SVOLGIMENTI PROVA SCRITTA
ANALISI 2 del 28/1/2021

1) $\sum_{k=0}^{+\infty} \left| \frac{\cos(kx)}{k!} \right| \leq \sum_{k=0}^{+\infty} \frac{1}{k!}$ convergente (al valore e).

$\Rightarrow$ la serie converge TOTALMENTE, UNIFORMEMENTE, ASSOLUTAMENTE e PUNTUALMENTE in $\mathbb{R}$.

SOMMA DELLA SERIE:

$$e^z = \sum_{k=0}^{+\infty} \frac{z^k}{k!}$$

$$e^{(e^{ix})} = e^{\cos x + i \sin x} = e^{\cos x}\left[\cos(\sin x) + i \sin(\sin x)\right]$$

$$\Rightarrow \sum \frac{(e^{ix})^k}{k!} = \sum \frac{e^{ikx}}{k!} = \sum \frac{1}{k!}\left[\cos(kx) + i \sin(kx)\right]$$

Confrontando le parti reali, si ha

$$\sum_{k=0}^{+\infty} \frac{\cos(kx)}{k!} = e^{\cos x} \cos(\sin x).$$

CONTINUITÀ

2) In $(0,0)$:

$$\lim_{(x,y)\to(0,0)} |f(x,y)| = \lim_{(x,y)\to(0,0)} \frac{|xy|}{|x|+|y|}$$

$$\le \lim_{(x,y)\to(0,0)} \frac{|xy|}{|x|} = \lim_{(x,y)\to(0,0)} |y| = 0 \Rightarrow$$ CONTINUA in $(0,0)$.

Negli altri punti, f è quoziente di funzioni $C^0 \Longrightarrow f \in C^0(\mathbb{R}^2)$.

DERIVABILITÀ PARZIALE:

In $(0,0)$: $\quad f(0,y) = f(x,0) = 0$

$$\Rightarrow f_x(0,0) = f_y(0,0) = 0.$$

~~Nel primo quadrante:~~

~~$f(x,y)$~~

N.B.: f DISPARI rispetto a entrambi gli assi.

Quindi $f(-x,-y) = f(x,y)$. SIMMETRICA RISPETTO ALLA RETTA $y=x$.

Nel primo quadrante:

$$f(x,y) = \frac{xy}{x+y} \Longrightarrow \frac{\partial f}{\partial x} = y\left[\frac{x+y-x}{(x+y)^2}\right] = \frac{y^2}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{x^2}{(x+y)^2} \quad \text{per simmetria.}$$

Nel secondo quadrante: $f(x,y) = \dfrac{xy}{-x+y}$ (3)

$$\Rightarrow \frac{\partial f}{\partial x} = y\left[\frac{-x+y+x}{(-x+y)^2}\right] = \frac{y^2}{(-x+y)^2}$$

$$\frac{\partial f}{\partial y} = x\left[\frac{\cancel{-x+y}\cancel{-y}}{(-x+y)^2}\right] = \frac{-x^2}{(-x+y)^2}$$

Analogamente, per il terzo quadrante:

$$f(x,y) = \frac{xy}{-(x+y)}$$ e per il quarto quadrante:

$$f(x,y) = \frac{xy}{x-y}.$$

In generale,

$$\frac{\partial}{\partial x}\left(\frac{xy}{|x|+|y|}\right) = y\left(\frac{|x|+|y|-x\cdot\frac{|x|}{x}}{(|x|+|y|)^2}\right)$$

$$= \frac{y|y|}{(|x|+|y|)^2}$$

$$\frac{\partial}{\partial y}\left(\frac{xy}{|x|+|y|}\right) = x\left(\frac{|x|+|y|-y\frac{|y|}{y}}{(|x|+|y|)^2}\right) = \frac{x|x|}{(|x|+|y|)^2}$$

In $(x_0, 0)$: $f(x,0)=0$

$x_0 \neq 0$ $\Rightarrow \frac{\partial f}{\partial x}(x_0, 0) = 0$

$$\frac{\partial f}{\partial y}(x_0,0) = \lim_{k\to 0}\left[\frac{x_0 \cancel{k}}{|x_0|+|k|}\right]\frac{1}{\cancel{k}} = \frac{x_0}{|x_0|}$$

$$= \begin{cases} +1 & \text{se } x_0 > 0 \\ -1 & \text{se } x_0 < 0 \end{cases} = \text{sign}(x_0)$$

