Bridging the gap between high school and university mathematics

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Abstract In this paper we explain the structure of a CD aimed at filling the gap between high school mathematics and university mathematics and discuss the information technologies we have employed to make it and the reasons of our choices.

1. Introduction

In [Rogora, 2001A] and [Rogora, 2001B] it was presented a system for archiving electronic teaching resources in a coherent format (multiple choice guizzes, exercises, notes, text of exams, etc.) and some tools for updating and reusing. A better system has now been developed, based on a wider standard, IMS Question and Test Interoperability specifications (http://www.imsglobal.org). A rich database of items conformant to that standard has been created under the joint efforts of the project Campus ONE (http://151.100.50.238/orientamento/) at the University "La Sapienza" of Roma and the project *Didattica in rete* (http://didnet.mat.uniroma1.it/) at the Department of Mathematics of the same University. Many of the items in the database have been prepared by a pool of mathematics teachers within the project of the Italian Research Council La costruzione del sapere matematico (building mathematical knowledge) aimed at helping students to bridge the gap between high school and university mathematics. One of the goals of this last project was to prepare a CD to be distributed at high school students and teachers containing materials for self-study and class activities. Instead of working directly for the realization of this CD, a more ambitious tour has been taken. The material has been archived in a Data Base according to IMS specifications, various abstract formats have been elaborated for presenting collection of items, finally an automated procedure have been prepared to fill the formats with the material inside the database. In this paper we report on the final outcome of the process. The advantage of this tour de force is a much easier collaborative maintenance and updating of the material and simple mechanisms to make use of it in different contexts. In few words, the production of a CD can be seen as a particular view of the material in the database and the advantage is that many other views are possible on the same material. The material has been organized in thematic paths based on open and multiple-choice questions with context sensitive comments to each answer. These paths are dynamically linked and initial tests help students to choose their personal path. Material is provided for personal study and for instructors. High school teachers and university teachers have shared their experiences in order to provide problems, solutions and comments.

The CD consists of a collection of linked interactive PDF documents.

A distinctive point of our project is that we try to use only GPL (or GPL-like) software and our tools for preparing and modifying the CD and all the software we produce are under GPL license (We are in the process of putting software and documentation on the web.).

The rest of the paper is organized as follows. In section 2 we discuss the problem for which the CD has been created. In section 3 we briefly present the structure of the CD from the user point of view. In sections 4 and 5 we describe the interaction between the user and the learning environment. In section 6 we digress on some of the technologies we have employed to prepare the CD and the reasons of our choices.

2. The problem

"The passage from secondary to tertiary mathematics education is determined by procedures varying considerably from one country to the other, and even within one country, from one institution to another. But whatever the context, this transition often presents major difficulties for an important part of those students who take mathematics courses at the tertiary level. This is true whether the students being considered are specialising in mathematics or are registered in a program for which mathematics is a service subject." [De Guzmán, 1998]

A round-table discussion on this important problem was held at ICM'98.

The nature of the difficulties is manifold:

- a) epistemological and cognitive
- b) sociological and cultural
- c) didactical

They are all discussed in [De Guzmán, 1998] where one can also find many references on this subject.

In Italy almost all the universities do not have any selective entrance examination.

The only selection faced by the students is "Esame di stato", the examination at the end of the secondary school. This examination is not selective (less 2% of students fail it) and doesn't help the students to choose the track that is more suitable for them since there is no an evaluation for each subject, but only a global evaluation.

The results of it is that in the universities only one student out of two is successful in the final exams of first year's courses, and only one student out of three graduates.

In [Accascina, 1998] are analysed the epistemological and cognitive difficulties in mathematics encountered by italian students at the beginning of tertiary level.

It steps out that students:

- a) do not know
- b) do not know that they do not know

c) do not know that their ignorance will influence their studies.

The final section of [De Guzmán, 1998] suggests 12 possible actions that might help alleviating these difficulties. The first three are:

1. Establish a better dialogue between secondary educators and tertiary educators

2. Provide students with orientations activities

3. Provide students with individualized help.

In order to help Italian students we have prepared a CD, partially following the above three suggestions under the project of the Italian Research Council: "La costruzione del sapere matematico" (building mathematical knowledge).

The authors are mathematicians teaching to students of secondary schools (final-year), university students (first-year) and preservice secondary school mathematics teachers.

The CD is based on *Syllabus* (<u>http://www.dm.unibo.it/umi/italiano/Didattica/syllabus.pdf</u>), an official document written by the Union of the Italian Mathematicians (U.M.I), which gives suggestions to the students concerning the mathematics knowledge, which is essential for the universities studies in scientific subjects.

3. The structure of the CD

The U.M.I Syllabus (see previous section) had diffusion among teachers but not among students. One reason could be that the students, not aware of their weakness, didn't realized the necessity to strengthen their knowledge in mathematics.

For these reasons our CD has been prepared in such a way that students have to face their knowledge. In fact the student is asked to solve a "general test" of 30 closed-items. After completing it the student knows the items that she-he did correctly and the ones she-he did wrongly. For all of them she-he can have comments to her-his answer.

