SVOLGIMENTI PROVA SCRITTA di ANALISI 2 del 26/10/2021

1) Criterio radice:

$$\sqrt[n]{|a_n|} = \frac{2}{\sqrt[n]{\sqrt{n}}} \xrightarrow[n\to\infty]{} 2$$

$$\Rightarrow R = \frac{1}{2}$$

Controlliamo negli estremi

$x = \frac{1}{2}$: $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} \left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ divergente

$x = -\frac{1}{2}$: $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} \left(-\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

convergente semplicemente, divergente assolutamente

$\Rightarrow$ CONV. SEMPL. in $\left[-\frac{1}{2}, \frac{1}{2}\right)$

CONV. ASS. in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

CONV. UNIF. in $\left[-\frac{1}{2}, b\right]$; $-\frac{1}{2} < b < \frac{1}{2}$

CONV. TOT. in $[-h, h]$ $0 < h < \frac{1}{2}$

2)

$$\vec{r}\,'(t) = \begin{cases} x'(t) = \cos t - t \,\text{sen}\, t \\ y'(t) = \text{sen}\, t + t\cos t \\ z'(t) = 1 \end{cases}$$

$$v(t) = \sqrt{(\cos t - t\,\text{sen}\, t)^2 + (\text{sen}\, t + t\cos t)^2 + 1}$$

$$= \sqrt{\cos^2 t + t^2 \text{sen}^2 t \cancel{- 2t\cos t\,\text{sen}\, t} + \text{sen}^2 t + t^2\cos^2 t + \cancel{2t\,\text{sen}\, t\cos t} + 1}$$

$$= \sqrt{1 + t^2 + 1} = \sqrt{2 + t^2}$$

$$\int_\gamma \left(2z - \sqrt{x^2+y^2}\right) ds = \int_{-\pi}^{\pi} \left[2t - \sqrt{t^2}\right] \sqrt{2+t^2}\, dt$$

$$= \int_{-\pi}^{\pi} (2t - |t|)\sqrt{2+t^2}\, dt = \int_{-\pi}^{0} 3t\sqrt{2+t^2}\, dt + \int_{0}^{\pi} t\sqrt{2+t^2}\, dt$$

$$= \left.(2+t^2)^{\frac{3}{2}}\right|_{-\pi}^{0} + \left.\frac{1}{3}(2+t^2)^{\frac{3}{2}}\right|_{0}^{\pi} =$$

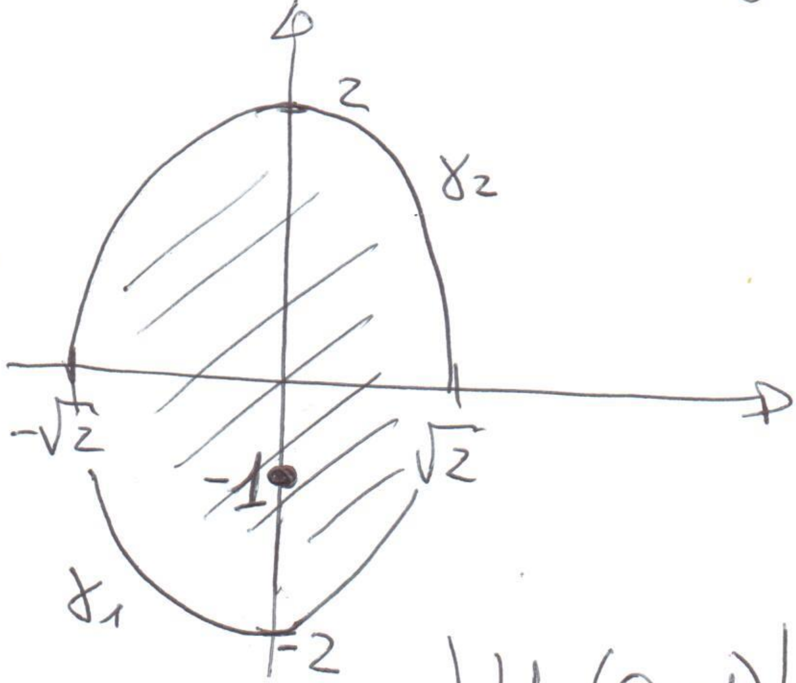
$$= (2)^{\frac{3}{2}} - (2+\pi^2)^{\frac{3}{2}} + \frac{1}{3}(2+\pi^2)^{\frac{3}{2}} - \frac{1}{3}\left(2^{\frac{3}{2}}\right)$$