Analogamente in $(0, y_0)$, $y_0 \neq 0$:

$$\frac{\partial f}{\partial x}(0,y_0) = \frac{y_0}{|y_0|} = \text{sign}(y_0) = \begin{cases} 1 & \text{se } y_0 > 0 \\ -1 & \text{se } y_0 < 0. \end{cases}$$

DERIVATE DIREZIONALI: 4

In $\mathbb{R}^2 - \{\text{assi coordinati}\}$, f è quoziente di funzioni C^∞ $\Rightarrow$ differenziabile $\Rightarrow$ vale la formula del gradiente.

In $(0,0)$:

$$\frac{df}{d\vec{v}}(0,0) = \lim_{t\to 0^{\pm}} \frac{\alpha\beta t^{\cancel{2}}}{(|\alpha|+|\beta|)|t|\cancel{t}} = \pm\frac{\alpha\beta}{(|\alpha|+|\beta|)}$$

$\Rightarrow \exists \frac{df}{d\vec{v}}(0,0)$ solo per $\alpha = 0$ $\left(\frac{\partial f}{\partial y}(0,0)=0\right)$

e per $\beta = 0$ $\left(\frac{\partial f}{\partial x}(0,0) = 0\right)$.

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In $(x_0, 0)$, $x_0 \neq 0$:

$$\frac{df}{d\vec{v}}(x_0,0) = \lim_{t\to 0} \left[\frac{(x_0+\alpha t)\beta \cancel{t}}{|x_0+\alpha t| + |\beta t|}\right]\frac{1}{\cancel{t}} = \frac{x_0}{|x_0|}\beta$$

$$= \begin{cases} 0 & \text{se } \beta = 0 \quad \left(\frac{\partial f}{\partial x}(x_0,0) = 0\right) \\ \beta \cdot \text{sign}(x_0) & \text{se } \beta \neq 0 \end{cases}$$

Analogamente:

$$\frac{df}{d\vec{v}}(0,y_0) = \begin{cases} 0 & \text{se } \alpha = 0 \quad \left(\frac{\partial f}{\partial y}(0,y_0) = 0\right) \\ \alpha \, \text{sign}(y_0) & \text{se } \alpha \neq 0. \end{cases}$$

Si osservi che, sia in $(x_0, 0)$, sia in $(0, y_0)$

$$\frac{df}{d\vec{v}} = \alpha f_x + \beta f_y. \quad \text{(formula del gradiente)}$$

Questa formula è CONDIZIONE NECESSARIA, MA NON SUFFICIENTE, per la differenziabilità, che dovremo studiare direttamente.

DIFFERENZIABILITÀ:

In $(0,0)$ già stabilito. Riverifichiamolo:

$$\lim_{(h,k)\to(0,0)} \frac{\left(\frac{hk}{|h|+|k|}\right)}{\sqrt{h^2+k^2}} = \lim_{(h,k)\to(0,0)} \left(\frac{hk}{|h|+|k|}\right)\frac{1}{\sqrt{h^2+k^2}}$$

$$= \lim_{\rho\to 0} \frac{\rho^2\cos\vartheta\,\sin\vartheta}{\rho\left[|\cos\vartheta|+|\sin\vartheta|\right]}\frac{1}{\rho} = \frac{\cos\vartheta\,\sin\vartheta}{\left[|\cos\vartheta|+|\sin\vartheta|\right]}$$

DIPENDENTE DA ϑ.

$\Rightarrow$ NO DIFF.BILE in $(0,0)$.

In $\mathbb{R}^2 - \{assi\}$ DIFFERENZIABILE perché C^∞.

