USING CABRI3D: FIRST IMPRESSIONS[1]

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Cabri3D is a very exciting new software for learning and teaching 3D geometry. However, since it represents 3 dimensional object on the screen of a computer, some care is needed for avoiding that serious misconceptions arise from its use

INTRODUCTION

Teaching 3D geometry is difficult. Some of the reasons of these difficulties are that:

i) Students do not have enough intuitive background;

ii) Students need to imagine objects and constructions only by reasoning since suitable models are not available and representing 3D objects with drawings is difficult (for them and their teachers).

Cabri3D is a dynamic-geometry software that can be used to help students and teachers to overcome some of these difficulties and making the learning of 3D geometry easier and more appealing. However the software is not as intuitive as a 2D dynamic-geometry software and can be misleading. The main problem is that while a 2D software can be used to *construct* the objects of two dimensional geometry, Cabri3D can only be used to *represent* the objects of three dimensional geometry on a two dimensional screen. One needs to face the same kind of difficulties for drawing three dimensional figures on paper. There is however a big advantage of Cabri3D with respect to paper drawings. It is possible to change dynamically the point of view on a 3D scene and to open different vistas by employing different projection methods in order to recover the complete information about a 3D object from its projections. However, the ability to reconstruct properties of a 3D objects by its projections needs a good deal of ingenuity. This paper is a report of some experiments we made with Cabri3D with a group of perspective high school teachers.

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EXPERIMENTS

Subjects Our experiments were made with eight perspective high school teachers who already got a degree in mathematics and were attending the second year of a two years course of postgraduate school in secondary teaching (SSIS).

All students already got acquainted with CabriII the year before (they are part of one of the groups analyzed in [AMR05]).

Before making the experiments with Cabri3D we tested our students with questions about 3D geometry taken from multiple choice tests used for helping high school students in the choice of university studies ([AMBO], [AMR]). Moreover we asked them to write the definition of perpendicularity of two lines, of two planes and of a line and a plane.

We discussed the results of the test with the students themselves and it turned out that, as we expected, our students' ideas about the notion of perpendicularity of planes and lines were quite confused, their ability to imagine 3D objects was weak and their awareness of the reasons behind the definitions of 3D geometry was low. We decided to plan some discussions in order to clarify these points. We discussed, among other things, how to define perpendicularity between planes, lines, planes and lines and proved some basic facts before beginning the experiments with Cabri3D. We wanted to be sure that students would have not begun the experiment without having had the opportunity to clarify their ideas about the mathematical prerequisites needed for the experiments.

Organization We run the experiment in two distinct meetings of three hours each in a computer lab. Each student had access to a PC and we used of a projector linked to a PC. In the first meeting each student worked by him/herself. In the second meeting we grouped the students into two groups. The activities were described in labsheets, which are available on request. Each student received also a plastic model of a cube since we were interested to see if and for which questions students would have preferred to use models instead of Cabri3D.

Data collection Students were asked to provide written answers to all questions in the labsheets. We planned time for discussion at the end of the first meeting and during the second meeting. Both authors took notes of the discussion during the experiments.

Activities The topics of the activities were chosen with two main goals: to help students to deal again with the mathematical difficulties which emerged during the test and to investigate the following questions about Cabri3D.

i) Do students like using Cabri3D to explore 3D geometry? Is the software easy and intuitive? Are students confused by using Cabri3D? What kind of problems do they find? What misconceptions can arise from the use of Cabri3D?

ii) Do students use Cabri3D for investigating combinatorial, topological and affine properties of 3D objects, or do they prefer to manipulate models? Is Cabri3D useful for understanding combinatorial, topological and affine properties of a construction?

iii) Do students use Cabri3D for investigating Euclidean properties of 3D objects, or do they prefer to manipulate models? Is Cabri3D useful for understanding Euclidean properties of a construction?

First Meeting activities

An introductory experiment It is described in [AMR05].

Introduction to Cabri3D Students quickly learned the basic usage of the program and were able to perform easily the simple tasks required in their first labsheet. We put care in preparing a labsheet in which potentially confusing phenomena would not occur. Then, we showed a square based pyramid inside a cube and asked the students to decompose the cube into three pyramids equivalent to the given one. We already discussed how to do this in classroom during the discussion of their tests and students found very hard to visualize the construction. The activity in the lab was completed quickly and easily by all of them using Cabri3D. No one used the plastic model.

Plane sections of the cube We planned this activity for checking if students still had problems with lines contained in perpendicular planes and if Cabri3D helped them to grasp these ideas. The results of this activity will be discussed thoroughly in the next session.