$$= \frac{2}{3} 2\sqrt{2} - \frac{2}{3}(2+\pi^2)^{3/2}$$

3) $f(x,y) = y^2 \cancel{-x} + (2-x^2)y$

$$\begin{cases} f_x = -2xy = 0 \\ f_y = 2y + 2 - x^2 = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = -1 \end{cases} \cup \begin{cases} y = 0 \\ x = \pm\sqrt{2} \end{cases}$$

appartengono alla frontiera di D



$f_{xx} = -2y$

$f_{xy} = f_{yx} = -2x$

$f_{yy} = 2$

$$|H_f(0,-1)| = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$

definita positiva $\Rightarrow$ PUNTO DI MIN RELATIVO

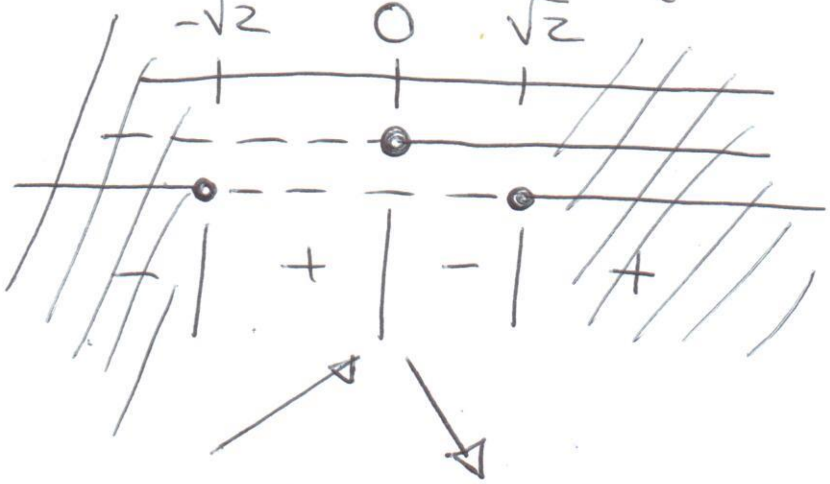
$$\gamma_1 : \begin{cases} y = x^2 - 2 \\ x \in [-\sqrt{2}, \sqrt{2}] \end{cases} \qquad \gamma_2 : \begin{cases} y = 2 - x^2 \\ x \in [-\sqrt{2}, \sqrt{2}] \end{cases}$$

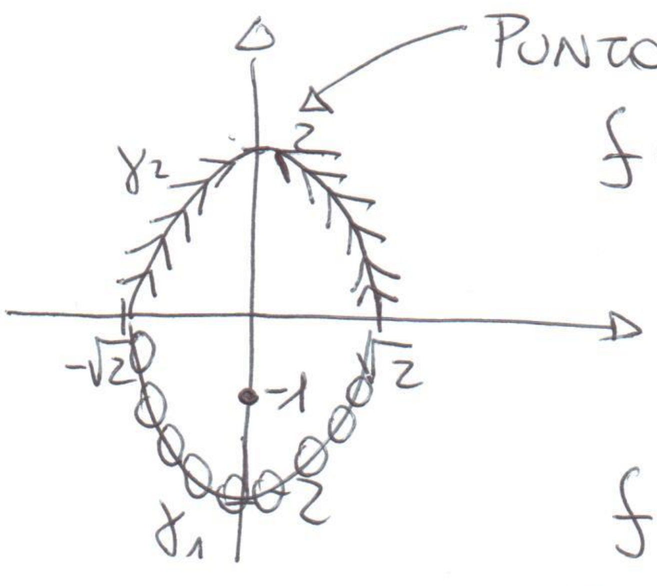
4

$$f\big|_{\gamma_1} \equiv 0$$

$$f\big|_{\gamma_2} = (2-x^2)\left[4-2x^2\right] = 2(2-x^2)^2$$

$$\left(f\big|_{\gamma_2}\right)' = -8x(2-x^2) = 8x(x^2-2)$$





PUNTO DI MAX. REL. e ASS.