In $(x_0,0)$, $x_0\neq 0$:

$$\lim_{(h,k)\to(0,0)} \left[\frac{(x_0+h)(\cancel{0+}k)}{|x_0+h|+|\cancel{0+}k|} - \frac{|x_0|}{x_0}k\right]\frac{1}{k}$$

Se $x_0>0$: $\exists h$ t.c. $x_0+h>0 \Rightarrow$

$$= \lim_{(h,k)\to(0,0)} \left[\frac{x_0+h}{x_0+h+|k|} - 1\right]\frac{\cancel{k}}{\cancel{k}} = 0$$

Analogamente per $x_0<0$:

$$= \lim_{(h,k)\to(0,0)} \left[\frac{x_0+h}{-(x_0+h)+|k|} + 1\right]\frac{\cancel{k}}{\cancel{k}} = 0.$$

Stesse considerazioni per i punti $(0, y_0)$, $y_0 \neq 0$.
Dunque f è DIFFERENZIABILE su tutto $\mathbb{R}^2 - \{(0,0)\}$.

3) $\frac{\partial f}{\partial x} = 2x = 0 \iff x = 0$ ⑥

$$\frac{\partial f}{\partial y} = \frac{-2y}{1+y^2} - 2y e^{-y^2} = -2y \underbrace{\left(\frac{1}{1+y^2} + e^{-y^2} \right)}_{>0}$$

$\boxed{\;= 0 \iff y = 0\;}$

$\Rightarrow (0,0)$ unico punto stazionario.

$f(0,0) = 1$

Lungo asse x: $\Delta f = f(x,0) - f(0,0) = x^2 + 1 - 1$
$= x^2 \geq 0$

Lungo asse y: $\Delta f = f(0,y) - f(0,0)$
$= -\log(1+y^2) + e^{-y^2} - 1$
$= -y^2 - y^2 + o(y^2) \sim -2y^2 \leq 0$

$\Rightarrow (0,0)$ è punto di sella.

In alternativa:

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$$f_{xx}=2 \quad ; \quad f_{xy}=f_{yx}=0$$

$$f_{yy}=-2\left(\frac{1}{1+y^2}+e^{-y^2}\right)+$$
$$-2y\left(\frac{-2y}{(1+y^2)^2}-2y\,e^{-y^2}\right)$$

$$|H_f(0,0)| = \begin{vmatrix} 2 & 0 \\ 0 & -4 \end{vmatrix} = -8$$

$\Rightarrow (0,0)$ punto di sella

Lungo la frontiera: utilizziamo i moltiplicatori di Lagrange.

$$\mathcal{L}(x,y,\lambda)=x^2-\log(1+y^2)+e^{-y^2}$$
$$-\lambda(x^2+y^2-1)$$

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$$\mathcal{L}_x = 2x - 2x\lambda = 2x(1-\lambda) = 0$$

$$\mathcal{L}_y = \frac{-2y}{1+y^2} - 2ye^{-y^2} - 2y\lambda$$

$$= -2y\left(\frac{1}{1+y^2} + e^{-y^2} + \lambda\right) = 0$$

$$\mathcal{L}_\lambda = 1 - x^2 - y^2 = 0 \quad \text{(VINCOLO)}$$

$$\begin{cases} x = 0 \\ y = \pm 1 \\ \underbrace{\frac{1}{2} + e^{-1} + \lambda = 0}_{\text{NON ESSENZIALE}} \end{cases} \cup \begin{cases} \lambda = 1 \\ -2y\underbrace{\left(\frac{1}{1+y^2} + e^{-y^2} + 1\right)}_{>0} = 0 \\ x^2 + y^2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} x = 0 \\ y = \pm 1 \\ \lambda = -\frac{1}{2} - \frac{1}{e} \end{cases} \cup \begin{cases} \lambda = 1 \\ y = 0 \\ x = \pm 1 \end{cases}$$

$\Rightarrow P_1 = (0,1)$; $P_2 = (0,-1)$; $P_3 = (1,0)$;

$P_4 = (-1,0)$.

