SVOLGIMENTI PROVA SCRITTA DI ANALISI 1 DEL 16/2/2023

COMPITO A

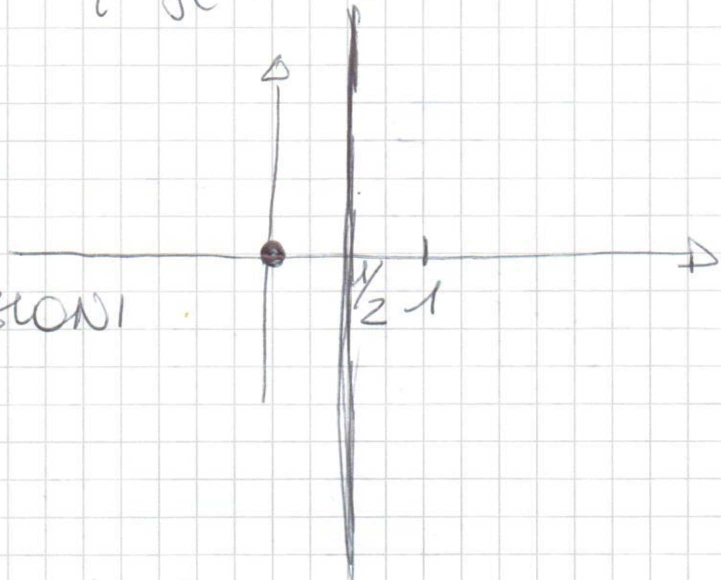
1) $(x-iy)^2+(x^2+y^2)-(x-iy)=0$

$$x^2-\cancel{y^2}-2ixy+x^2+\cancel{y^2}-x+iy=0$$

$$\begin{cases} 2x^2-x=0 \\ -2xy+y=0 \end{cases} \Rightarrow \begin{cases} x(2x-1)=0 \\ -y(2x-1)=0 \end{cases}$$

$$\begin{cases} x=0 \\ y=0 \end{cases} \cup \begin{cases} x=\frac{1}{2} \\ y\in\mathbb{R} \end{cases}$$

INFINITE SOLUZIONI

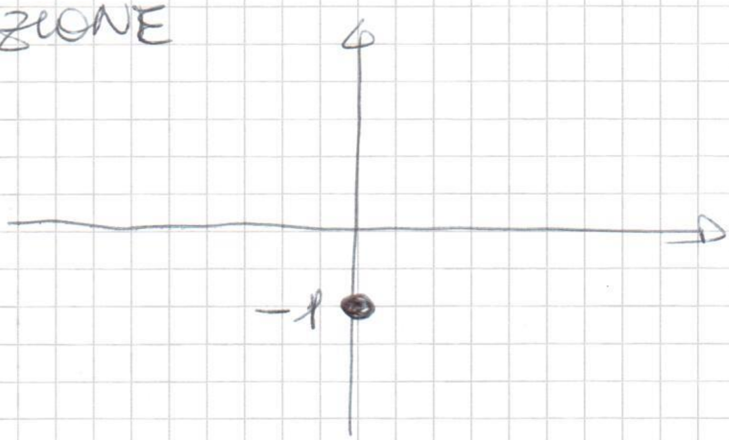


$(x-iy)^2+(x^2+y^2)-(x-iy)+i=0$

$$\begin{cases} 2x^2-x=0 \\ -2xy+y+1=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=-1 \end{cases} \cup \cancel{\begin{cases} x=\frac{1}{2} \\ 1=0 \end{cases}}$$

IMPOSSIBILE

UNA SOLUZIONE



2) Criterio del rapporto: 2

$$\frac{a_{n+1}}{a_n} = \frac{[2(n+1)]!}{(n+1)!\cdot(n+1)!}\cdot\frac{1}{2^{\cancel{n+1}}}\cdot\frac{n!\cdot n!}{(2n)!}\cdot\cancel{2^n}$$

$$= \frac{(2n+2)(2n+1)\cancel{(2n)!}}{(n+1)\cancel{n!}\,(n+1)\cancel{n!}}\cdot\frac{\cancel{n!\cdot n!}}{\cancel{(2n)!}}\cdot\frac{1}{2}$$

$$\underset{n\to\infty}{\sim}\frac{4n^2}{2n^2}\xrightarrow[n\to\infty]{} 2>1$$

la serie diverge.

3) $D=\mathbb{R}$

NON ABBIAMO SIMMETRIE

Segno: $e^{-x}>0 \quad \forall x\in\mathbb{R} \Rightarrow f(x)>0 \quad \forall x\in D$

NO intersezioni asse x

Asse y: $f(0)=\operatorname{arctg} 1=\frac{\pi}{4}$

$\lim_{x\to+\infty} f(x)=\operatorname{arctg}(0)=0$ AS. ORIZZ. a $+\infty$ $y=0$

$\lim_{x\to-\infty} f(x)=\operatorname{arctg}(+\infty)=\frac{\pi}{2}$ AS. ORIZZ. a $-\infty$ $y=\frac{\pi}{2}$

$f'(x)=\frac{1}{1+e^{-2x}}\left(-e^{-x}\right)<0 \quad \forall x\in\mathbb{R}$

f sempre decrescente.

NO MAX o MIN, REL. e ASS.