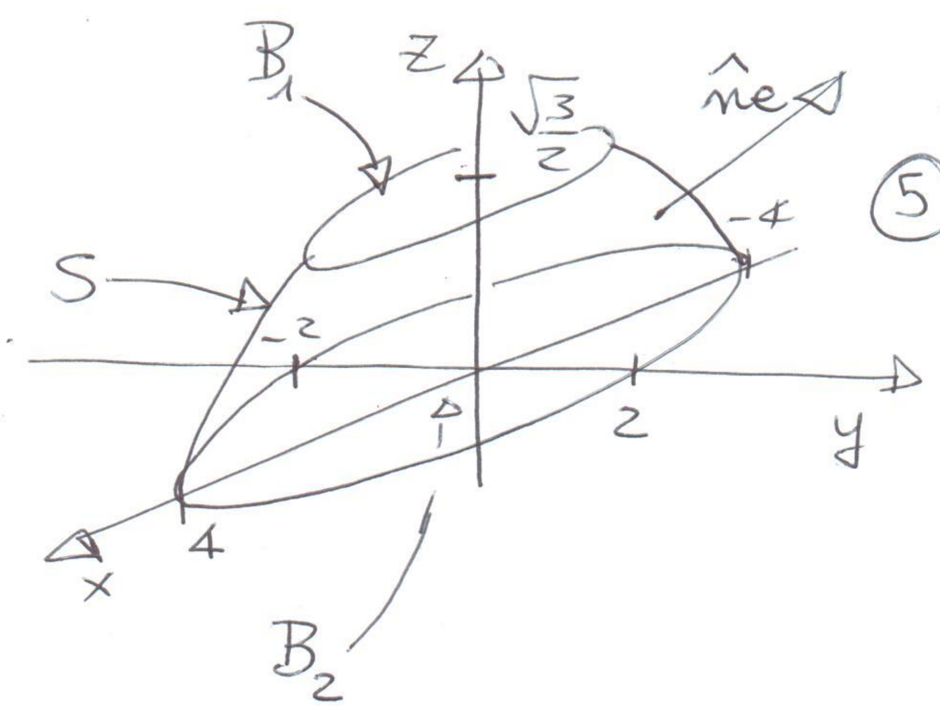
$$f(0,2) = 8$$

$$f\big|_{\gamma_1} \equiv 0$$

$$f(0,-1) = -1$$

PUNTO DI MIN. REL. e ASS.

4)



5

$$\Phi_S(\vec{F}) = \iiint_D \operatorname{div} \vec{F} \, dx\,dy\,dz - \Phi_{B_2} - \Phi_{B_1}$$

$$\operatorname{div} \vec{F} = 1$$

$$= \operatorname{vol} D - \Phi_{B_2} - \Phi_{B_1}$$

$$\operatorname{Vol} D = \int_0^{\frac{\sqrt{3}}{2}} dz \operatorname{Area} S(z)$$

$$S(z) = \left\{ \frac{x^2}{16} + \frac{y^2}{4} \leq 1 - z^2 \right\} = \left\{ \frac{x^2}{16(1-z^2)} + \frac{y^2}{4(1-z^2)} \leq 1 \right\}$$

$$\operatorname{Area}(S(z)) = \pi \, 4\sqrt{1-z^2} \; 2\sqrt{1-z^2}$$

$$= 8\pi(1-z^2)$$

⑥

$$\Rightarrow \mathrm{vol}\, D = \int_0^{\sqrt{3}/2} 8\pi\left(1-z^2\right)dz$$

$$= 8\pi\left[z-\frac{z^3}{3}\right]_0^{\sqrt{3}/2} = 8\pi\left[\frac{\sqrt{3}}{2}-\frac{\cancel{3}\sqrt{3}}{\cancel{3}\cdot 8}\right]$$

$$= \cancel{8}\pi\sqrt{3}\left(\frac{4-1}{\cancel{8}}\right) = 3\sqrt{3}\,\pi$$

Φ_{B_2} : $\hat{n}_e = (0,0,1) \Rightarrow \vec{F}\cdot\hat{n}_e = \frac{\sqrt{3}}{2}$

$$\Rightarrow \Phi_{B_2} = \frac{\sqrt{3}}{2}\,\mathrm{Area}(B_2)$$

$$B_2 = \left\{\frac{x^2}{16}+\frac{y^2}{4}\leq 1-\frac{3}{4}\right\} = \left\{\frac{x^2}{4}+y^2\leq 1\right\}$$

