SVOLGIMENTI PROVA SCRITTA di ANALISI 1 del 3/11/2016

1) $$\frac{x^3 e^{x^2} + 2(\cos x - 1)\sin x}{\log^3(1+x)\sin^2 x} =$$

$$= \frac{x^3\left[1+x^2+o(x^2)\right] + 2\left[-\frac{x^2}{2}+\frac{x^4}{4!}+o(x^4)\right]\cdot\left(x - \frac{x^3}{3!}+o(x^3)\right)}{\left(x+o(x)\right)^3\left(x+o(x)\right)^2}$$

$$= \cancel{x^3+x^5-x^2+\frac{x^2}{12}}\ \frac{x^3+x^5+2\left(-\frac{x^3}{2}+\frac{x^5}{12}+\frac{x^5}{24}\right)+o(x^5)}{x^5+o(x^5)}$$

$$= \frac{\cancel{x^3}-\cancel{x^3}+x^5\left(1+\frac{1}{6}+\frac{1}{12}\right)+o(x^5)}{x^5+o(x^5)} \sim \frac{\frac{5}{4}\cancel{x^5}}{\cancel{x^5}} \to \frac{5}{4}$$

2) $$\int_0^{\pi/4}\left(\frac{\sin x + \cos x - \operatorname{tg} x}{\cos x}\right)dx =$$

$$= \int_0^{\pi/4}\left[\frac{\sin x}{\cos x} + 1 - \frac{\sin x}{\cos^2 x}\right]dx =$$

$$= \cancel{\int_0^{\pi/4}} -\ln|\cos x| + x - \frac{1}{\cos x}\Big|_0^{\pi/4}$$

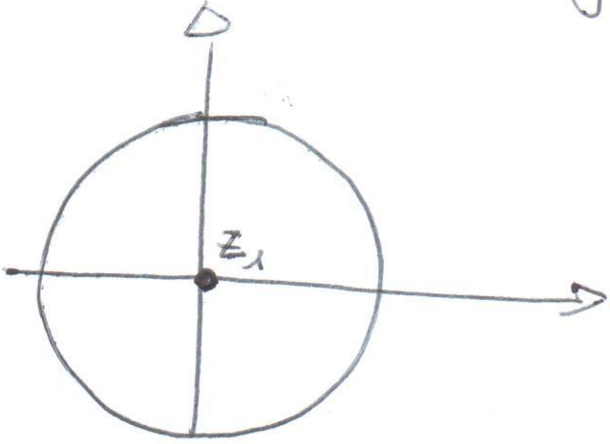
$$= -\ln\left|\frac{\sqrt{2}}{2}\right| + \frac{\pi}{4} - \sqrt{2} - \left[\ln|1| + 0 - 1\right]$$

$$= \ln(\sqrt{2}) + \frac{\pi}{4} - \sqrt{2} + 1.$$

3) $$|z|^4 - |z|^2 = |z^2|\left[|z|^2 - 1\right] = 0$$

$$\Rightarrow \{z_1 = 0\} \cup \{|z| = 1\}$$

z appartenenti alla circonferenza unitaria.



4) Criterio del rapporto:

$$\frac{a_{n+1}}{a_n} = \frac{(n!)^3}{[(n-1)!]^3} \cdot \frac{[3(n-1)]!}{(3n)!}$$

$$= \frac{[n \cdot \cancel{(n-1)!}]^3}{\cancel{[(n-1)!]^3}} \cdot \frac{\cancel{(3n-3)!}}{3n(3n-1)(3n-2)\cancel{(3n-3)!}}$$

$$= \frac{n^3}{3n(3n-1)(3n-2)} \xrightarrow[n\to\infty]{} \frac{1}{27} < 1.$$

La serie converge.

5) $I_{def} = \{x \in \mathbb{R} \mid x^3 - x^2 > 0\}$

$= \{x \in \mathbb{R} \mid x^2(x-1) > 0\}$

$= \{x \in \mathbb{R} \mid x > 1\} = (1, +\infty).$

$f(x)$ è una radice quadrata, quindi è sempre strettamente positiva.

No intersezioni con gli assi.

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{1}{\sqrt{x^2(x-1)}} = \frac{1}{\sqrt{0^+}} = +\infty$$

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AS. VERTICALE $x = 1$.

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \frac{1}{\sqrt{x^3-x^2}} = 0$$

AS. ORIZZONTALE a $+\infty$

$y = 0$.

CRESCENZA - DECRESCENZA:

$$f'(x) = \frac{-(3x^2-2x)}{2(x^3-x^2)^{3/2}} = \frac{-x(3x-2)}{2(x^3-x^2)^{3/2}} \geq 0$$

$$\iff 3x-2 \leq 0 \iff x \leq \frac{2}{3}$$

Ma $x = \frac{2}{3} \notin I_{def}$

$\Rightarrow f$ sempre decrescente in $(1, +\infty)$

$$f''(x) =$$

$$-\frac{1}{2}\left[\frac{(6x-2)(x^3-x^2)^{3/2} - \frac{3}{2}(3x^2-2x)\cancel{(x^3-x^2)^{1/2}}(3x^2-2x)}{(x^3-x^2)^{\cancel{3}\,5/2}}\right]$$

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$$= -\frac{1}{2}\left[\frac{\cancel{x^2}(6x-2)(x-1) - \frac{3}{2}\cancel{x^2}(3x-2)^2}{x^{\cancel{5}\,3}(x-1)^{5/2}}\right]$$

$$= -\frac{1}{2}\left[\frac{6x^2-6x-2x+2-\frac{3}{2}(9x^2-12x+4)}{x^3(x-1)^{5/2}}\right]$$

$$= -\frac{1}{2}\left[\frac{6x^2-\frac{27}{2}x^2-8x+18x+2-6}{x^3(x-1)^{5/2}}\right]$$

$$= \frac{1}{4}\left[\frac{15x^2-20x+8}{x^3(x-1)^{5/2}}\right] > 0 \quad \forall x \in D$$

perché $15x^2-20x+8>0 \quad \forall x \in \mathbb{R}$

f sempre convessa.

Grafico: