SVOLGIMENTI PROVA SCRITTA di MATEMATICA per CTF - LT del 18/7/22

1)
$$y(x) = e^{-\int_0^x 2t\,dt} \cdot \left[\int_0^x e^{\int_0^t 2s\,ds}\, t e^{-t^2}\, dt \right]$$
$$= e^{-t^2 \big|_0^x} \left[\int_0^x e^{s^2 \big|_0^t} \cdot t e^{-t^2}\, dt \right] =$$
$$= e^{-x^2} \left[\int_0^x t\, dt \right] = \frac{x^2}{2} e^{-x^2}.$$

2)
$$\lim_{x \to 1} f(x) = \frac{0}{0}$$

Per il Teorema di Ruffini studiato in Algebra entrambi i polinomi sono divisibili per $(x-1)$.
Infatti
$$f(x) = \frac{\cancel{(x-1)}(x^2+x+2)}{\cancel{(x-1)}(x^2-1)} \quad \text{per } x \neq 1$$
$$= \frac{(x^2+x+2)}{(x+1)(x-1)} \xrightarrow[x \to 1^{\pm}]{} \frac{4}{2 \cdot 0^{\pm}} = \pm\infty.$$

Dunque $\nexists \lim_{x \to 1} f(x)$.

Altrimenti, applicando il Teo. di de L'Hôpital,

$$\lim_{x \to 1} f(x) \overset{H}{=} \lim_{x \to 1} \frac{3x^2+1}{3x^2-2x-1} =$$ ②

$$= \lim_{x \to 1} \frac{3x^2+1}{(3x+1)(x-1)} = \frac{4}{4 \cdot 0^{\pm}} = \pm\infty.$$

3) $D = \{x \neq \pm 1\}$. Né pari, né dispari.

$$f(x) = \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)} \qquad \forall x \in D$$

$$\lim_{x \to 1^{\pm}} f(x) = \frac{3}{2}$$ NO AS. VERT. (SINGOLARITÀ ELIMINABILE)

$$\lim_{x \to -1^{\pm}} f(x) = \frac{1}{0^{\pm}} = \pm\infty$$ AS. VERT. di equazione $x = -1$

$$f(x) = \frac{x^2+x+1}{x+1} = x + \frac{1}{x+1} \Rightarrow y = x$$ AS. OBLIQUO per $x \to \pm\infty$

4) Chiamati con X_i $(i = 1, \dots, 11)$ i dati sperimentali si ha

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MEDIA: $X_M = \frac{\sum_{i=1}^{11} X_i}{11} \cong 79{,}45$

$$\sigma = \sqrt{\frac{1}{11}\sum_{i=1}^{11}(x_i - x_M)^2} \cong 12,61.$$

5) Poniamo $Z = \frac{x-\mu}{\sigma} = \frac{x-10}{2}$

con $\mu = 10$; $\sigma^2 = 4$; $\sigma = 2$

$\Rightarrow$ $Z(8) = -1$; $Z(13) = \frac{3}{2} = 1,5$

$Z(9) = -\frac{1}{2} = -0,5$; $Z(10,7) = \frac{0,7}{2} = 0,35$.

$P\{x<8\} = P\{Z<-1\} = \Phi(-1) = 1 - \Phi(1) =$
$= 1 - 0,8413 = 0,1587.$

$P\{x>13\} = P\{Z>1,5\} = 1 - \Phi(1,5) =$
$1 - 0,9332 = 0,0668$

$P\{9<x<10,7\} = P\{-0,5<Z<0,35\} =$
$= \Phi(0,35) - \Phi(-0,5) =$
$= \Phi(0,35) - 1 + \Phi(0,5) =$
$= 0,6368 + 0,6915 - 1 = 0,3283$

5) AUTOVALORI:

$$\det\begin{pmatrix} 2-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 2 & 0 & 1-\lambda \end{pmatrix}$$ REGOLA DI LAPLACE LUNGO LA 2ª RIGA

$$= (1-\lambda)\left[(2-\lambda)(1-\lambda)-2\right]$$

$$= (1-\lambda)\left[\lambda^2-3\lambda\right] = -\lambda(\lambda-1)(\lambda-3)$$

3 autovalori distinti $\Rightarrow$ la matrice è diagonalizzabile.

$\lambda_1=0$: $$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 2x+z=0 \\ y=0 \end{cases} \Rightarrow S_1 = \left\{\begin{pmatrix} x \\ 0 \\ -2x \end{pmatrix}, x\in\mathbb{R}\right\}$$

$\lambda_2=1$: $$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} x+z=0 \\ 2x=0 \end{cases} \Rightarrow S_2 = \left\{\begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}, y\in\mathbb{R}\right\}$$

$$\lambda_3 = 3 : \begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 0 \\ 2 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ 5

$$\Rightarrow \begin{cases} -x + z = 0 \\ -2y = 0 \end{cases} \Rightarrow S_3 = \left\{ \begin{pmatrix} x \\ 0 \\ x \end{pmatrix}, x \in \mathbb{R} \right\}$$

7) $\sqrt{x} = t \Rightarrow dt = \frac{1}{2\sqrt{x}} dx \Rightarrow dx = 2t\,dt$

$t(1) = 1 \; ; \; t(4) = 2$

$$\int_1^2 \frac{2\cancel{t}}{\cancel{t}(1+t)} dt = 2\ln(|1+t|) =$$
$$= 2\ln\left(\frac{3}{2}\right).$$