$$= \frac{\sqrt{3}}{2}\pi 2 = \sqrt{3}\pi$$

Φ_{B_1} : $\hat{n}_e = (0,0,-1) \Rightarrow \vec{F}\cdot\hat{n}_e = 0$

$$\Rightarrow \Phi_{B_1} = 0$$

$$\Rightarrow \Phi_S = \mathrm{vol}\,D - \Phi_{B_1} - \Phi_{B_2} = 3\sqrt{3}\pi - \sqrt{3}\pi$$

$$= 2\sqrt{3}\pi.$$

In alternativa, calcolando direttamente il flusso:

$$S = \begin{cases} x = 4 \sin\varphi \cos\vartheta \\ y = 2 \sin\varphi \sin\vartheta \\ z = \cos\varphi \end{cases} \qquad z \in \left[0, \frac{\sqrt{3}}{2}\right] \Rightarrow \varphi \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$$

$$W(\varphi, \vartheta) = \begin{pmatrix} 4\cos\varphi\cos\vartheta & 2\cos\varphi\sin\vartheta & -\sin\varphi \\ -4\sin\varphi\sin\vartheta & 2\sin\varphi\cos\vartheta & 0 \end{pmatrix}$$

$$L = 2\sin^2\varphi\cos\vartheta \; ; \; M = 4\sin^2\varphi\sin\vartheta \; ;$$

$$N = 8\cos\varphi\sin\varphi\cos^2\vartheta + 8\cos\varphi\sin\varphi\sin^2\vartheta = 8\cos\varphi\sin\varphi > 0$$

(normale rivolto verso l'alto)

$$\vec{F} \cdot \vec{n}_e = \left[4\sin^2\varphi\sin^2\vartheta - 1\right] 2\sin^2\varphi\cos\vartheta + 8\cos^2\varphi\sin\varphi$$

$$\cancel{\Rightarrow \Phi_S(\vec{F}) = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left[8\sin^4\varphi\cos\vartheta\sin^2\vartheta \right.}$$

$$\Rightarrow \Phi_S(\vec{F}) = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} d\varphi \int_0^{2\pi} d\vartheta \begin{bmatrix} 8\sin^4\varphi \sin^2\vartheta \cos\vartheta \\ -2\sin^2\varphi \cos\vartheta \\ +8\cos^2\varphi \sin\varphi \end{bmatrix} \quad (8)$$

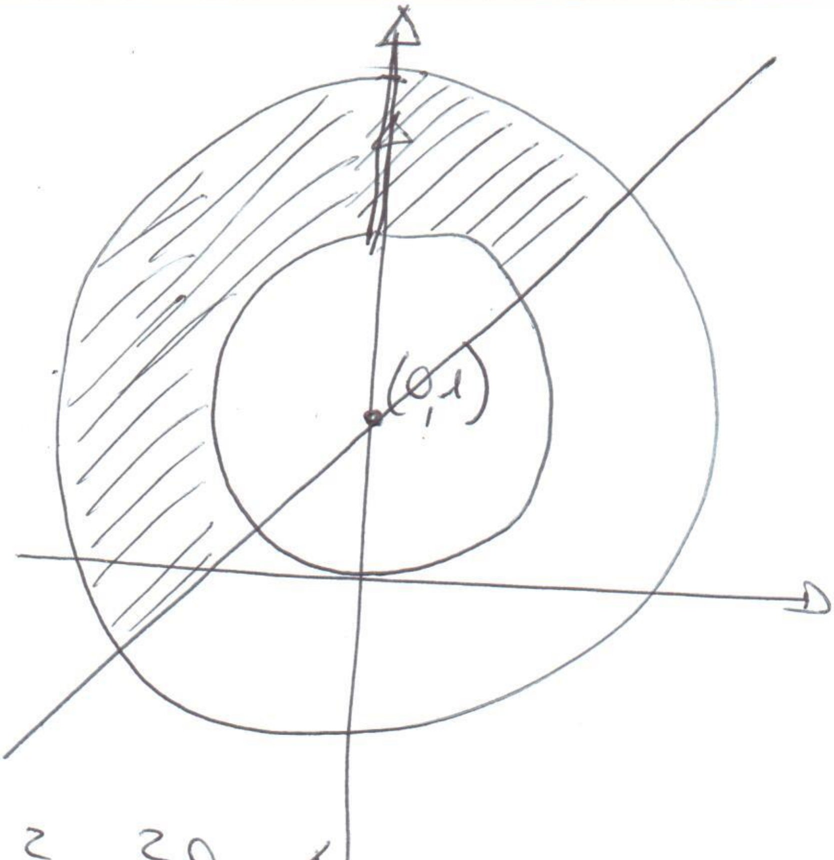
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} d\varphi \left[\cancel{8\sin^4\varphi \frac{\sin^3\vartheta}{3}} - \cancel{2\sin^2\varphi \sin\vartheta} + 8\cos^2\varphi \sin\varphi \, \vartheta \right]_0^{2\pi}$$

