## THE ASSESSMENT OF GEOMETRICAL EFFECTS ON COMPTON PROFILE MEASUREMENTS

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Energy-dispersive measurements of Compton-scattered X-rays for analyses have become important with the development o high-efficiency, high-energy-resolution detectors for 50-200 keV X-rays. The Compton-scattered X-ray peak is an inherently broad peak resulting in part from the momentum distribution of the target electrons, referred to as the Compton profile. Hence information from the width of the Compton peak can be used to study the momentum distribution. The actual peak width also depends on geometrical and attenuation effects. We have begun to assess the geometrical effects and present some of the results

#### 1. The assessment

One of the basic interactions between X-rays and electrons is Compton scattering. This scattering is incoherent, resulting in a phase change of the X-ray along with a transfer of energy. When the target electron is free and stationary, the energy of a Compton-scattered X-ray can be given by

$$\left(\frac{E}{E_0}\right) = \frac{1}{1 + \gamma (1 - \cos \theta)},\tag{1}$$

where E is the energy of the scattered X-ray,  $E_0$  the energy of the incident X-ray,  $\gamma = E_0/m_0c^2$ , and  $\theta$  is the scattering angle. The scattering of X-rays by bound electrons is greatly complicated by the Compton profile of the bound electrons. The Compton profile is the distribution of electron momenta relative to the momentum of the incident X-ray. The scattering then results in a distribution of scattered-X-ray energies with respect to the most probable energy, E, of eq. (1).

Eq. (1) illustrates that there is an angular dependence on the energy of the scattered X-rays. Because of this angular dependence, we expect a geometry contribution to the spread in the energy of the observed scattered X-rays, resulting from the solid angle subtended by the detector. One must be aware of this "geometrical spread" so that it is not attributed to the Compton profile being measured. The contribution from the geometrical spread ( $\Delta E_{geom}$ ) resulting from the detector accepting  $\Delta \theta$  about  $\theta$  can be determined with the equation

$$\Delta E_{\text{geom}} = \frac{\mathrm{d}E}{\mathrm{d}\theta} \Delta \theta. \tag{2}$$

In order to evaluate eq. (2) we must either be able to describe the sample in  $\theta$ -space or describe  $\theta$  in an "easy" geometry. An appropriate geometry was pre-

0168-9002/89/\$03.50 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division) sented in ref. [1] and is reproduced in fig. 1. In this geometry the axis of symmetry is the line between the source and the detector. We can then relate the angle of scattering,  $\theta$ , to r and z through the relationship

$$\tan \theta = \frac{\frac{r}{b}}{\frac{z}{b}\left(1 - \frac{z}{b}\right) - \left(\frac{r}{b}\right)^2}.$$
(3)

As was illustrated in ref. [1] we can generate contours of constant scattering angle in the two dimensions of iand z. This is of interest in this work since there is a 1:1 correlation, through eq. (1), between the contours of constant scattering angle and those of constant scattering energy. Drawing the sample in relation tc contours of constant scattering energy is useful since it helps to determine the number of contour lines subtended by the sample and hence  $\Delta E$ .

The *R*- and *Z*-axes are Cartesian and hence we can easily describe two of the three dimensions of the sample in r and z. This approach is quite useful since now we can describe  $\theta$  in terms of r and z and can describe



Fig. 1. The geometry illustrating the source-detector line as the axis of symmetry.

**II. SOURCES AND DETECTORS** 



Fig. 2. A contour plot of  $\Delta E/E$  for 60 keV X-rays as a function of r/b and z/b. This plot was generated assuming  $\Delta r/b = 0.023$  and  $\Delta z/b = 0.03$ .

the dimensions of the sample in terms of  $\Delta r$  and  $\Delta z$  instead of  $\Delta \theta$ .  $\Delta E$  is then calculated by differentiating E with respect to r and z.

The third dimension is dealt with by recognizing that we have azimuthal symmetry about the source-detector axis [1,2]. Therefore the three-dimensional surfaces of constant scattering angle are toroids. The third dimension is dealt with by projecting along the toroidal surface to the R-Z plane while maintaining the same scattering angle.

As an illustration, fig. 2 shows a contour plot of constant  $\Delta E/E$  for 60 keV X-rays as a function of r/b and z/b. This plot was generated assuming  $\Delta r/b = 0.023$  and  $\Delta z/b = 0.03$ . These values were selected since



Fig. 3. A contour plot of isoefficiency as a function of r/b and z/b for a source area, normalized to  $b^2$ , of 0.01 and a detector area, normalized to  $b^2$ , of 0.023.

they are related to measurements on plexiglass that are being carried out and will be reported separately. These calculations are for plexiglass with a thickness of 1.6 mm. These calculations are strictly from geometrical considerations and great care must be taken in interpreting their meaning since they are totally independent of the attenuation of the X-ray beam. If the geometry of the measurement is such that z = 0 and the sample is mounted normally to the incident X-ray beam, then  $\Delta r/b$  is the sample thickness normalized to b.  $\Delta z/b$  is determined by the source and detector aperture sizes.

In order to determine the most appropriate geometry, we must balance the geometrical spread with the efficiency of the system. If  $A_s$  and  $A_D$  are the areas of the source and detector apertures respectively, we can write the efficiency as

$$\epsilon = \left(\frac{A_{\rm S}}{r^2 + z^2}\right) \left[\frac{A_{\rm D}}{r^2 + (b - z)^2}\right].$$
 (4)

Fig. 3 shows a contour plot of isoefficiency as a function of r/b and z/b for a source area, normalized to  $b^2$ , of 0.01 and a detector area, normalized to  $b^2$ , of 0.023.

## 2. Conclusions

We have been working on methods of assessing the contributions to the already naturally broad Compton scatter peak resulting from geometrical effects. This work will be useful in Compton profile measurements so that the broadening of the scatter peak due to geometrical effects will not be included in the measurement of the actual Compton profile. This work should also prove useful X-ray analytical techniques where the Compton-scattered X-rays are used as part of the analysis. In analytical techniques it is important to understand the Compton peak in order to accurately measure the peak.

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