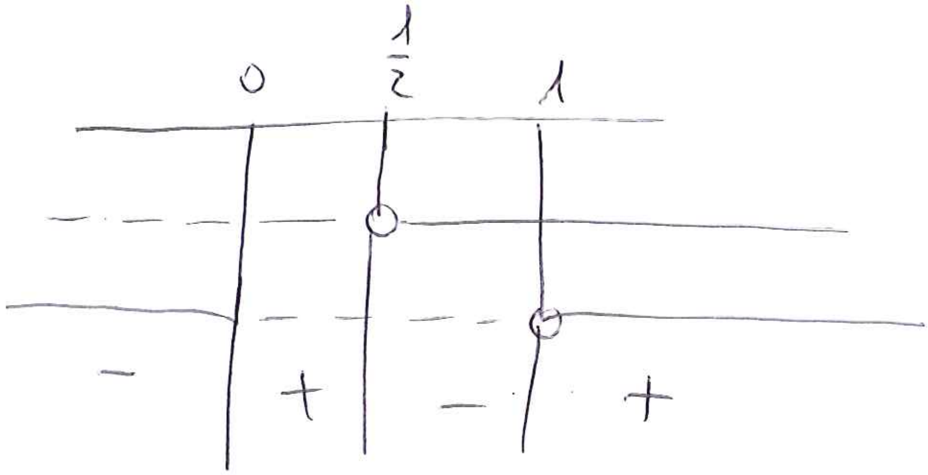
$$4x^2-4x+1 \leq 0$$

$$(2x-1)^2 \leq 0 \quad \Longleftrightarrow x=\frac{1}{2}$$

$\Rightarrow \quad f(x)<0 \quad$ in $\left(0,\frac{1}{2}\right)$ e in $\left(\frac{1}{2},1\right)$

$f(x)=0$ in $x=\frac{1}{2}$ (punto di massimo relativo)

$$f'(x)=\frac{2x-1}{x^2-x} \qquad \forall x \in I_{def}$$



f decresce in $(-\infty,0)$; cresce in $\left(0,\frac{1}{2}\right)$;

decresce in $\left(\frac{1}{2},1\right)$; cresce in $(1,+\infty)$.

Poiché $\lim\limits_{x\to\pm\infty} \log\left[4|x^2-x|\right]=+\infty$

e $\lim\limits_{x\to 0^{\pm}} f(x)=\lim\limits_{x\to 1^{\pm}} f(x)=-\infty$

non esistono MAX o MIN. ASSOLUTI

ASINTOTI OBLIQUI:

$$\lim_{x \to \pm\infty} \frac{\log\left[4|x^2-x|\right]}{x} = 0$$

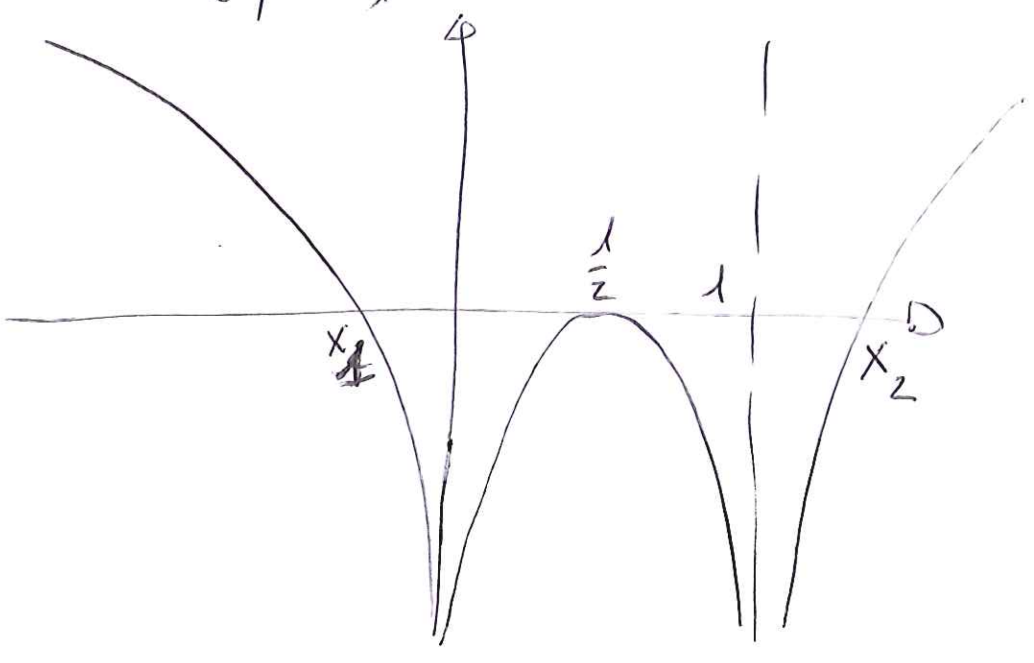
perché il logaritmo è un infinito di ordine inferiore rispetto a x.

