SVOLGIMENTI PROVA SCRITTA di ANALISI 2 - 19/2/2019

1) CONV. ASS. e PUNTUALE:

$$\sum_{n=1}^{\infty} \left| \frac{\cos(e^{nx})}{n^3} \right| \leq \sum_{n=1}^{\infty} \frac{1}{n^3} \text{ che converge}$$

$\Rightarrow \forall x \in \mathbb{R}$ c'è conv. puntuale e assoluta.

Dalla maggiorazione, segue che c'è anche CONV. TOTALE (e quindi UNIFORME) su tutto $\mathbb{R}$.

$$\sum_{n=1}^{\infty} f_n'(x) = \sum_{n=1}^{\infty} \frac{1}{n^3}\left(-n e^{nx} \sin(e^{nx})\right)$$

$$= -\sum_{n=1}^{\infty} \frac{1}{n^2}\left[e^{nx} \sin(e^{nx})\right]$$

In $(-\infty, 0]$ $\quad |e^{nx}\sin(e^{nx})| \leq 1$

$\Rightarrow \sum |f_n'(x)| \leq \sum \frac{1}{n^2}$ convergente

$\Rightarrow \sum f_n'$ converge totalmente in $(-\infty, 0]$.

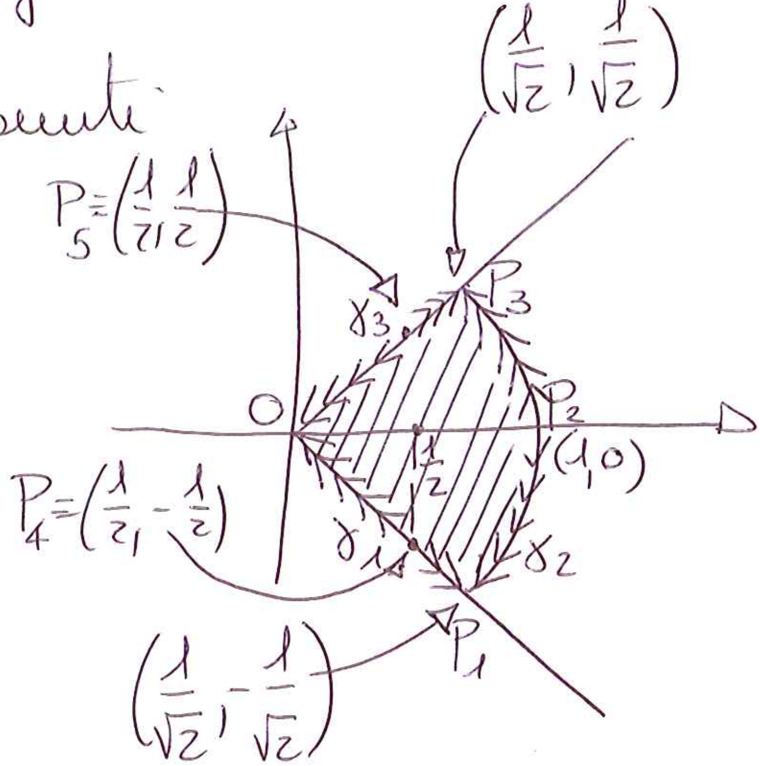
2) $$f(x,y) = \int_0^y e^{-t^2}\,dt - \int_0^x e^{-t^2}\,dt$$

$$\Rightarrow \quad f_x = -e^{-x^2} \; ; \quad f_y = e^{-y^2}$$

$$f_{xx} = +2x e^{-x^2} \; ; \quad f_{yy} = -2y e^{-y^2}$$

$$f_{xy} = f_{yx} = 0.$$

3) Cerchiamo punti stazionari in $\mathring{D}$:



$$\begin{cases} f_x = 2(x-1) \\ f_y = 2y \end{cases}$$

$$\vec{\nabla} f(x,y) = \vec{0}$$

$$\Leftrightarrow \quad (x,y) = (1,0) \overset{P_2}{=}. \text{ Il punto } P_2 \notin \mathring{D}.$$

Su $\partial D = \gamma_1 \cup \gamma_2 \cup \gamma_3$, con

$$\gamma_1 : \begin{cases} x \in \left[0, \frac{1}{\sqrt{2}}\right] \\ y = -x \end{cases} ; \quad \gamma_2 : \begin{cases} x = \cos\vartheta \\ y = \sin\vartheta \end{cases}, \ \vartheta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right].$$

$$\gamma_3 : \begin{cases} x \in \left[0, \frac{1}{\sqrt{2}}\right] \\ y = x \end{cases}$$

Si osservi che $f(x,y) \geq 0$ e $f(1,0) = 0$
$\Rightarrow (1,0)$ punto di MIN. ASS.