N.B. f è SIMMETRICA RISPETTO A ENTRAMBI GLI ASSI

$$f(P_1) = f(P_2) = -\log 2 + e^{-1} \leftarrow \text{MIN. ASS.}$$

$$f(P_3) = f(P_4) = 2 \leftarrow \text{MAX. ASS.}$$ 8

Altrimenti, parametrizziamo la frontiera:

$$x^2 = 1 - y^2$$

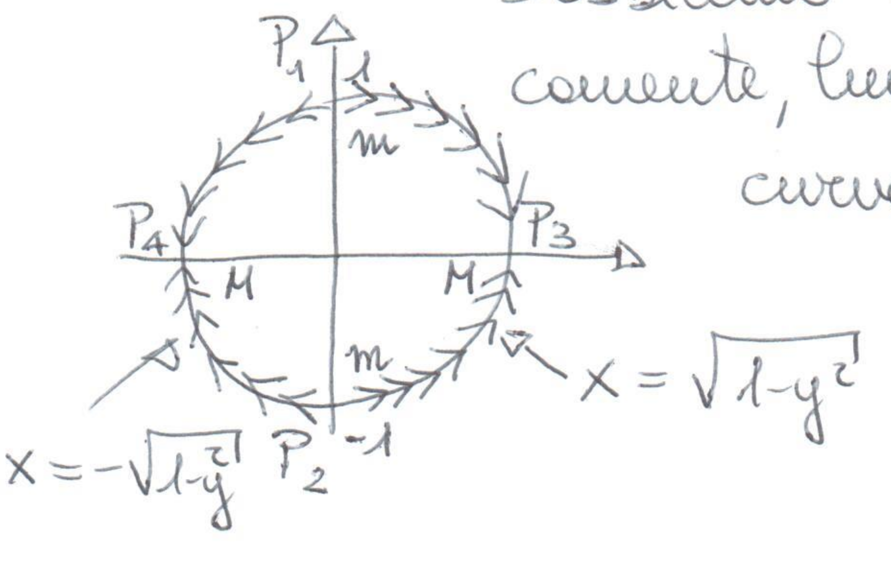
$$\Rightarrow f(x^2 = 1-y^2) = 1 - y^2 - \log(1+y^2) + e^{-y^2}$$

$$\Rightarrow f'_y = -2y - \frac{2y}{1+y^2} - 2y\, e^{-y^2}$$

$$= -2y \underbrace{\left[1 + \frac{1}{1+y^2} + e^{-y^2}\right]}_{>0} \geq 0$$

$$\Leftrightarrow y \leq 0$$

Dobbiamo muoverci simmetricamente, lungo le due curve $x = \pm\sqrt{1-y^2}$



Ulteriore svolgimento:

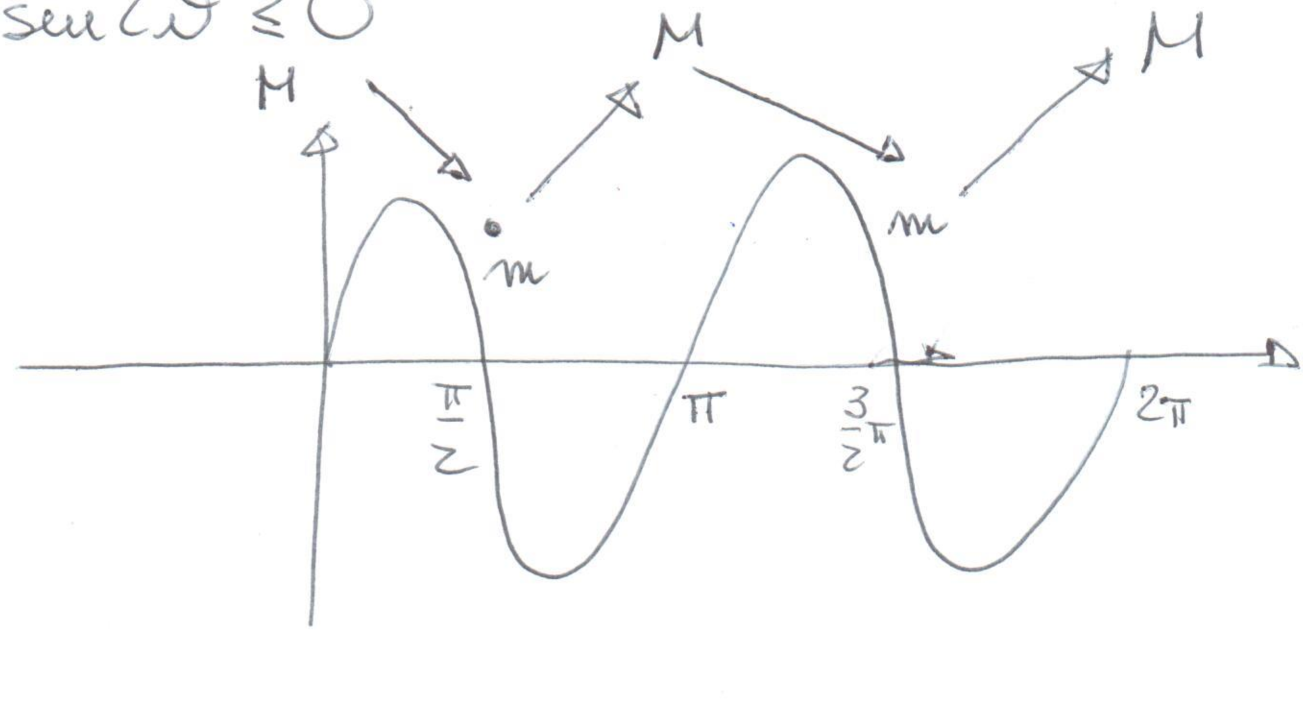
$$\begin{cases} x(\vartheta)=\cos\vartheta \\ y(\vartheta)=\sin\vartheta \end{cases} \quad ; \quad \vartheta\in[0,2\pi]$$