Second Meeting activities

We planned the activities for the second meeting, which was a week later, after having read the students'answers on the labsheet of the first meeting.

More on plane sections of the cube We investigated more thoroughly if Cabri3D helped students to deal with perpendicularity or if Cabri3D confused them.

Placing points in space We believe that this is a potentially confusing point and wanted to test our claim.

EXPERIMENT ON THE PLANE SECTIONS OF THE CUBE

The study of properties of plane sections of a cube is a classical elementary problem in 3D geometry, which has received attention both from teachers, as an intriguing elementary example, and from researchers in the didactics of mathematics (see [Ba], [Er]). We prepared a Cabri3D file in order to make students capable to explore plane sections of a cube (see Figure 1). The interaction with the construction in the Figure is through three controls. The first, provided by Cabri3D, allows one to change the point of view on the figure. The second depends on the position of the point V which is bounded to move on a sphere of center O and controls the orthogonal direction to the section plane. The third depends on the position of the point *P* of the section plane, bounded to move on the line *r* joining the centre of the sphere to *V*. By choosing *V* and *P* it is possible to choose any plane. The interaction with the figure has been considered very intuitive by the students. It took them some practice however to be able to position planes a satisfactory way (for example it is not easy to place exactly a plane through the mid points of three sides). However we expected these difficulties and were interested to observe how students were able to cope with them.

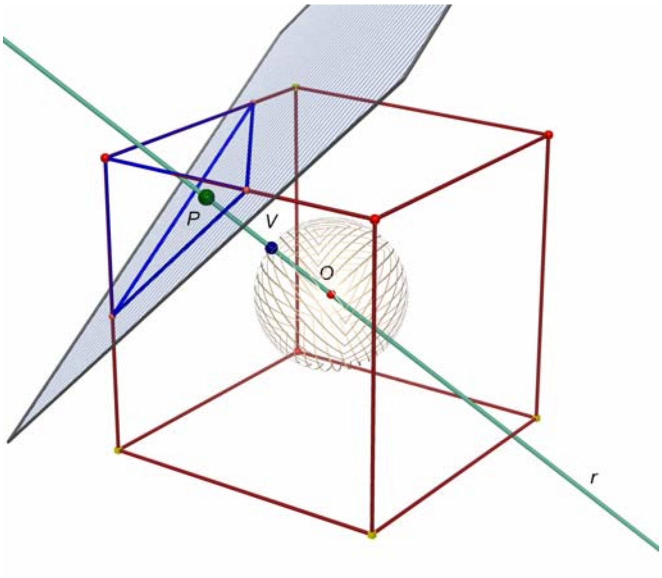

Figure 1 *The plane section is controlled using the point V on the sphere and the point P on the line r*

First Meeting During the first meeting we asked the following questions.

What kind of remarks can you make about the polygons which can be obtained as plane sections of a cube? *Goals:* Which properties can easily be detected with Cabri3D? Do students prefer to use Cabri3D or a model to answer this question? *Analysis:* All students decided to work only with Cabri3D and not with the plastic model of the cube we provided them with. All of them found polygons of 3, 4, 5 and 6 sides. Most of the remarks were about different types of quadrilaterals: they saw square, rectangles, trapezoids but only one of them saw parallelograms. The only remark about triangles was about the existence of equilateral triangular sections. No remark was made about pentagons except raising the question of existence of regular pentagonal sections (one student). Two students made the remark that regular hexagonal sections exist. No one claimed to have seen a right triangle, which cannot occur as a plane section of the cube but it is often claimed to be seen when one of the vertices of a triangular section (like the one in Figure 1) approaches the vertex of the cube (see [Ba]). Two of the students made some remarks about the topological properties of the function *n(V,P)* which counts the number of sides of the section with the plane corresponding to the position *P* and *V* of the controls. They asked themselves how this number varies when the sections are obtained by fixing V and moving P, i.e. in a pencil of parallel sections, and what kind of symmetries it has.

Can a heptagon be the plane section of a cube? Can a plane section have more than seven sides? *Goals:* Which argument, if any, is given to answer this question? Do students need to use Cabri3D? *Analysis*: All students said that no polygons with more than 6 sides can be cut by a plane on a cube. Four students gave a proof of this (the number of sides is bounded by the number of faces of the cube).