At the end the student is given the suggestion to solve a "theme test" (10 items with closed answer) on those themes where she-he appears weak.

The themes are "Arithmetic", "Algebra", "2D and 3D Euclidean Geometry", "Trigonometry", "and Functions".

Each theme is subdivided in topics. For example "Arithmetic" is subdivided in the following topics "Integer numbers", "Real numbers", "Powers and logarithms".

After completing each theme test and reading the comments, the student is given the suggestion to follow the "self-instruction paths" concerning the topics where she-he appears weak.

Every self instruction (or learning) path is a succession of questions (most of them with open answer). Each question is followed by a commented solution.

At the end of all this work the student is asked to solve another "general test".

Most of the materials have been tested in various contexts and the statistics relative to the various items are also archived in the DB (see introduction). Their analysis will give valuable suggestions on which items include and which exclude in future versions of the CD.

We have put much effort to create a flexible tool, which can be used in different contexts and explored without any guide. A suggested path exists but we have tried to arrange things in such a way that is possible to explore the CD without any guidance. The single resource (quiz, exercise, etc.) with its hints, its solutions and its comments is self-contained and this for two reasons:

- to make the CD attractive also for scarcely motivated students and useful also for a minimal use
- to make the material reusable in different contexts.

4. Self Evaluation

In this and in the following paragraph we describe two typical interactions between a hypothetical user of our CD and the learning environment we have programmed (briefly "the system"). To describe the CD we use some standard UML nomenclature [Fowler, 2000]. The interaction described in this section is the *Multiple Choice Based Self Evaluation Scenario* with a single actor (a student) interacting with the system. It is used when a student takes either a "general test" or a "theme test" (see section 3). In the next section we describe the *Self Instruction Scenario* (actor; a single student) that is used when a student takes a self-instruction path.

The main steps in the Multiple Choice Based Self Evaluation Scenario are the following

1. Student opens the system and System recognizes the student

- 2. Student chooses a Test
- 3. Student takes a Test
- 4. System scores the Test
- 5. Student browses comments and suggestion to her-his answers and to the other possible answers
- 6. System gives suggestions for a Self Instruction Path.

We digress a little bit about each step.

- 1. When the student opens the CD Rom the system asks for her-his name and loads the information about the Tests and the Learning Paths she-he has already taken, and the corresponding scores.
- 2. The first screen contains compact information about the material contained in the CD Rom, suggests taking a general test and links to the index of the Tests (general tests about all topics of the program and specific tests about specific topics) and to the index of the Learning Paths. In both indexes the already taken tests are marked. When the student chooses a test, the system displays the *Main Test Page*. (In *Fig. 1* you can see the bottom part of it). In this screen there is a button and a small white window for each question of the test. The button opens the question (see *Fig.2*). In the white window appears the state of the question: yet to be answered, already answered, intended not to be answered. At any moment during the test the student can go back to the Main Test Page.

8.	16.
	Tempo Rimanente:
	Inizio Test Pausa Fine Test

Fig 1 Bottom part of the Main Test Page. You can see the buttons that links to question 8 and 16

The window at the bottom displays the time remaining to the end of the test. The buttons in the last line are used for beginning the test, pausing, ending the test.

3. In Figure 2 you can see a portion of a multiple choice quiz screen. The first line (in the figure) is reserved to general information about the quiz (for example, if the use of a calculator is allowed). In the next lines is contained the text of the question. Then there is the list of the answers with a sensible box for choosing. In the last line (of the figure) there is a box for choosing not to answer. In the bottom line (not visible in the figure) are links to: next question, previous question, Main Test Page.



Fig. 2 A multiple-choice quiz about the decimal representation of a fraction

- 3. When the student finishes the test, the system displays the obtained score and a short report.
- 4. From the short report page the student can go to the Solutions Main Page where one can find a button for each question: green if answered correctly, red if answered not correctly, grey if not answered. When you press the button the system displays a screen similar to *Fig 2* (see *Fig. 3*) with a colored box corresponding to the chosen answer.



Fig 3 The chosen answer was D and it was correct (the button is green)

Each box links to a comment in which the student can find either a complete solution if the answer is correct (see Fig 4),

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Domanda : Il numero \frac{3}{1000} si può an

Risposta Data :

3 \cdot 10^{-3}

RISPOSTA ESATTA

Infatti 3 \cdot 10^{-3} = \frac{3}{10^3} = \frac{3}{1000}.
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Fig 4 Comment to right answer is the complete solution of the exercise

or a context sensitive comment to each wrong answer (explanations of why the answer is not correct, remarks on why the student may have chosen the answer, suggestions for not repeating the same mistake) (see *Fig 5*).



Fig 5 Comment to a wrong answer tries to explain the reason of the mistake

5. From the Solutions Main Page the student can ask the system for help in planning his learning path. Depending on the score, the given answers and the other Tests already taken, the system suggests one or more learning paths.