$$f(\vartheta)=\cos^2\vartheta-\log(1+\sin^2\vartheta)+e^{-\sin^2\vartheta}$$

$$f'(\vartheta)=-2\sin\vartheta\cos\vartheta-\frac{2\sin\vartheta\cos\vartheta}{1+\sin^2\vartheta}-2\sin\vartheta\cos\vartheta\cdot e^{-\sin^2\vartheta}$$

$$=-\sin2\vartheta\underbrace{\left[1+\frac{1}{1+\sin^2\vartheta}+e^{-\sin^2\vartheta}\right]}_{>0}\geq 0$$

$$\Longleftrightarrow \sin2\vartheta\leq 0$$



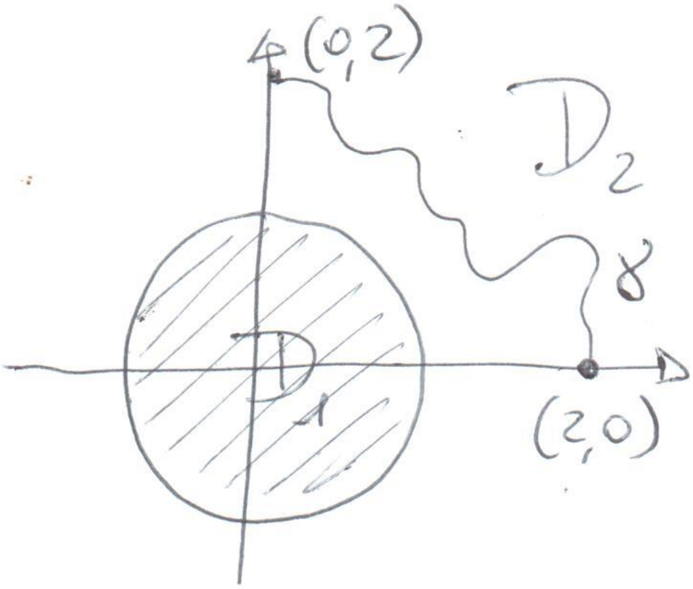
4) ω è definita per $x^2+y^2 \neq 1$ (8c)

Quindi $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \neq 1\}$

$$= \underbrace{\{x^2+y^2<1\}}_{D_1} \cup \underbrace{\{x^2+y^2>1\}}_{D_2} =$$

D è SCONNESSO. Poiché $(0,2),(2,0) \in D_2$

$\Rightarrow$ ci mettiamo in D_2



ω è CHIUSA:

$$\frac{\partial X}{\partial y} = x\left[\frac{(1-x^2-y^2)^{\cancel{2}} - y\,2\cancel{(1-x^2-y^2)}(-2y)}{(1-x^2-y^2)^{\cancel{4}\,3}}\right]$$

