SVOLGIMENTI PROVA SCRITTA ANALISI 2 del 15/9/2020.

1) $\sum \frac{1}{n^{x^2+1}}$ è una serie armonica.

$\Rightarrow$ converge puntualmente se

$$x^2+1>1 \Rightarrow x^2>0 \Rightarrow x\neq 0$$

$$\Phi_{punt} = \mathbb{R}-\{0\}$$

$$\sup_{\Phi_{punt}} \left|\frac{1}{n^{x^2+1}}\right| = \frac{1}{n} \quad \cancel{\Rightarrow \text{NO CONV.}}$$

$\sum \frac{1}{n}$ diverge $\Rightarrow$ NO CONV. TOT. e UNIF. su Φ_{punt}.

Consideriamo

$$\Phi_{tot} = (-\infty, -\alpha] \cup [\alpha, +\infty)$$

con $\alpha > 0$

$$\sup_{\Phi_{tot}} \left|\frac{1}{n^{x^2+1}}\right| = \frac{1}{n^{\alpha^2+1}} \quad ; \quad \sum \frac{1}{n^{\alpha^2+1}}$$

converge perché $\alpha^2+1>1$.

$\Rightarrow$ CONV. TOT. e UNIF. in Φ_{ass}.

2

2) $f \in C^{\infty}(\mathbb{R}^2 - \{(0,0)\})$

$\Rightarrow$ diff.bile, derivabile, continua

$$\frac{\partial f}{\partial x} = y^2 \left[\frac{2x \sinh(x^2+y^2) - x^2 \cosh(x^2+y^2) 2x}{\sinh^2(x^2+y^2)} \right]$$

$$= \frac{2xy^2}{\sinh^2(x^2+y^2)} \left[\sinh(x^2+y^2) - x^2 \cosh(x^2+y^2) \right]$$

Analogamente

$$\frac{\partial f}{\partial y} = \frac{2yx^2}{\sinh^2(x^2+y^2)} \left[\sinh(x^2+y^2) - y^2 \cosh(x^2+y^2) \right]$$

In $(0,0)$:

$$\lim_{(x,y)\to(0,0)} |f(x,y)| = \lim_{(x,y)\to(0,0)} \left| \frac{x^2y^2}{x^2+y^2} \right|$$

$$\leq \lim_{(x,y)\to(0,0)} \frac{\cancel{x^2}y^2}{\cancel{x^2}} = 0 = f(0,0).$$

$\Rightarrow f \in C^0(\mathbb{R}^2)$.

$$f(0,y) = f(x,0) = 0$$

$$\Rightarrow \frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$$

DER. DIR:

$$\frac{df}{d\vec{v}}(0,0) = \lim_{t\to 0} \frac{t^4\alpha^2\beta^2}{t\sinh(t^2)} = \lim_{t\to 0} \frac{t^4\alpha^2\beta^2}{t^3}$$

$$= 0 \qquad \forall \vec{v}.$$

DIFF. BILITA':

$$\lim_{(h,k)\to(0,0)} \left| \frac{f(h,k) - \overset{=0}{f(0,0)} - \cancel{f_x(0,0)}\overset{=0}{}h - \cancel{f_y(0,0)}\overset{=0}{}k}{\sqrt{h^2+k^2}} \right|$$

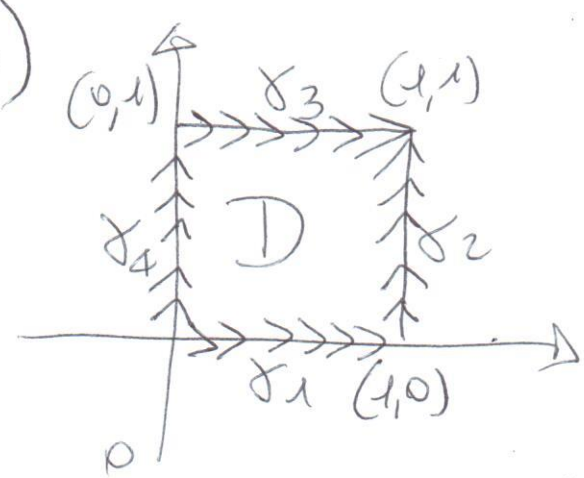
$$= \lim_{(h,k)\to(0,0)} \left| \frac{h^2k^2}{\sqrt{h^2+h^2}\,\sinh(h^2+k^2)} \right|$$

$$= \lim_{\rho\to 0} \left| \frac{\rho^4 \cos^2\vartheta \sin^2\vartheta}{\rho\cdot\rho^2} \right| = \lim_{\rho\to 0} \rho\cos^2\vartheta\sin^2\vartheta$$

$$\leq \lim_{\rho\to 0}\rho = 0 \qquad \text{UNIF. RISPETTO A } \vartheta$$

$\Rightarrow$ differenziabile anche in $(0,0)$.

