COMPITO D

D1

1)

$$1-\frac{1}{n^2}+\sinh\left(\frac{1}{n^2}\right)=$$

$$=1-\frac{1}{n^2}+\frac{1}{n^2}+\frac{1}{3!\,n^6}+o\left(\frac{1}{n^6}\right)$$

$$\sim 1+\frac{1}{6n^6}$$

$$\Rightarrow \quad a_n=\left(1+\frac{1}{6n^6}\right)^{n^6}\xrightarrow[n\to\infty]{} e^{\frac{1}{6}}$$

2)

$$f(x) = \frac{2\sin x - x\left[\cosh(x^2)+1\right]\cdot\cos x}{2x\left[\cosh(x^2)+1\right]\cdot\cos x}$$

$$\lim_{x\to0} f(x) = \lim_{x\to0}\frac{1}{2\cos x\left[\cosh(x^2)+1\right]}\cdot$$

$$\lim_{x\to0}\frac{2\sin x - x\left[\cosh(x^2)+1\right]\cos x}{x}$$

$$= \lim_{x \to 0} \frac{2\left[x - \frac{x^3}{6} + o(x^3)\right] - x\left[2 + \frac{x^2}{2} + o(x^2)\right]\left[1 - \frac{x^2}{2} + o(x^2)\right]}{x}$$

D2

$$= \lim_{x \to 0} \frac{2x - \frac{x^3}{3} + o(x^3) - x\left[2 - x^2 + \frac{x^2}{2} + o(x^2)\right]}{x}$$

$$= \lim_{x \to 0} \frac{x^3\left(-\frac{1}{3} + 1 - \frac{1}{2}\right) + o(x^3)}{x} = \lim_{x \to 0} \frac{\frac{1}{6}x^3}{x}$$

$$= \lim_{x \to 0} \frac{1}{6}x^2 = 0.$$

3) $|1-i| = \sqrt{2}$

$$\Rightarrow 1 - i = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right]$$

$$(1-i)^8 = 16\left[\cos(-2\pi) + i \sin(-2\pi)\right]$$

$$= 16\left[\cos(0) + i \sin(0)\right]$$

D3

$$(1-i)^{\frac{8}{3}} = \sqrt[3]{(1-i)^8} =$$

$$\sqrt[3]{16}\left[\cos\left(\frac{2k\pi}{3}\right) + i\,\text{sen}\left(\frac{2k\pi}{3}\right)\right].$$

$$w_0 = \sqrt[3]{16}\left[\cos(0) + i\,\text{sen}(0)\right] = \sqrt[3]{16}$$

$$w_1 = \sqrt[3]{16}\left[\cos\left(\frac{2\pi}{3}\right) + i\,\text{sen}\left(\frac{2\pi}{3}\right)\right]$$

$$= \sqrt[3]{16}\left[-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right]$$

$$w_2 = \sqrt[3]{16}\left[\cos\left(\frac{4\pi}{3}\right) + i\,\text{sen}\left(\frac{4\pi}{3}\right)\right]$$

$$= \sqrt[3]{16}\left[-\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right].$$

D4

4) Per $x \to 0$ $\quad f(x) > 0 \quad \forall x \in (0, +\infty)$

$$f(x) \sim \frac{1}{\sqrt{x}}$$

Poiché $\frac{1}{\sqrt{x}}$ è integrabile in 0, lo è anche $f(x)$.

Per $x \to +\infty$

$$f(x) = \frac{1 + \frac{e^{\sqrt{x}} + e^{-\sqrt{x}}}{2}}{\sqrt{x}\, e^{\sqrt{x}}} \underset{x \to +\infty}{\sim} \frac{e^{\sqrt{x}}}{2\sqrt{x}\, e^{\sqrt{x}}} = \frac{1}{2\sqrt{x}}$$

Poiché $\frac{1}{2\sqrt{x}}$ NON è integrabile a $+\infty$, $f(x)$ NON è integrabile a $+\infty$ e quindi in $(0, +\infty)$.