$$= x\left[\frac{1-x^2+3y^2}{(1-x^2-y^2)^3}\right]$$

$$\frac{\partial Y}{\partial x} = \frac{1}{2}\left[\frac{-2x(1-x^2-y^2)^{\cancel{2}} - 2(1-x^2+y^2)\cancel{(1-x^2-y^2)}\cdot(-2x)}{(1-x^2-y^2)^{\cancel{4}\,3}}\right.$$

$$= \frac{-x}{(1-x^2-y^2)^3}\left[1-x^2-y^2-2+2x^2-2y^2\right]$$

$$= \frac{x(1-x^2+3y^2)}{(1-x^2-y^2)^3} \quad \Rightarrow \quad X_y = Y_x.$$

D_2 è molteplicemente connesso, però, essendo chiusa, la forma è LOCALMENTE esatta.

Potenziale:

$$V(x,y) = \int \frac{xy}{(1-x^2-y^2)^2}\,dx =$$

$$+\frac{1}{2}\,\frac{y}{(1-x^2-y^2)} + \varphi(y)$$

$$V_y = \frac{1}{2}\left[\frac{1-x^2-y^2+2y^2}{(1-x^2-y^2)^2}\right] + \varphi'(y)$$

$$= \frac{1}{2}\left[\frac{1-x^2+y^2}{(1-x^2-y^2)^2}\right] + \varphi'(y) = Y = \frac{1-x^2+y^2}{2(1-x^2-y^2)^2}$$

$$\Rightarrow \varphi'(y) = 0 \quad \Rightarrow \quad \varphi(y) = C$$

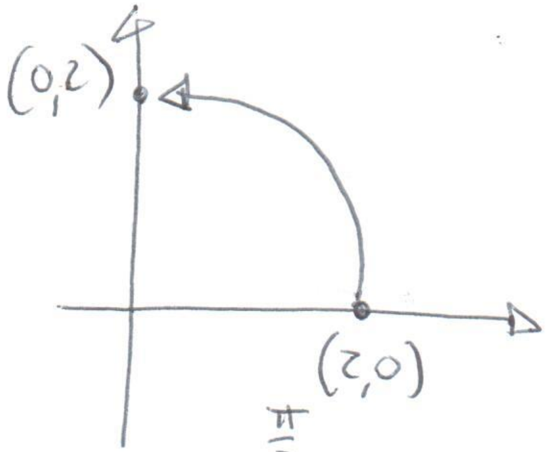
$$\Rightarrow V(x,y) = \frac{1}{2}\,\frac{y}{(1-x^2-y^2)} + C$$

$$\Rightarrow \int_\gamma \omega = V(0,2) - V(2,0)$$

$$= \frac{1}{2}\left[\frac{2}{1-4}\right] = -\frac{1}{3}.$$

In alternativa, calcoliamo l'integrale lungo la circonferenza:

$$\begin{cases} x(\vartheta) = 2\cos\vartheta \\ y(\vartheta) = 2\sin\vartheta \end{cases} \qquad \vartheta \in \left[0, \frac{\pi}{2}\right]$$



$$\int_\gamma \omega = \int_0^{\frac{\pi}{2}} \left[\frac{4\cos\vartheta\sin\vartheta}{(1-4)^2}(-2\sin\vartheta) + \frac{1-4(\cos^2\vartheta - \sin^2\vartheta)}{\cancel{2}(1-4)^2}(\cancel{2}\cos\vartheta) \right] d\vartheta$$

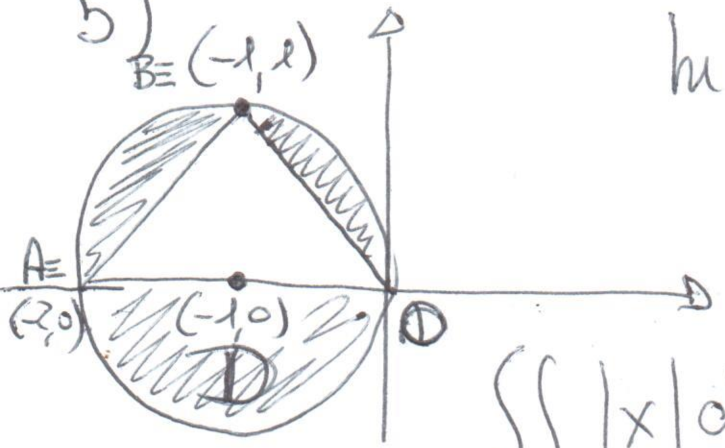
$$= \frac{1}{9}\int_0^{\frac{\pi}{2}} \left[-8\sin^2\vartheta\cos\vartheta + \cos\vartheta - 4(1-2\sin^2\vartheta)\cos\vartheta \right] d\vartheta$$

