SVOLGIMENTI PROVA SCRITTA del 26/1/2022.

1)
$$y(x) = e^{-2\int_0^x t\,dt}\left[2+\int_0^x e^{2\int_0^t s\,dt}\, t e^{-t^2}\,dt\right]$$

$$= e^{-t^2\big|_0^x}\left[2+\int_0^x e^{s^2\big|_0^t}\, t e^{-t^2}\,dt\right]$$

$$= e^{-x^2}\left[2+\int_0^x e^{t^2}\, t e^{-t^2}\,dt\right] =$$

$$= e^{-x^2}\left[2+\int_0^x t\,dt\right] = e^{-x^2}\left(2+\frac{t^2}{2}\Big|_0^x\right)$$

$$= \left(\frac{x^2}{2}+2\right)e^{-x^2}.$$

2) $D = \left\{x \neq \frac{1}{3}\right\} = \left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, +\infty\right).$

$$\lim_{x\to \frac{1}{3}^\pm} f(x) = \frac{\frac{4}{9}+\frac{2}{3}+1}{0^\pm} = \pm\infty$$

$x = \frac{1}{3}$ ASINTOTO VERTICALE da DX e SX.

$$f(x) = \left(\frac{4}{3}x + \frac{10}{9}\right) + \frac{\frac{19}{9}}{3x-1} \xrightarrow[x\to\pm\infty]{} \pm\infty$$

Inoltre, poiché $\dfrac{\frac{19}{9}}{3x-1} \xrightarrow[x\to\pm\infty]{} 0$,

$f(x)$ ha come ASINTOTO OBLIQUO a $\pm\infty$ $\quad y = \frac{4}{3}x + \frac{10}{9}$.

Segno: $\quad 4x^2+2x+1>0 \quad \forall x\in\mathbb{R}$

$\Rightarrow \quad f(x)>0 \iff 3x-1>0$

$\Rightarrow \quad f(x)>0$ in $\left(\frac{1}{3}, +\infty\right)$

$\qquad f(x)<0$ in $\left(-\infty, \frac{1}{3}\right)$.

f NON SI ANNULLA MAI.

3) f sempre definita.

$$f'(x) = 2\left[(x^2-x+1)^{-2}\right]' = 2\cdot(-2)(x^2-x+1)^{-3}\cdot(2x-1)$$

$$= \frac{-4(2x-1)}{(x^2-x+1)^3}.$$

Poiché $x^2-x+1>0 \quad \forall x\in\mathbb{R}$

$\Rightarrow \quad f'(x)>0 \iff 2x-1<0$

$\iff \quad x < \frac{1}{2}$

f cresce in $\left(-\infty, \frac{1}{2}\right)$ e decresce in $\left(\frac{1}{2}, +\infty\right)$

$\Rightarrow \quad x = \frac{1}{2}$ PUNTO DI MASSIMO ASSOLUTO

$$f\left(\frac{1}{2}\right) = \frac{2}{\frac{1}{4} - \frac{1}{2} + 1} = \frac{8}{3}$$

Poichè $f(x) > 0 \quad \forall x \in \mathbb{R}$ e $\lim_{x \to \pm\infty} f(x) = 0$, non abbiamo MIN. REL. o ASS. ma ESTREMO INFERIORE pari a 0.

4) $P(\text{obesi}) = 0{,}3$

$P(\text{diabetici}) = 0{,}03$

$P(\text{obesi} \cap \text{diabetici}) = 0{,}02$

$$P(\text{diabetici} \mid \text{obesi}) = \frac{P(\text{diabetici} \cap \text{obesi})}{P(\text{obesi})}$$

$$= \frac{0{,}02}{0{,}3} = 0{,}0\overline{6}$$

$$P(\text{obesi} \mid \text{diabetici}) = \frac{P(\text{diabetici} \wedge \text{obese})}{P(\text{diabetici})}$$

$$= \frac{0,02}{0,03} = 0,\overline{6} .$$

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5) MEDIA $\cong$ 170,9

MEDIANA = 171

MODA = 170

6) $$\det M = \begin{vmatrix} 2 & 2 & 0 \\ 1 & k & -1 \\ 0 & 1 & 2 \end{vmatrix} = 4k - 4 + 2$$

$$= 4k - 2 = 0$$

$$\Leftrightarrow k = \frac{1}{2} .$$

Autovalori (per $k=2$):

$$\det(M - \lambda \mathbb{I}) = \begin{vmatrix} 2-\lambda & 2 & 0 \\ 1 & 2-\lambda & -1 \\ 0 & 1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)^3 + (2-\lambda) - 2(2-\lambda)$$

$$= (2-\lambda)\left[(2-\lambda)^2 - 1\right] = 0$$

$$\Rightarrow \lambda_1 = 2 \quad ; \quad (2-\lambda)^2 = 1$$

$$\Rightarrow (2-\lambda) = \pm 1$$

$$\Rightarrow \quad \lambda_2 = 1 \quad ; \quad \lambda_3 = 3$$

$\lambda_1 = 2$:

$$\begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} y = 0 \\ -z = -x \end{cases} \Rightarrow \begin{pmatrix} x \\ 0 \\ x \end{pmatrix}$$

$\lambda_2 = 1$:

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 2y = -x \\ y - z = -x \end{cases} \quad \begin{cases} y = -\frac{x}{2} \\ z = y + x = \frac{x}{2} \end{cases}$$

$$\Rightarrow \begin{pmatrix} x \\ -\frac{x}{2} \\ \frac{x}{2} \end{pmatrix}$$

$\lambda_3 = 3$:

$$\begin{pmatrix} -1 & 2 & 0 \\ 1 & -1 & -1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 2y = x \\ -y - z = -x \end{cases} \Rightarrow \begin{cases} y = \frac{1}{2}x \\ z = x - y = \frac{1}{2}x \end{cases}$$

$$\Rightarrow \begin{pmatrix} x \\ \frac{1}{2}x \\ \frac{1}{2}x \end{pmatrix}$$