Infatti:

D5

$$\int_0^{+\infty} \frac{1+\cosh\sqrt{x}}{\sqrt{x}\, e^{\sqrt{x}}}\,dx$$

$\sqrt{x} = t$

$dt = \frac{1}{2\sqrt{x}}\,dx \Rightarrow \frac{dx}{\sqrt{x}} = 2dt$

$t(0) = 0$ "$t(+\infty) = +\infty$"

$$= \int_0^{+\infty} \frac{1+\cosh(t)}{e^t}\,2dt$$

$$= \int_0^{+\infty} \left[2e^{-t} + 1 + e^{-2t}\right] dt$$

$$= \left[-2e^{-t} + t - \frac{1}{2}e^{-2t}\right]_0^{+\infty} =$$

$$\lim_{t\to+\infty} \left[-2e^{-t} + t - \frac{1}{2}e^{-2t}\right] - \left[-2 - \frac{1}{2}\right]$$

$$= \lim_{t\to+\infty} t + \frac{5}{2} = +\infty.$$

5) $$f(x) = \begin{cases} \log(x^2-2x) & \text{per } x \leq 0;\ x \geq 2. \\ \log(2x-x^2) & \text{per } 0<x<2 \end{cases}$$

f NON è definito per $2x-x^2=0$, cioè per

$x=0$; $x=2$

$\Rightarrow I_{def} = (-\infty, 0) \cup (0,2) \cup (2,+\infty)$.

$$\lim_{x\to 0^\pm} \log(|x^2-2x|) = \lim_{x\to 2^\pm} \log(|x^2-2x|) = -\infty.$$

ASINTOTI VERTICALI DX e SX:

$x=0$; $x=2$.

$$\lim_{x\to\pm\infty} f(x) = +\infty$$

NO ASINTOTI ORIZZONTALI

$$\lim_{x\to\pm\infty} \frac{\log(|x^2-2x|)}{x} = 0$$

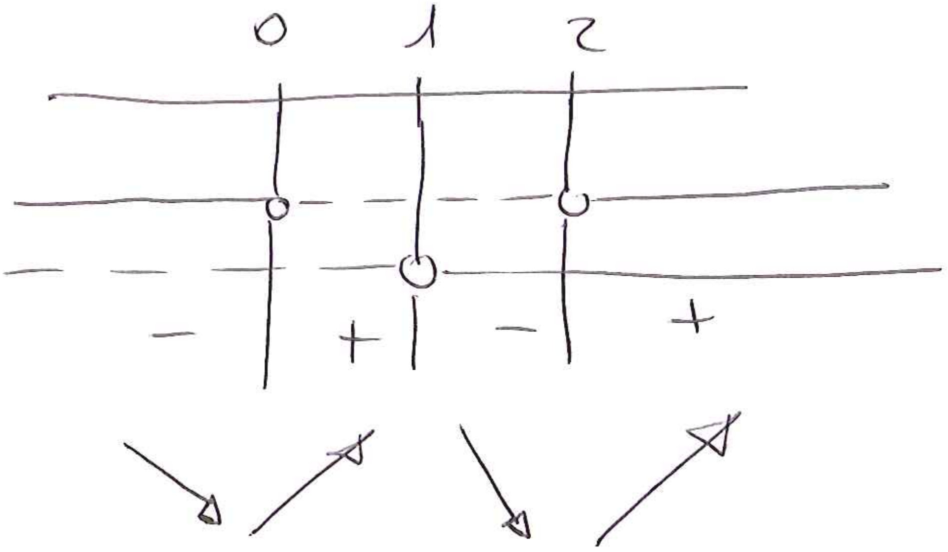
perché il logaritmo è un infinito di ordine inferiore rispetto a x.

NO ASINTOTI OBLIQUI.

Dx

$$f'(x) = \frac{2x-2}{x^2-2x} = \frac{2(x-1)}{x^2-2x} \qquad \forall x \in I_{dy}$$

~~$f'(x) > 0$~~



$f'(x) = 0 \iff x = 1$ (punto di MAX. REL.).

Poichè $\lim_{x \to 0} f(x) = -\infty$ e $\lim_{x \to \pm\infty} f(x) = +\infty$

NON CI SONO MAX. o MIN. ASS.

$$f''(x) = \frac{2\left[x^2-2x-2(x-1)^2\right]}{(x^2-2x)^2}$$

$$= \frac{2\left[-x^2+2x-2\right]}{(x^2-2x)^2}$$

$f''(x) > 0 \iff x^2-2x+2 < 0$

MAI VERIFICATO

f concava in $(-\infty, 0)$, in $(0, 2)$, in $(2, +\infty)$.