Inoltre f simmetrica rispetto all'asse x

$$f\big|_{\gamma_1} = (x-1)^2 + x^2 = f\big|_{\gamma_3}$$
$$= 2x^2 - 2x + 1$$

$$\left(f\big|_{\gamma_1}\right)' = 2(2x-1) \geq 0 \iff x \geq \frac{1}{2}$$

$\Rightarrow$ f decresce lungo γ_1 e γ_3 fino a $x = \frac{1}{2}$

poi cresce.

$\Rightarrow P_4 = \left(\frac{1}{2}, -\frac{1}{2}\right)$ e $P_5\left(\frac{1}{2}, +\frac{1}{2}\right)$

punti di MIN. REL.

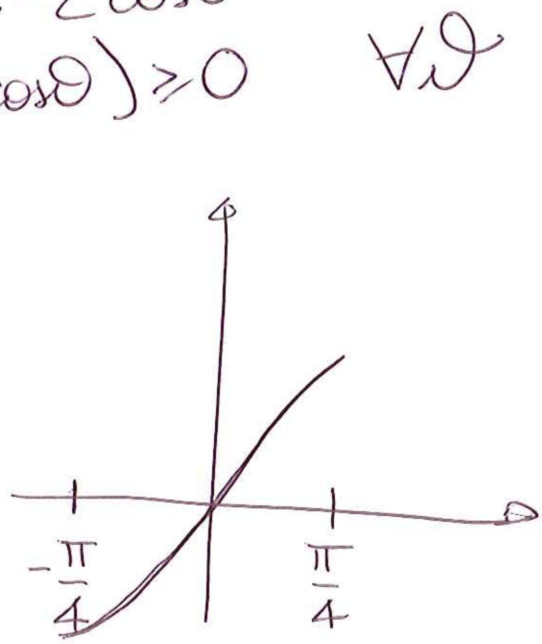
$$f(P_4) = f(P_5) = \frac{1}{2}$$

$$f\big|_{\gamma_2} = (\cos\vartheta - 1)^2 + \sin^2\vartheta = 2 - 2\cos\vartheta$$
$$= 2(1-\cos\vartheta) \geq 0 \quad \forall\vartheta$$

$$\cancel{\left(f\big|_{\gamma_2}\right)} \quad \left(f\big|_{\gamma_2}\right)_\vartheta = 2\sin\vartheta$$

$f\big|_{\gamma_2}$ decresce in $\left[-\frac{\pi}{4}, 0\right]$,

cresce in $\left[0, \frac{\pi}{4}\right]$



Quindi, come previsto, $P_2=(1,0)$ punto di MIN. REL. e ASS.

Dal grafico si osserva che

$$P_1=\left(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right) \text{ e } P_3=\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) \text{ sono}$$

punti di MAX. REL., così come $(0,0)$.

$$f(P_1)=f(P_3)=\left(f|_{\gamma_2}\right)\left(\pm\frac{\pi}{4}\right)=2\left(1-\frac{1}{\sqrt{2}}\right)$$

$$=\sqrt{2}(\sqrt{2}-1)$$

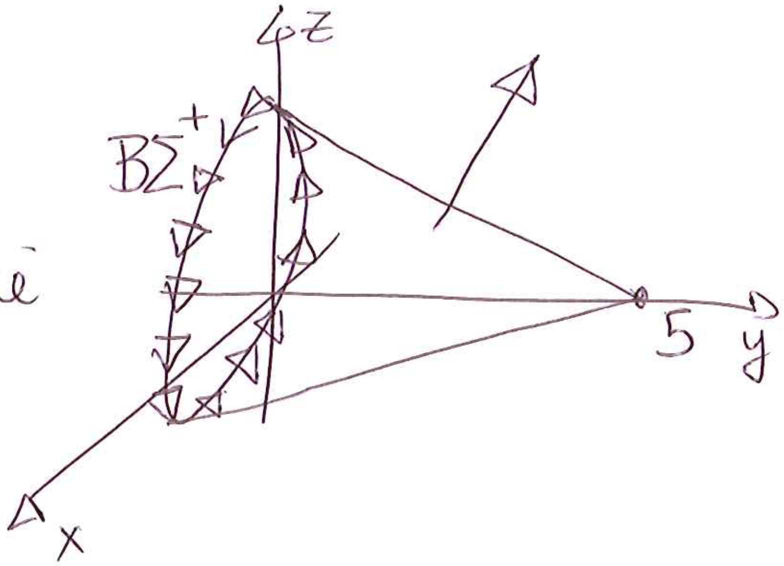
$$=2-\sqrt{2}<1$$

$$f(0,0)=1$$

$\Rightarrow$ $(0,0)$ è punto di MAX. ASS.