$$f''(x) = \frac{2(x^2-x)-(2x-1)^2}{(x^2-x)^2} = \frac{2x^2-2x-4x^2+4x-1}{(x^2-x)^2}$$

$$= \frac{-2x^2+2x-1}{(x^2-x)^2}$$

$f''(x) > 0 \iff 2x^2-2x+1 < 0$ MAI

$\Rightarrow$ f è concava in $(-\infty, 0)$, in $(0,1)$, in $(1,+\infty)$



C4

2) $$1-\frac{1}{n^2}+\sin\left(\frac{1}{n^2}\right)=$$

$$1-\frac{1}{n^2}+\left[\frac{1}{n^2}-\frac{1}{6n^6}+o\left(\frac{1}{n^6}\right)\right]\underset{n\to\infty}{\sim}1-\frac{1}{6n^6}$$

$$\Rightarrow a_n\sim\left(1-\frac{1}{6n^6}\right)^{n^6}\xrightarrow[n\to\infty]{}e^{-\frac{1}{6}}$$

3) $$e^x+1\xrightarrow[x\to 0]{}2\ ;\quad \sqrt{1+x^2}\xrightarrow[x\to 0]{}1$$

$$\Rightarrow \lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{2\log(1+x)-x(e^x+1)\left(\sqrt{1+x^2}\right)}{2x\underbrace{(e^x+1)}_{2}\sqrt{1+x^2}}$$

$$=\lim_{x\to 0}\frac{2\left(x-\frac{x^2}{2}+o(x^2)\right)-x\left(2+x+\frac{x^2}{2}+o(x^2)\right)\left(1+\frac{x^2}{2}+o(x^2)\right)}{4x}$$

$$=\lim_{x\to 0}\frac{\cancel{2x}-x^2-\cancel{2x}-x^2+o(x^2)}{4x}=\lim_{x\to 0}\frac{-2x^2}{4x}=0.$$

4) $$(i-1)=\sqrt{2}\left[\cos\left(\frac{3}{4}\pi\right)+i\sin\left(\frac{3}{4}\pi\right)\right]$$

$$\Rightarrow (i-1)^8=16\left[\cos(6\pi)+i\sin(6\pi)\right]=16$$

$$\Rightarrow (i-1)^{\frac{8}{3}}=\sqrt[3]{16}=\sqrt[3]{16}\cdot\left[\cos(0)+i\sin(0)\right]$$

C5

$$\Rightarrow (i-1)^{\frac{8}{3}} = \sqrt[3]{16}\left[\cos\left(\frac{2k\pi}{3}\right) + i\sin\left(\frac{2k\pi}{3}\right)\right]$$

da cui le soluzioni:

$$w_0 = \sqrt[3]{16}$$

$$w_1 = \sqrt[3]{16}\left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right] =$$

$$\sqrt[3]{16}\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$$

$$w_2 = \sqrt[3]{16}\left[\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right]$$

$$= \sqrt[3]{16}\left[-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right].$$

5) ~~Per $x \to 0$~~ $f(x) > 0 \quad \forall x \in (0, +\infty)$

Per $x \to 0$

$$f(x) \sim \frac{1}{\sqrt{x}}$$

integrabile in 0.

Per $x \to +\infty$

$$f(x) = \frac{1 + \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2}}{\sqrt{x}\, e^{\sqrt{x}}} \sim \frac{\frac{e^{\sqrt{x}}}{2}}{\sqrt{x}\, e^{\sqrt{x}}} = \frac{1}{2\sqrt{x}}$$

NON integrabile a $+\infty$.

C_6

$$\int_0^{+\infty} f(x)\,dx = +\infty$$

Infatti: $$\int_0^{+\infty} \frac{1+\sinh\sqrt{x}}{\sqrt{x}\,e^{\sqrt{x}}}\,dx$$

$$\left(t=\sqrt{x} \;;\; dt = \frac{1}{2\sqrt{x}}\,dx \;;\; t(0)=0 \;;\; \text{"}t(\infty)=\infty\text{"} \right)$$

$$= \int_0^{+\infty} \frac{\left[1+\frac{e^{t}-e^{-t}}{2}\right]2}{e^{t}}\,dt$$

$$= \int_0^{+\infty} \left[2e^{-t} + \cancel{e^{t}}\,1 - e^{-2t}\right] dt$$

$$= -2e^{-t} + t + \frac{1}{2}e^{-2t} \Big|_0^{+\infty}$$

$$= \lim_{t\to+\infty}\left(-2e^{-t}+t+\frac{1}{2}e^{-2t}\right) - \left(-2+\frac{1}{2}\right) = +\infty.$$