PROVA SCRITTA di ANALISI MAT. 1 del 25/3/2014

1) $\frac{x-2}{1-2x} > 0 \iff x \in \left(\frac{1}{2}, 2\right).$

$f(x) = 0 \iff \frac{x-2}{1-2x} = 1 \iff x-2 = 1-2x$

$\iff x = 1$

$f(x) > 0 \iff \frac{x-2}{1-2x} > 1 \iff x-2 < 1-2x$

$\iff \frac{1}{2} < x < 1$

$$\lim_{x \to 2^-} f(x) = \log\left(\frac{0^-}{-3}\right) = \log(0^+) = -\infty$$

$$\lim_{x \to \frac{1}{2}^+} f(x) = \log\left(\frac{-\frac{3}{2}}{0^-}\right) = \log(+\infty) = +\infty$$

AS. VERTICALE DX: $x = \frac{1}{2}$

AS. VERTICALE SX: $x = 2$.

$$f'(x) = \cancel{\frac{1}{x-2}} \left(\frac{1-2x}{x-2}\right) \cdot \left[\frac{1-2x+2(x-2)}{(1-2x)^2}\right]$$

$$= \frac{-3}{(x-2)(1-2x)} < 0 \qquad \forall x \in \left(\frac{1}{2}, 2\right).$$

f decrescente in $\left(\frac{1}{2}, 2\right)$.

②

$$f''(x) = \frac{3}{(x-2)^2(1-2x)^2}\left[-2x^2+5x-2\right]'$$

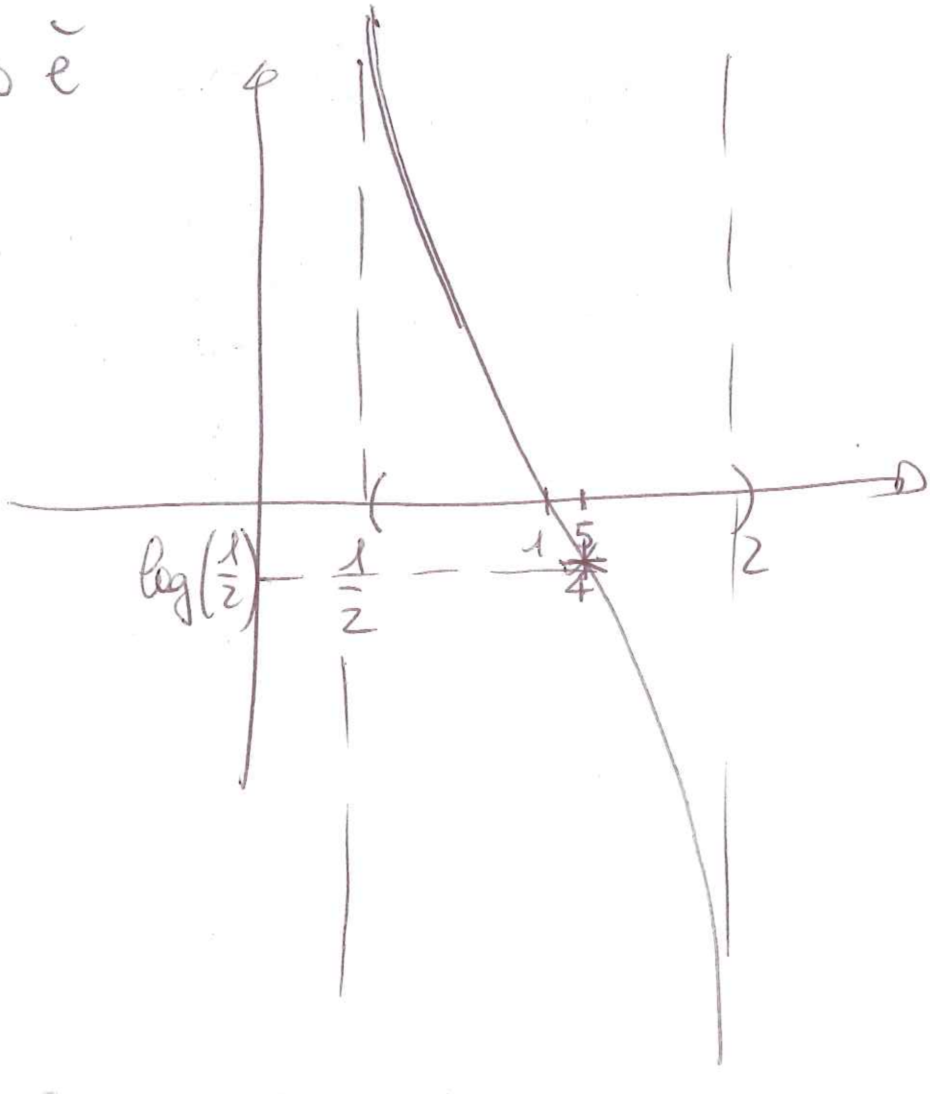
$$= \frac{3}{(x-2)^2(1-2x)^2}(-4x+5) > 0 \iff x < \frac{5}{4}$$

f convessa in $\left(\frac{1}{2}, \frac{5}{4}\right)$; concava in $\left(\frac{5}{4}, 2\right)$.