3)



In $\mathring{D}$:

$$f_x = e^{-x^2} > 0$$
$$f_y = e^{-y^2} > 0$$
$\Rightarrow$ NO PUNTI STAZIONARI

In ∂D:

$$f\big|_{\gamma_1} = f(x,0) = \int_0^x e^{-t^2}\,dt + \cancel{\int_0^0 e^{-t^2}\,dt}_{=0}$$

$$\left(f\big|_{\gamma_1}\right)' = e^{-x^2} > 0 \Rightarrow \text{crescente}$$

$$f\big|_{\gamma_2} = f(1,y) = \underbrace{\int_0^1 e^{-t^2}\,dt}_{=\text{costante}} + \int_0^y e^{-t^2}\,dt$$

$$\left(f\big|_{\gamma_2}\right)' = e^{-y^2} > 0 \Rightarrow \text{crescente}$$

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$$f\big|_{\gamma_3} = f(x,1) = \int_0^x e^{-t^2}\,dt + \underbrace{\int_0^1 e^{-t^2}\,dt}_{= \text{costante}}$$

$$\left(f\big|_{\gamma_3}\right)' = e^{-x^2} > 0 \quad \Rightarrow \text{crescente}$$

$$f\big|_{\gamma_4} = f(0,y) = \underbrace{\cancel{\int_0^0 e^{-t^2}\,dt}}_{=0} + \int_0^y e^{-t^2}\,dt$$

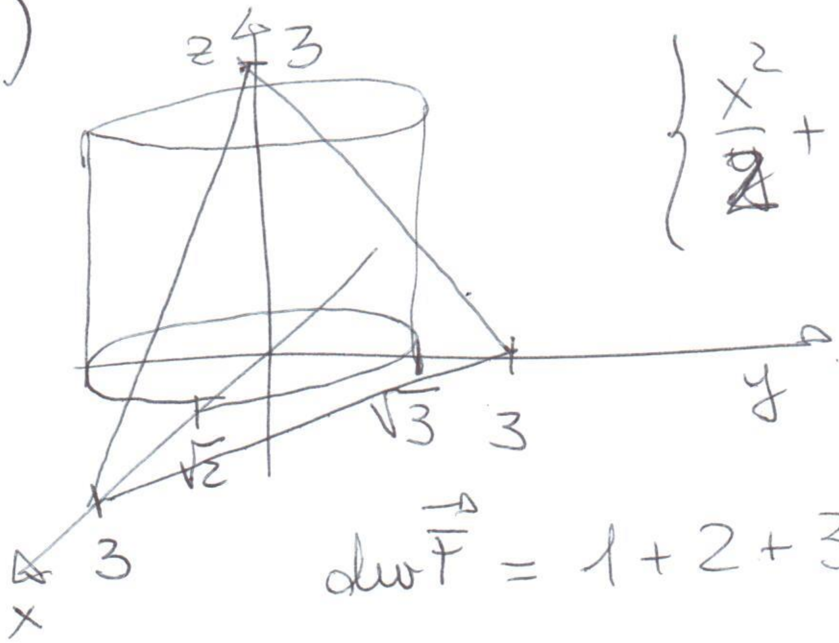
$$\left(f\big|_{\gamma_4}\right)' = e^{-y^2} > 0 \quad \Rightarrow \text{crescente}$$

MIN: $f(0,0) = 0$

MAX: $f(1,1) = \int_0^1 e^{-t^2}\,dt + \int_0^1 e^{-t^2}\,dt$

$$= 2\int_0^1 e^{-t^2}\,dt.$$

4)



$$D = \left\{ \frac{x^2}{2} + \frac{y^2}{3} = 1 \ ; \quad 0 \le z \le 3 - x - y \right\}$$

$$\operatorname{div} \vec{F} = 1 + 2 + 3 = 6$$

$$\Phi_{\partial D}(\vec{F}) = \iiint_D \operatorname{div} \vec{F} \, dx\,dy\,dz = 6 \operatorname{vol} D$$

$$= 6 \iint_{\tilde{D}(x,y)} \left[3 - x - y \right] dx\,dy$$

Coordinate ellittiche:

$$\begin{cases} x = \sqrt{2}\,\rho \cos\vartheta \\ y = \sqrt{3}\,\rho \sin\vartheta \end{cases} \qquad \begin{matrix} 0 \le \rho \le 1 \\ 0 \le \vartheta \le 2\pi \end{matrix}$$

$$J = \sqrt{6}\,\rho$$

$$= 6 \int_0^{2\pi} d\vartheta \int_0^1 \sqrt{6}\,\rho \left[3 - \sqrt{2}\rho\cos\vartheta - \sqrt{3}\rho\sin\vartheta \right] d\vartheta$$

