SVOLGIMENTI AN. MAT. 1 del 28/10/15

1) $2\rho^4 = \rho^4 \left[\cos(4\vartheta) + i \sin(4\vartheta)\right]$

$$\begin{cases} 2\rho^4 = \rho^4 \cos(4\vartheta) \\ \rho^4 \sin(4\vartheta) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \rho = 0 \\ 0 = 0 \end{cases} \cup \begin{cases} \sin(4\vartheta) = 0 \quad (\rho \neq 0) \\ \underbrace{\rho^4}_{\neq 0} \underbrace{\left[2 - \cos(4\vartheta)\right]}_{\neq 0} = 0 \end{cases}$$

IMPOSSIBILE

$\Rightarrow \rho = 0 \Rightarrow z = 0.$

2)

$$1 - \cos\left(\frac{n+2}{n^2-4}\right) = 1 - \cos\left(\frac{1}{n-2}\right)$$

$$\sim \frac{1}{2(n-2)^2} \sim \frac{1}{2n^2}$$

$\sum \frac{1}{2n^2}$ converge $\Rightarrow \sum \left[1 - \cos\left(\frac{n+2}{n^2-4}\right)\right]$ converge.

3) Per $x \to +\infty$

$$f(x) \sim \frac{x}{x^4} = \frac{1}{x^3}$$

che è integrabile in $[a, +\infty)$.

$$\int_0^{+\infty} \frac{x}{(1+x^2)^2}\,dx \qquad x^2 = t \quad dt = 2x\,dx$$

$$t(0) = 0 \quad t(+\infty) = +\infty$$

$$= \frac{1}{2}\int_0^{+\infty} \frac{dt}{(1+t)^2} = -\frac{1}{2(1+t)}\Big|_0^{+\infty}$$

$$= \frac{1}{2}.$$

4) $I_{def} = \{x \neq \pm 2\} = (-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$.

$f(x) > 0 \quad \forall x \in I_{def}$

$$\lim_{x \to \pm\infty} f(x) = e^0 = 1$$

$$\lim_{x \to 2^\pm} e^{\frac{x+1}{(x+2)(x-2)}} = e^{\frac{3}{4 \cdot 0^\pm}} = e^{\pm\infty}$$

$$\Rightarrow \lim_{x \to 2^+} f(x) = +\infty \ ; \ \lim_{x \to 2^-} f(x) = 0^+$$

$$\lim_{x \to -2^{\pm}} f(x) = e^{\frac{-1}{-4\cdot 0^{\pm}}} = e^{\pm\infty}$$

$$\Rightarrow \lim_{x \to -2^{+}} f(x) = +\infty \quad ; \quad \lim_{x \to -2^{-}} f(x) = 0^{+}$$