$P_2=(1,0)$ è punto di MIN. ASS.

4) Falda inferiore di cono, con vertice in $(0,5,0)$.



Bordo $B\Sigma$: $\begin{cases} 5-\sqrt{x^2+z^2}=0 \\ y=0 \end{cases} = \begin{cases} y=0 \\ x^2+z^2=25 \end{cases}$

$B\Sigma^+$: $\begin{cases} x = 5\sin\vartheta \\ y = 0 \\ z = 5\cos\vartheta \end{cases}$ $\quad \vartheta \in [0, 2\pi]$

$$\Rightarrow \underset{\Sigma}{\Phi}(\overrightarrow{\mathrm{rot} F}) = \oint_{B\Sigma^+} \vec{F}\,\vec{ds}$$

$$= \int_0^{2\pi} \left[(-2\cdot 5\cos\vartheta)(5\cos\vartheta) - (2\cdot 5\sin\vartheta)(5\sin\vartheta)\right] d\vartheta$$

$$= -50 \cdot 2\pi = -100\pi.$$

Si osservi che la circuitazione è $\neq 0$ perchè il campo NON è irrotazionale.

Infatti, ad esempio,

$$(F_2)_z = y^6 e^{x+z} \neq (F_3)_y = 6yz$$

5) $\begin{cases} x'(t)=2t \\ y'(t)=\frac{3}{2}\sin t \\ z'(t)=\frac{3}{2}\cos t \end{cases}$; $\begin{cases} x''(t)=2 \\ y''(t)=\frac{3}{2}\cos t \\ z''(t)=-\frac{3}{2}\sin t \end{cases}$

$$\begin{cases} x'''(t)=0 \\ y'''(t)=-\frac{3}{2}\sin t \\ z'''(t)=-\frac{3}{2}\cos t \end{cases}$$

$$v(t)=\|\vec{r}\,'(t)\|=\sqrt{4t^2+\frac{9}{4}\sin^2 t+\frac{9}{4}\cos^2 t}$$

$$=\sqrt{4t^2+\frac{9}{4}}=\frac{1}{2}\sqrt{16t^2+9}$$

$$\Rightarrow \ell(\gamma)=\int_0^2\sqrt{4t^2+\frac{9}{4}}\,dt=2\int_0^2\sqrt{t^2+\frac{9}{16}}\,dt$$

$$\left[\text{Ma} \quad \int\sqrt{a^2+x^2}\,dx=\frac{1}{2}\left[x\cdot\sqrt{a^2+x^2}+a^2\log\left(x+\sqrt{a^2+x^2}\right)\right]+c\right]$$

$$\Rightarrow \ell(\gamma)=\left[t\sqrt{\frac{9}{16}+t^2}+\frac{9}{16}\log\left(t+\sqrt{\frac{9}{16}+t^2}\right)\right]_0^2$$

$$=2\sqrt{\frac{73}{16}}+\frac{9}{16}\left[\log\left(2+\sqrt{\frac{73}{16}}\right)-\log\left(\frac{3}{4}\right)\right]$$

$$= \frac{\sqrt{73}}{2} + \frac{9}{16} \log\left(\frac{8+\sqrt{73}}{3}\right).$$

$$k(t) = \frac{\|\vec{r}\,' \wedge \vec{r}\,''\|}{v^3(t)}$$

$$\vec{r}\,' \wedge \vec{r}\,'' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2t & \frac{3}{2}\sin t & \frac{3}{2}\cos t \\ 2 & \frac{3}{2}\cos t & -\frac{3}{2}\sin t \end{vmatrix}$$

$$= -\frac{9}{4}\vec{i} - \left(-3t\sin t - 3\cos t\right)\vec{j} + \left(3t\cos t - 3\sin t\right)\vec{k}$$

$$= -\frac{9}{4}\vec{i} + 3\left(t\sin t + \cos t\right)\vec{j} + 3\left(t\cos t - \sin t\right)\vec{k}$$