5. Learning Paths

Learning paths are different from Tests. Most of the questions in a learning path are open questions and the system does not allow, for didactical reasons, to choose the order in which to answer the questions.

In this case it is used the *Self Instruction Scenario* (actor; a single student). The main steps are the following

- 1. Student opens the system and System recognizes the student
- 2. Student chooses a Self Instruction Path
- 3. Student reads and answers to a single question of the Path
- 4. Student reads the correct answer and its comments
- 5. Student goes to the next (or proceeding question).

We digress a little bit about each step.

1. This step is skipped if the student has already done a test (see above section).

2. When the student begins the test the first question is shown. In Figure 6 you can see a portion of an open question screen. In the first line there is a general information (in this case, for example, the use of a calculator is not allowed).

Then there is the text of the question. In the red line the student is asked to write down the solutions. In the bottom line (non visible in the figure) are links to: hint, answer, quit the path, next question, previous question.

Uso della calcolatrice: vietato **Domanda 5:** Scrivi sotto forma di numeri decimali le seguenti frazioni: $\frac{1}{100}; \frac{7}{5}; \frac{3}{25}; \frac{21}{70}$ $\frac{1}{3}; \frac{2}{3}; \frac{5}{6}$ Rispondi alla domanda per iscritto prima di vedere la soluzione.

Fig 6 An open question taken from a learning path about the decimal representation of a fraction

In Fig. 7 you can see the hint given to the question in Fig. 6 when the relative button is pressed.



Fig 7 Before going to the solution the student can get a hint from the system

4. The student can see the correct answer by pressing the relative button (Fig 8).

Risposta esatta :	$\frac{1}{100} = 0,01$; $\frac{7}{5} = 1,4$; $\frac{3}{25} = 0,12$; $\frac{21}{70} = 0,3$
	$\frac{1}{3} = 0, \overline{3}; \frac{2}{3} = 0, \overline{6}; \frac{5}{6} = 0, 8\overline{3}$

Fig 8 Solution to the open question. The solution is short. A detailed comment is provided later. The student is invited to think of the procedure to get the solution before reading the comment.

In the bottom line (not visible in the figure) are links to: comment to the answer (see *Fig 9*); go back to the question. The student can read the comments by pressing the relative button.

Commento

Dividendo con virgola il numeratore con il denominatore otteniamo i risultati. In alcuni di questi casi però conviene seguire una strada più veloce:

$$\frac{7}{5} = \frac{7 \cdot 2}{5 \cdot 2} = \frac{14}{10} = 1,4$$
$$\frac{3}{25} = \frac{3 \cdot 4}{25 \cdot 4} = \frac{12}{100} = 0,12$$
$$\frac{21}{70} = \frac{7 \cdot 3}{7 \cdot 10} = \frac{3}{10} = 0,3$$

Notiamo che, in ogni caso, abbiamo sempre ottenuto o numeri decimali limitati o numeri periodici. Sarà un caso?

Fig 9 Comments to the exercise. In this comment the procedure to get the answer is detailed, some comments are made and further questions are asked to the student.

6. Technologies

The structure of the materials contained in the CD conforms to IMS Question and Test Interoperability standards in order to promote archiving and reusing the materials in conformant DB's. The standards promote in particular the separation between content and presentation, which is crucial for allowing different presentations of the same content (e.g. with the same material we prepare also traditional tests (on paper) and on line quizzes).

The presentation of the material on the CD has been realized as a collection of PDF documents. The choice of PDF has many reasons, among which the need to:

- generate automatically the content of the CD from the archived material;
- get high quality rendering of formulas and graphics, both in print and on screen;
- have traditional navigation and visual tools like buttons, etc;
- have the possibility to interact with the user with a powerful scripting language;
- use only open standard and free software.

We want to stress that content and presentation has been completely separated and therefore the same materials can be used in different projects and presented in different ways.

To prepare our CD we have used only GPL like software (TeX, LaTeX, perl, etc.) and all software produced during the project (formats, scripts for the automation of the CD building, libraries for the scripting of documents) is distributed under GPL license

We have prepared four formats for the presentation of the material, each of which has a paper version and an on line version. The paper version is moreover divided in three typologies, only questions, questions and hints, questions, solutions and comments. The formats are used to present

- a single multiple choice quiz
- a single open question
- a collection of multiple choice quizzes
- a collection of multiple choice quizzes and open questions

These formats are used to convert an object IMS conformant into a PDF document by using PDF LaTeX

Each format, in its onscreen version generates an interactive document that uses the usual interface elements to interact with the user. In particular user have conditioned access to hints, comments and solutions and corrections.

Interactivity is based on an automatic management of a user profile that is dynamically updated during each session, and allows, for example, limited access to hints and the possibility to change the answer to some questions.

Interactivity management is based on Java Script.

The enclosure of *Java Script* in PDF documents is made by using the package *insdljs* of the bundle *Acrotex* (<u>http://www.ctan.org</u>), and by a new GPL library.

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