N.B.: l'immagine di f è $R_f = \left(0, \frac{\pi}{2}\right)$.

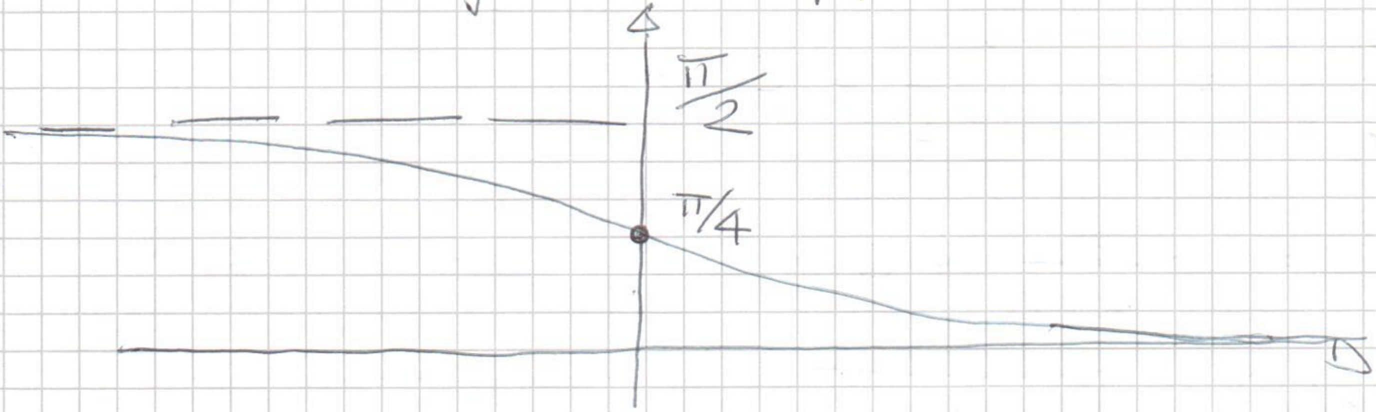
$$f''(x) = -\left[\frac{-e^{-x}\left(1+e^{-2x}\right) + e^{-x}\left(2e^{-2x}\right)}{\left(1+e^{-2x}\right)^2}\right]$$

$$= \frac{e^{-x}}{\left(1+e^{-2x}\right)^2}\left[e^{-2x} - 1\right] \geq 0$$

$$\iff e^{-2x} \leq 1 \quad \iff \quad x \geq 0$$

f concava in $(-\infty, 0)$ e convessa in $(0, +\infty)$.

In $x=0$ flesso obliquo ascendente.



4) $a(x) = \cot g\, x$ è continua $\forall x \neq k\pi$

$f(x) = e^{\cos x} \in C^0(\mathbb{R})$

Poiché $x_0 = \frac{\pi}{2}$, studiamo il problema in

$I = (0, \pi)$. $a, f \in C^0(I)$

$\Rightarrow \exists!$ sol. GLOBALE $y \in C^1(0, \pi)$.

$$y(x) = e^{-\int_{\pi/2}^{x} \operatorname{cotg} t \, dt} \left[\int_{\pi/2}^{x} e^{+\int_{\pi/2}^{t} \operatorname{cotg} s \, ds} \cdot e^{\cos t} \, dt + 1 \right]$$

$$= e^{-\ln|\sin t| \,\big|_{\pi/2}^{x}} \left[\int_{\pi/2}^{x} e^{\ln|\sin s| \,\big|_{\pi/2}^{t}} \cdot e^{\cos t} \, dt + 1 \right]$$

In $I = (0, \pi)$ $\quad \sin x > 0$

$$\Rightarrow \; = e^{-\ln(\sin t) \,\big|_{\pi/2}^{x}} \left[\int_{\pi/2}^{x} e^{\ln(\sin s) \,\big|_{\pi/2}^{t}} \cdot e^{\cos t} \, dt + 1 \right]$$

$$= e^{-\ln(\sin x) + \cancel{\ln 1}} \left[\int_{\pi/2}^{x} e^{\ln(\sin t) - \cancel{\ln 1}} \cdot e^{\cos t} \, dt + 1 \right]$$

$$= \frac{1}{\sin x} \left[\int_{\pi/2}^{x} e^{\cos t} \cdot \sin t \, dt + 1 \right]$$

$$= \frac{1}{\sin x} \left[1 - e^{\cos t} \Big|_{\pi/2}^{x} \right] = \frac{1}{\sin x} \left[1 - e^{\cos x} + 1 \right]$$

$$= \frac{1}{\sin x} \left(2 - e^{\cos x} \right)$$

5) $$f(x) \underset{x\to 0}{\sim} \frac{\left(\cancel{1}+\cancel{x}+\cancel{\frac{x^2}{2}}+\cancel{\frac{x^3}{6}}+\frac{x^4}{24}\right)-\cancel{\frac{x^3}{6}}-\cancel{\frac{x^2}{2}}-\cancel{x}-\cancel{1}}{\left(\cancel{1}-\cancel{\frac{x^2}{2}}+\frac{x^4}{24}\right)+\cancel{\frac{x^2}{2}}-\cancel{1}}$$

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$$= \underline{1}$$

La funzione ha pertanto in $x=0$ una singolarità eliminabile ed è dunque integrabile in $(0,1]$.