$$f(0) = e^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{e}}.$$

$$f'(x) = e^{\frac{x+1}{x^2-4}} \left[\frac{x^2-4-2x(x+1)}{(x^2-4)^2} \right]$$

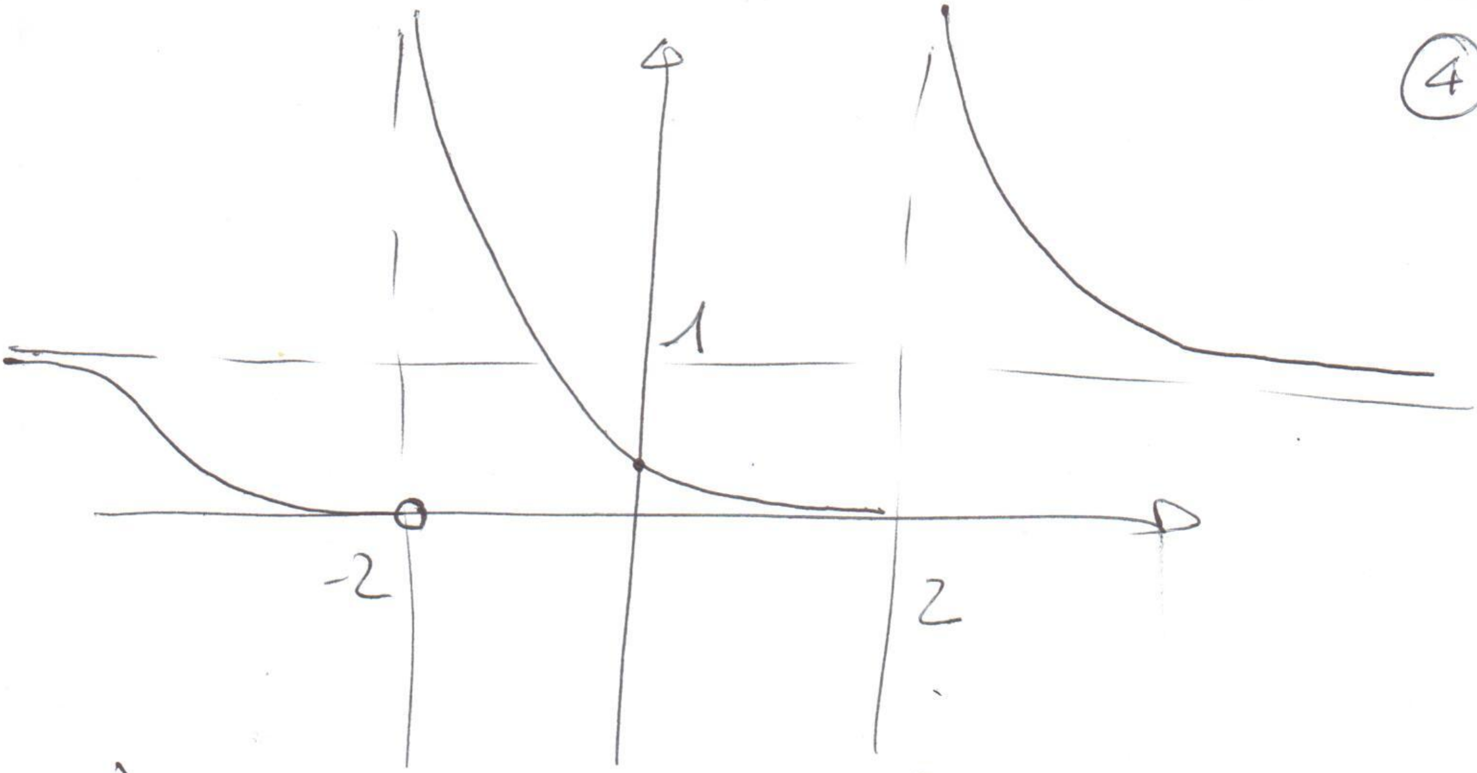
$$= e^{\frac{x+1}{x^2-4}} \left[\frac{-x^2-2x-4}{(x^2-4)^2} \right]$$

$$= -e^{\frac{x+1}{x^2-4}} \underbrace{\left[\frac{x^2+2x+4}{(x^2-4)^2} \right]}_{>0} < 0 \quad \forall x \in I_{def}$$

f decresce in $(-\infty, -2)$; in $(-2, 2)$, in $(2, +\infty)$.

$$\lim_{x \to -2^{-}} f'(x) = 0 \quad ; \quad \lim_{x \to 2^{-}} f'(x) = 0$$

Supponendo un numero minimo di flessi, si ha il grafico



5) omogenea associata: $\alpha^2 - 4\alpha + 4 = 0$

$(\alpha - 2)^2 = 0$

$\Rightarrow y_0(x) = C_1 e^{2x} + C_2 x e^{2x}$

$y_p(x) = A x^2 e^{2x}$; $y_p'(x) = A e^{2x}\left[2x + 2x^2\right]$

; $y_p''(x) = A e^{2x}\left[2 + 8x + 4x^2\right]$

$\Rightarrow \cancel{A e^{2x}}\left[\cancel{4x^2} + \cancel{8x} + 2 - \cancel{8x} - \cancel{8x^2} + \cancel{4x^2}\right] = \cancel{e^{2x}}$

$\Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2}$

$\Rightarrow y(x) = C_1 e^{2x} + C_2 x e^{2x} + \frac{1}{2} x^2 e^{2x}$

$$y(0) = C_1 = 0 \quad \Rightarrow C_1 = 0$$

$$y(x) = C_2 x e^{2x} + \frac{1}{2} x^2 e^{2x}$$

$$y'(x) = C_2 e^{2x} + C_2 2x e^{2x} + x e^{2x} + x^2 e^{2x}$$

$$y'(0) = C_2 = 0 \quad \Rightarrow C_2 = 0$$

$$\Rightarrow \quad \underline{y(x) = \frac{1}{2} x^2 e^{2x}}$$