$$\|\vec{r}\,' \wedge \vec{r}\,''\| = 3\sqrt{\frac{9}{16} + (t\sin t + \cos t)^2 + (t\cos t - \sin t)^2}$$

$$= 3\sqrt{\frac{9}{16} + t^2 + \cancel{2t\sin t\cos t} + 1 - \cancel{2t\cos t\sin t}}$$

$$= 3\sqrt{\frac{25}{16} + t^2} = \frac{3}{4}\sqrt{25+16t^2}$$

$$\Rightarrow k(t) = \frac{\frac{3}{4}\sqrt{25+16t^2}}{\frac{1}{8}\left(\sqrt{16t^2+9}\right)^3} = \frac{6\,\cancel{8}\sqrt{25+16t^2}}{\cancel{2}\left(\sqrt{16t^2+9}\right)^3}$$

$$\tau(t) = \frac{-(\vec{r}\,' \wedge \vec{r}\,'') \cdot \vec{r}\,'''}{\|\vec{r}\,' \wedge \vec{r}\,''\|^2}$$

$$(\vec{r}\,' \wedge \vec{r}\,'') \cdot \vec{r}\,''' = \begin{vmatrix} 2t & \frac{3}{2}\sin t & \frac{3}{2}\cos t \\ 2 & \frac{3}{2}\cos t & -\frac{3}{2}\sin t \\ 0 & -\frac{3}{2}\sin t & -\frac{3}{2}\cos t \end{vmatrix}$$

$$= -\frac{9}{2}t\cos^2 t - \cancel{\frac{9}{2}\cos t \sin t} - \frac{9}{2}t\sin^2 t + \cancel{\frac{9}{2}\cos t \sin t}$$

$$= -\frac{9}{2}t$$

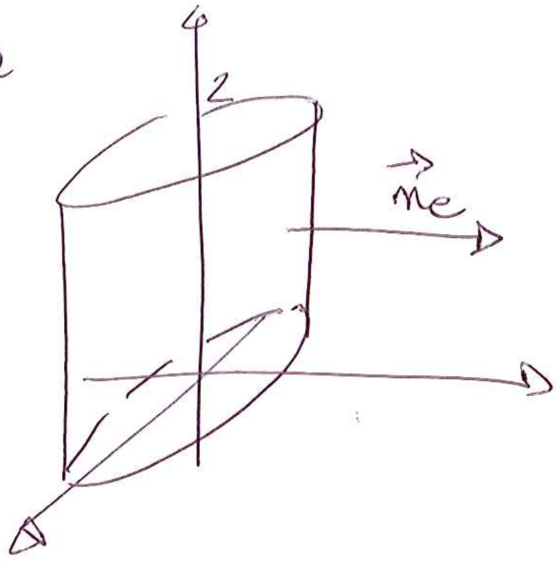
$$\|\vec{r}\,' \wedge \vec{r}\,''\|^2 = \frac{9}{16}(25+16t^2)$$

$$\Rightarrow \tau(t) = \frac{8t}{(25+16t^2)}.$$

6) Superficie cilindrica

$$\begin{cases} x = 2\cos\vartheta \\ y = 2\sin\vartheta \\ z = t \end{cases}$$

$\vartheta \in [0, 2\pi]$; $t \in [0, 2]$



$$\begin{pmatrix} x_\vartheta & y_\vartheta & z_\vartheta \\ x_t & y_t & z_t \end{pmatrix} = \begin{pmatrix} -2\sin\vartheta & 2\cos\vartheta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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$$\Rightarrow \vec{n}_e = \pm (2\cos\vartheta, 2\sin\vartheta, 0)$$

Poichè consideriamo la normale rivolta verso l'esterno $\Rightarrow$

$$\vec{n}_e = (2\cos\vartheta, 2\sin\vartheta, 0).$$

$$\Rightarrow W(\vartheta, t) = \sqrt{4\cos^2\vartheta + 4\sin^2\vartheta} = 2$$

$$\Rightarrow \int_S \frac{e^x y}{x^2+y^2} dS = \int_0^{2\pi} d\vartheta \int_0^2 dt \left[\frac{e^{2\cos\vartheta} 2\sin\vartheta}{4} 2 \right]$$

$$= \frac{1}{2} 2 \cdot \int_0^{2\pi} e^{2\cos\vartheta} 2\sin\vartheta \, d\vartheta$$

$$= \left[-e^{2\cos\vartheta} \right]_0^{2\pi} = 0$$