$$= \frac{1}{9}\int_0^{\frac{\pi}{2}} \left[\cancel{-8\sin^2\vartheta\cos\vartheta} + \cos\vartheta - 4\cos\vartheta + \cancel{8\sin^2\vartheta\cos\vartheta} \right] d\vartheta =$$

$$= \frac{1}{9}\int_0^{\frac{\pi}{2}} -3\cos\vartheta \, d\vartheta = -\frac{1}{3}\,\text{sen}\,\vartheta \Big|_0^{\frac{\pi}{2}} = -\frac{1}{3}$$

10_b

5)



In D $\quad x \leq 0$

$$\Rightarrow \iint_D |x|\,dx\,dy = -\iint_D x\,dx\,dy$$

Teorema di Guldino:

$$\iint x\,dx\,dy = x_B \cdot \text{Area } D$$

Per simmetria, $x_B = -1$

$$\text{Area } D = \cancel{\text{Area}}\ \text{Area}(C) - \text{Area}(T)$$

$$= \pi - \frac{2\cdot 1}{2} = \pi - 1$$

$$\Rightarrow \iint |x|\,dx\,dy = -(-1)(\pi-1) = \pi - 1.$$

Svolgimento alternativo (PROF.SSA TOSTI):

$$\iint_D |x|\,dx\,dy = -\left[\iint_{\mathcal{E}} x\,dx\,dy - \iint_T x\,dx\,dy\right]$$

$$\mathcal{E} = \begin{cases} x = -1 + \rho\cos\vartheta \\ y = \rho\,\mathrm{sen}\,\vartheta \end{cases} \quad ; \; \vartheta \in [0, 2\pi] \quad \rho \in [0, 1]$$

$$\overline{OB}: \begin{cases} y = -x \\ x \in [-1, 0] \end{cases} \iff \begin{cases} x = -y \\ y \in [0, 1] \end{cases}$$

$$\overline{AB}: \begin{cases} y = x + 2 \\ x \in [-2, -1] \end{cases} \iff \begin{cases} x = y - 2 \\ y \in [0, 1] \end{cases}$$

$$\Rightarrow T = \{ y \in [0, 1] \; ; \; y - 2 \leq x \leq -y \}$$

x-semplice

$$\iint_D |x|\,dx\,dy = -\left[\int_0^{2\pi} d\vartheta \int_0^1 (-1 + \rho\cos\vartheta)\,\rho\,d\rho - \int_0^1 dy \int_{y-2}^{-y} x\,dx\right]$$

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$$= -\left[\int_0^{2\pi} d\vartheta \left[-\frac{\rho^2}{2} + \frac{\rho^3}{3}\cos\vartheta \right]_0^1 \right.$$

$$\left. - \int_0^1 dy \left[\frac{x^2}{2} \right]_{(y-2)}^{-y} \right]$$

$$= -\left[\int_0^{2\pi} \left(-\frac{1}{2} + \frac{1}{3}\cos\vartheta \right) d\vartheta - \frac{1}{2} \int_0^1 \left[(-y)^2 - (y-2)^2 \right] dy \right]$$

$$= -\left[\left(-\frac{1}{2}\vartheta + \frac{1}{3}\sin\vartheta \right) \Big|_0^{2\pi} - \frac{1}{2} \int_0^1 \left[y^2 - (y-2)^2 \right] dy \right]$$

$$= -\left[-\pi - \frac{1}{2} \left[\frac{y^3}{3} - \frac{(y-2)^3}{3} \right]_0^1 \right]$$

$$= \pi + \frac{1}{6} \left[\left(1 - (-1)^3 \right) - \left(-(-2)^3 \right) \right]$$

$$= \pi + \frac{1}{6} \left[2 - 8 \right] = \pi - 1.$$

6)

$$\gamma'(t): \begin{cases} x' = -e^{-t} \\ y' = 2e^{\frac{t}{2}} \\ z' = 2e^{2t} \end{cases} \quad ; \quad \gamma''(t): \begin{cases} x'' = e^{-t} \\ y'' = e^{\frac{t}{2}} \\ z'' = 4e^{2t} \end{cases}$$