$$= 16\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\varphi \sin\varphi \, d\varphi = 16\pi \left[\frac{-\cos^3\varphi}{3} \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{16}{3}\pi \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{16\sqrt{3}\pi}{8} = 2\sqrt{3}\pi.$$

5)

$$\begin{cases} x = \rho\cos\vartheta \\ y = 1 + \rho\sin\vartheta \end{cases}$$



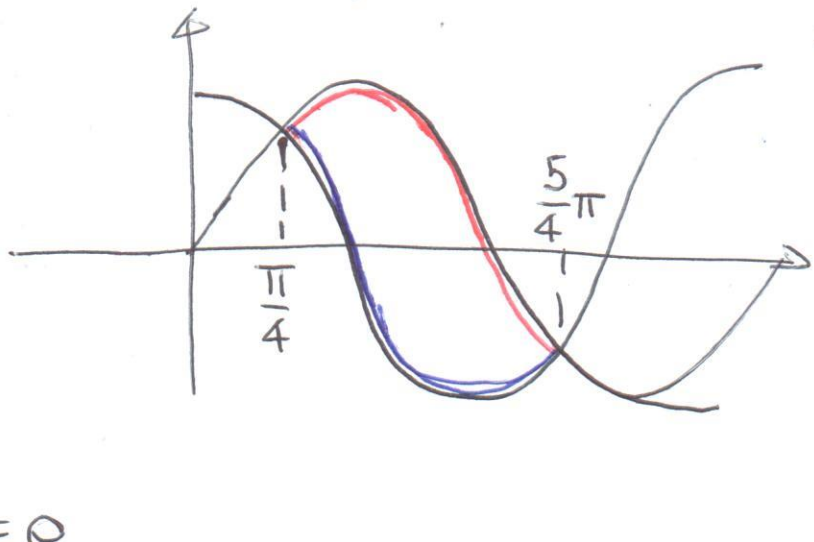
(9)

$$1 \le \rho^2\cos^2\vartheta + \rho^2\sin^2\vartheta \le 4 \implies$$

$$1 \le \rho^2 \le 4 \implies 1 \le \rho \le 2$$

$$1 + \rho\sin\vartheta \ge \rho\cos\vartheta + 1 \implies \sin\vartheta \ge \cos\vartheta$$

$$\implies \vartheta \in \left[\frac{\pi}{4}, \frac{5}{4}\pi\right]$$



$J = \rho$

$$\iint_D \frac{x^2+y^2}{x^2+(y-1)^2}\,dx\,dy =$$

$$\int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} d\vartheta \int_1^2 \frac{\rho^2\cos^2\vartheta + (\rho\sin\vartheta+1)^2}{\rho^{\cancel{2}}}\,\cancel{\rho}\,d\rho$$

$$= \int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} d\vartheta \int_1^2 \left[\frac{\rho^2 + 2\rho\sin\vartheta + 1}{\rho}\right] d\rho$$

$$= \int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} d\vartheta \int_1^2 \left[\rho + 2\sin\vartheta + \frac{1}{\rho}\right] d\rho$$

$$= \int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} d\vartheta \left[\frac{\rho^2}{2} + 2\rho\sin\vartheta + \ln(|\rho|)\right]_1^2$$

$$= \int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} d\vartheta \left[\frac{3}{2} + 2\sin\vartheta + \ln 2\right]$$

$$= \left[\left(\frac{3}{2} + \ln 2\right)\vartheta - 2\cos\vartheta\right]_{\frac{\pi}{4}}^{\frac{5}{4}\pi} =$$

$$\left(\frac{3}{2} + \ln 2\right)\pi + 2\sqrt{2}.$$