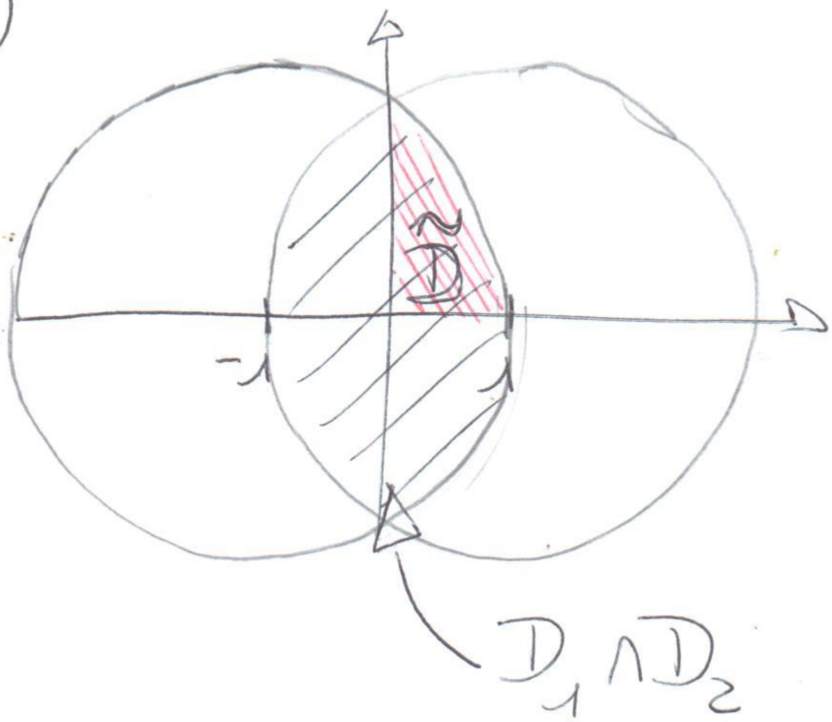
$$= 6\sqrt{6} \int_0^{2\pi} d\vartheta \left[\frac{3}{2}\rho^2 - \frac{\cancel{1}}{3}\rho^3 \left(\sqrt{2}\cos\vartheta - \sqrt{3}\sin\vartheta \right) \right]_0^1$$

$$6\sqrt{6}\int_0^{2\pi} d\vartheta \left[\frac{3}{2} - \frac{1}{3}\left(\sqrt{2}\cos\vartheta - \sqrt{3}\sin\vartheta\right)\right]$$

$$= 6\sqrt{6}\left[\frac{3}{2}\vartheta - \frac{1}{3}\sqrt{2}\sin\vartheta - \frac{\sqrt{3}}{3}\cos\vartheta\right]_0^{2\pi}$$

$$= 6\sqrt{6}\cdot 3\pi = 18\sqrt{6}\,\pi.$$

5)



Dominio simmetrico rispetto a entrambe gli assi.

Calcolo solo l'area del dominio $\tilde{D}$

$$\tilde{D} = \left\{ x \geq 0 \,;\, y \geq 0 \,;\, (x-1)^2 + y^2 \leq 4 \right\}$$

$$\text{Area}(D_1 \cap D_2) = 4\,\text{Area}(\tilde{D})$$

$$\begin{cases} x = -1 + \rho\cos\vartheta \\ y = \rho\sin\vartheta \end{cases}$$

$$\begin{cases} \rho\cos\vartheta \geq 1 \\ \rho\sin\vartheta \geq 0 \\ \rho \leq 2 \end{cases} \Rightarrow \begin{cases} \cos\vartheta > 0 \\ \sin\vartheta > 0 \\ \frac{1}{\cos\vartheta} \leq \rho \leq 2 \end{cases}$$

$$\begin{cases} \vartheta \in \left[0, \frac{\pi}{2}\right] \\ \cos\vartheta \geq \frac{1}{2} \end{cases} \Rightarrow \begin{cases} \vartheta \in \left[0, \frac{\pi}{3}\right] \\ \frac{1}{\cos\vartheta} \leq \rho \leq 2 \end{cases}$$

$$\text{Area}(D_1 \cap D_2) = 4\int_0^{\pi/3} d\vartheta \int_{\frac{1}{\cos\vartheta}}^{2} \rho\, d\rho$$

$$= 4\int_0^{\frac{\pi}{3}} d\vartheta \left[\frac{\rho^2}{2}\right]_{\frac{1}{\cos\vartheta}}^{2} = 2\int_0^{\pi/3} \left[4 - \frac{1}{\cos^2\vartheta}\right] d\vartheta$$

$$2\left[\frac{4\pi}{3} - \operatorname{tg}\vartheta \Big|_0^{\pi/3}\right] = 2\left(\frac{4\pi}{3} - \sqrt{3}\right).$$