La curva è regolare (gli esponenziali non si annullano mai), semplice e NON chiusa (le derivate sono di segno costante, quindi le coordinate sono monotòne).

$$v(t) = \sqrt{e^{-2t} + 4e^{t} + 4e^{4t}}$$

$$= \sqrt{(e^{-t} + 2e^{2t})^2} = |e^{-t} + 2e^{2t}| = e^{-t} + 2e^{2t}$$

$$\Rightarrow \ell(\gamma) = \int_0^1 [e^{-t} + 2e^{2t}]\,dt = \left[-e^{-t} + e^{2t}\right]_0^1$$

$$= -e^{-1} + e^{2} - (-1 + 1) = e^{2} - \frac{1}{e}.$$

$$\hat{T} = \frac{\vec{r}\,'}{\|\vec{r}\,'\|} = \frac{\vec{r}\,'(t)}{v(t)} =$$

$$\frac{1}{\left(e^{-t}+2e^{2t}\right)}\left(-e^{-t}, 2e^{\frac{t}{2}}, 2e^{2t}\right)$$

$$\hat{B} = \frac{\vec{r}\,' \wedge \vec{r}\,''}{\|\vec{r}\,' \wedge \vec{r}\,''\|}$$

$$\vec{r}\,' \wedge \vec{r}\,'' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -e^{-t} & 2e^{\frac{t}{2}} & 2e^{2t} \\ e^{-t} & e^{\frac{t}{2}} & 4e^{2t} \end{vmatrix}$$

$$= \left(8e^{\frac{5}{2}t} - 2e^{\frac{5}{2}t}\right)\vec{i} + \left(4e^{t} + 2e^{t}\right)\vec{j}$$

$$- \left(e^{-\frac{t}{2}} + 2e^{-\frac{t}{2}}\right)\vec{k}$$

$$= 6e^{\frac{5}{2}t}\,\vec{i} + 6e^{t}\,\vec{j} - 3e^{-\frac{t}{2}}\,\vec{k}$$

$$= 3e^{-\frac{t}{2}}\left(2e^{3t}; 2e^{\frac{3}{2}t}; -1\right)$$

$$\|\vec{r}\,' \wedge \vec{r}\,''\| = 3e^{-\frac{t}{2}}\sqrt{4e^{6t}+4e^{3t}+1}$$

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$$= 3e^{-\frac{t}{2}}\sqrt{(2e^{3t}+1)^2}$$

$$= 3e^{-\frac{t}{2}}\left|2e^{3t}+1\right| = 3e^{-\frac{t}{2}}\left(2e^{3t}+1\right)$$

$$\Rightarrow \hat{B} = \frac{\cancel{3e^{-\frac{t}{2}}}}{\cancel{3e^{-\frac{t}{2}}}(2e^{3t}+1)}\left(2e^{3t}, 2e^{\frac{3}{2}t}, -1\right)$$

N.B.:

$$\hat{T} = \frac{1}{e^{-t}(2e^{3t}+1)}\left[e^{-t}\left(-1, 2e^{\frac{3}{2}t}, 2e^{3t}\right)\right]$$

$$= \frac{1}{(2e^{3t}+1)}\left(-1, 2e^{\frac{3}{2}t}, 2e^{3t}\right)$$

Così si verifica più facilmente che $\hat{T}\cdot\hat{B} = 0$.

$$\hat{T}(0) = \frac{1}{3}(-1, 2, 2)$$

$$\hat{B}(0) = \frac{1}{3}(2, 2, -1)$$

N.B.:

$$\hat{N}(0) = \frac{1}{9}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{vmatrix}$$

$$= \hat{B}(0) \wedge \hat{T}(0)$$

$$= \frac{1}{9}\left[6\vec{i} - 3\vec{j} + 6\vec{k}\right]$$

$$= \frac{1}{3}(2, -1, 2).$$