flesso discendente in $x_1 = \frac{5}{4}$

$$f\left(\frac{5}{4}\right) = \log\left(\frac{\frac{5}{4}-2}{1-\frac{5}{2}}\right) = \log\left(\frac{1}{2}\right) < 0$$

Il grafico è



2) $\int_{\cancel{2}}^{3}$ in $(\cancel{2}, 3)$ $\quad x-2>0 \; ; \; 2x-1>0$

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$$\Rightarrow \int_2^3 \log\left(\frac{x-2}{2x-1}\right) dx =$$

$$\int_2^3 \left[\log(x-2) - \log(2x-1)\right] dx =$$

$$= (x-2)\log(x-2)\Big|_2^3 - \int_2^3 \frac{(x-2)}{(x-2)}\, dx$$

$$-\frac{1}{2}(2x-1)\log(2x-1)\Big|_2^3 + \frac{1}{\cancel{2}}\int_2^3 \frac{(2x-1)}{(2x-1)}\cdot\cancel{2}\,dx$$

$$= -\lim_{x\to 2^+}(x-2)\log(x-2) \cancel{-1}$$

$$-\frac{1}{2}5\log 5 + \cancel{\frac{1}{2}\lim} + \frac{1}{2}3\log 3 \cancel{+1}$$

$$\left[\lim_{x\to 2^+}(x-2)\log(x-2) = \lim_{t\to 0^+} t\log t = 0\right]$$

$$= \cancel{\frac{1}{2}\log(27} \; \frac{1}{2}\left[3\log 3 - 5\log 5\right].$$

3) $z \neq 0$

$$z^2 + 2iz - 1 + i\bar{z} = 0$$

$$(x^2 - y^2 + 2ixy) + 2i(x + iy) - 1 + i(x - iy) = 0$$

$$\begin{cases} x^2 - y^2 - 2y - 1 + y = 0 \\ 2xy + 2x + x = 0 \end{cases}$$

$$\begin{cases} x(2y+3) = 0 \\ x^2 - y^2 - y - 1 = 0 \end{cases}$$

$$\begin{cases} x = 0 \\ y^2 + y + 1 = 0 \end{cases} \cup \begin{cases} y = -\frac{3}{2} \\ x^2 - \frac{9}{4} + \frac{3}{2} - 1 = 0 \end{cases}$$

Δ

$\Delta < 0$

IMPOSSIBILE

$$\begin{cases} y = -\frac{3}{2} \\ x^2 = \frac{9 - 6 + 4}{4} = \frac{7}{4} \end{cases}$$

$$\Rightarrow \quad z_1 = \frac{-\sqrt{7}}{2} - \frac{3}{2}i$$

$$z_2 = \frac{\sqrt{7}}{2} - \frac{3}{2}i$$

4)

$$a_n = \left\{ n\left[\frac{1}{n} - \frac{1}{6n^3} + o\left(\frac{1}{n^4}\right)\right]\left[1 - \frac{1}{2n^2} + o\left(\frac{1}{n^3}\right)\right] - \left[1 - \frac{1}{2n^2} + o\left(\frac{1}{n^2}\right)\right]\right\} n^2$$

$$= \left\{ n\left[\frac{1}{n} - \frac{1}{2n^3} - \frac{1}{6n^3} + o\left(\frac{1}{n^4}\right)\right] - 1 + \frac{1}{2n^2} + o\left(\frac{1}{n^2}\right)\right\} n^2$$

$$= \left\{ \cancel{1} - \cancel{\frac{1}{2n^2}} - \frac{1}{6n^2} + o\left(\frac{1}{n^3}\right) \cancel{- 1} + \cancel{\frac{1}{2n^2}} + o\left(\frac{1}{n^2}\right)\right\} n^2$$

$$\cancel{= \frac{1}{6n^2} + o\left(\frac{1}{n^2}\right)} \quad \sim -\frac{1}{6} \xrightarrow[n\to\infty]{} -\frac{1}{6}$$

5)

$$\lim_{x\to 3^-} \operatorname{arctg}\left(\frac{x+2}{x-3}\right) = \operatorname{arctg}\left(\frac{5}{0^-}\right) = \operatorname{arctg}(-\infty) = -\frac{\pi}{2}$$

$$\lim_{x\to 3^+} (\alpha x + \beta) = 3\alpha + \beta = f(3).$$

f è continua in $x = 3 \iff \boxed{\beta = -\frac{\pi}{2} - 3\alpha}$

$$f'(x) = \begin{cases} \dfrac{1}{1+\left(\dfrac{x+2}{x-3}\right)^2}\left[\dfrac{x-3-(x+2)}{(x-3)^2}\right] & x<3 \\ \alpha & x>3 \end{cases}$$

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$$= \begin{cases} \dfrac{\cancel{x}-5}{(x-3)^2+(x+2)^2} & x<3 \\ \alpha & x>3 \end{cases}$$

$$\lim_{x\to 3^-} f'(x) = -\frac{1}{5} = \lim_{x\to 3^+} f'(x) = \alpha$$

$$\Longleftrightarrow \alpha = -\frac{1}{5}$$

f è derivabile in $x=3 \Longleftrightarrow \begin{cases} \alpha = -\dfrac{1}{5} \\ \beta = -\dfrac{\pi}{2} + \dfrac{3}{5} \end{cases}$