Segno:

$$f(x) \geq 0 \iff |x^2 - 2x| \geq 1$$

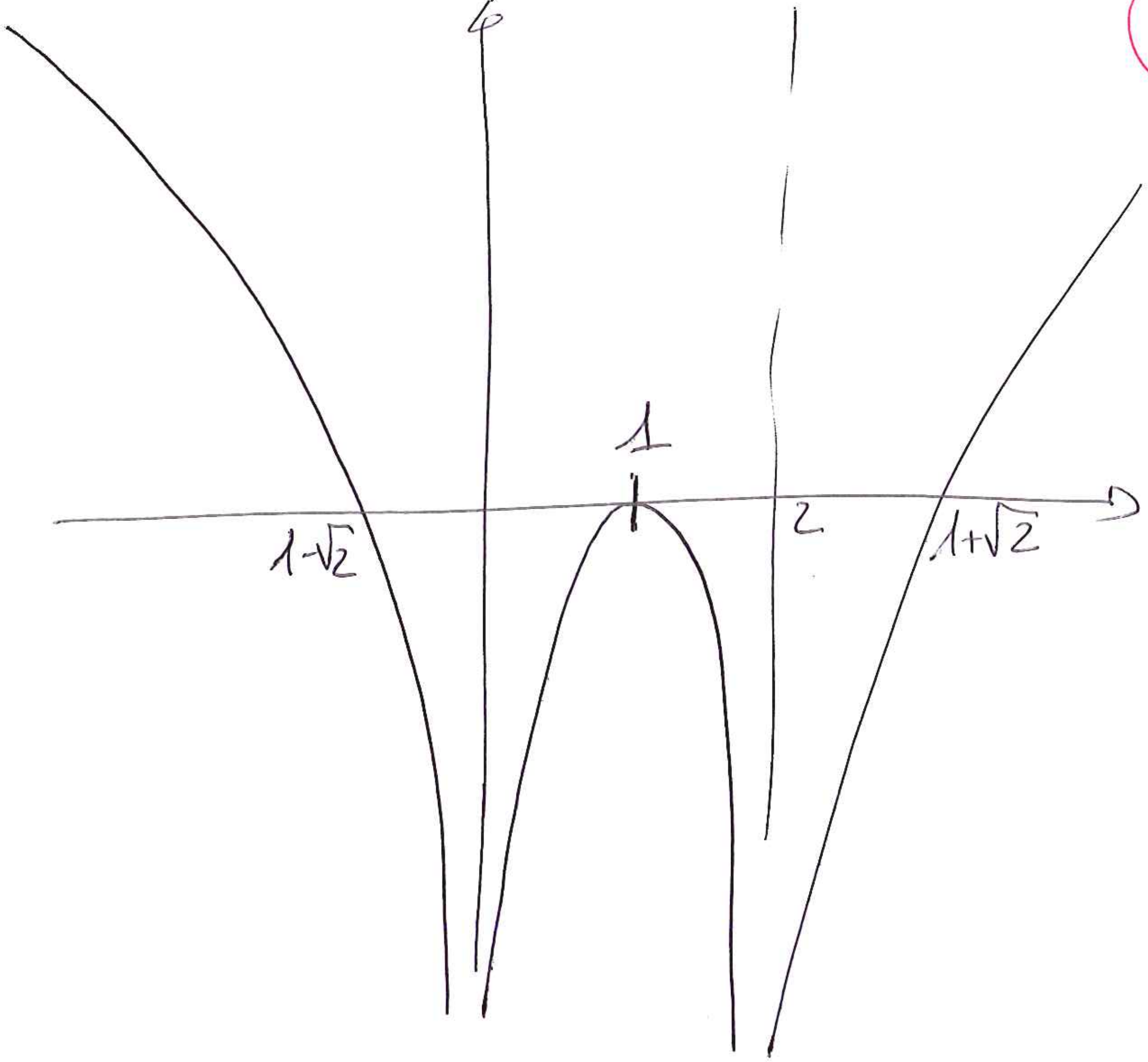
$$x^2 - 2x \geq 1 \quad \cup \quad x^2 - 2x \leq -1$$

$$x^2 - 2x \geq 1 \Downarrow x^2 - 2x - 1 \geq 0 \Downarrow x_{1,2} = 1 \pm \sqrt{2} \Downarrow x \leq x_1 = 1 - \sqrt{2} \; ; \; x \geq x_2 = 1 + \sqrt{2}$$

$$x^2 - 2x \leq -1 \Downarrow x^2 - 2x + 1 \leq 0 \Downarrow (x-1)^2 \leq 0 \quad x_3 = 1.$$

f si annulla in $x_1 = 1 - \sqrt{2}$; $x_2 = 1 + \sqrt{2}$; $x_3 = 1$

f positiva in $(-\infty, 1-\sqrt{2})$; negativa in $(1-\sqrt{2}, 0)$; negativa in $(0, 1)$; negativa in $(1, 2)$; negativa in $(2, 1+\sqrt{2})$, positiva in $(1+\sqrt{2}, +\infty)$.


Dg
1
1-√2
2
1+√2