Can you get a square? And a rectangle which is not a square? *Goals:* Is Cabri3D useful to answer this question or do students need a model? *Analysis:* All students saw square sections. All but one saw rectangular sections. Two students didn't use Cabri3D. Those who used Cabri3D were not confused, i.e. they correctly detected right angles in the plane section. No one used the plastic model.

Can you get a rectangle whose side ratio is $\sqrt{2}$ ***? And 2?*** *Goals:* Is Cabri3D useful for answering this question? How? *Analysis*: All students (also the one who did not see rectangles before) found how to cut rectangles with different side ratio. Four of them claimed to have been helped by the use of Cabri3D. The others, either did not use Cabri3D (1) or did not find it useful for this question. Those who used it noticed that once they got a rectangular section the section remained rectangular if the plane moved in the pencil of parallel planes. This shows that the dynamic geometry features offered by Cabri3D may help to suggest the correct answer to some questions of Euclidean nature by continuity. Only one student used the plastic model.

Can you get a quadrilateral which is not a parallelogram? *Goals:* Is Cabri3D useful for answering this affine question? *Analysis:* All students worked only with Cabri3D and found trapezoids.

Can you get a parallelogram which is not a rectangle*? Goals:* Is Cabri3D useful for answering this Euclidean question? Do misconceptions about lines in perpendicular planes persist when using Cabri3D or can it help to get rid of them? *Analysis:* This is the crucial question. The existence of parallelograms which are not rectangles cannot easily be detected with Cabri3D. We were interested to see if students would have used Cabri3D or if they would have begun to think without using it. Only one of the students claimed the existence of a rectangular section and she was the only one who found Cabri3D useful to answer this question, but she was unsure.

Second Meeting During the second meeting we decided to group students into two groups and ask them again if there are sections of the cube which are parallelograms and not rectangles. We wanted to follow the discussion within and among the groups and see if and Cabri3D would have helped them to get a convincing answer to the problem. The students of the first group begun to use Cabri3D very soon, trying to convince themselves that all the parallelogram sections were rectangles. They were not able to disprove this statement by using Cabri3D. In fact they were not able to decide if the angles they saw on the screen were not right because the angles in the 3D scene were not right or because of changes due to projection onto the computer screen When they tried to convince themselves that the sections were actually rectangles they made a basic mistake by assuming that a plane intersects two perpendicular planes in two perpendicular lines. This was quite amazing since a couple of week before we discussed at length about orthogonal planes and lines in orthogonal planes. It seemed that Cabri3D was not effective to help them recover from their basic misunderstanding. Moreover, when we explicitly reminded them that the plane sections of two perpendicular planes needs not to be perpendicular, one student could not convince herself just by looking at the figures produced by Cabri3D and changing the point of view. We were able to convince her only when we asked her to compute the sum of the internal angles of a triangular section of the cube in the hypothesis that every plane cuts two perpendicular planes in a right angle. No one of them felt the necessity to pick a model. This suggests that Cabri3D should not be used in a classroom in the same way as suggested in [Si] for a 2D dynamic-geometry software:

The dynamic-geometry supported classroom offers a challenge regarding the creation of the contexts mentioned by Papert. Students in such classes may spend much of their class time interacting with a computer program rather than communicating with a teacher.

We believe that students using Cabri3D should communicate frequently with their teachers in order to avoid the arising of basic misconceptions.

In the second group the situation was different. They did not make the mistake about sections of orthogonal planes and did not use Cabri3D for about 20 minutes during which they discussed until they got a clear idea of what they wanted to see, i.e. the most distorted (non rectangular) parallelogram they could conceive as a plane section of the cube. One of them understood how to find such plane sections by looking at a plastic model of the cube and then tried to reproduce it with Cabri3D. She was not convinced that the parallelogram was not a rectangle by looking at the model but she believed that reproducing the section with Cabri3D would have convinced her. Once she got the section with Cabri3D, she moved the point of view perpendicularly to the cutting plane and then she convinced herself and the other elements of her group that the section was not a rectangle. The section was obtained by taking two opposite vertices A and B of the cube and the midpoint C of any of the 6 edges adjacent to neither A nor B.

It seems therefore that Cabri3D may help to investigate also some Euclidean property of 3D objects but one needs to be able to get rid of basic 3D misconceptions before using it effectively.

EXPERIMENT ON THE PROBLEM OF PLACING POINTS ON 3D SPACE

We got further evidence to the fact that basic geometric properties can be hard to see with Cabri3D in our last labsheet. We made the following experiment. We asked them to open a blank file (not the default file but one with no elements), draw a tetrahedron and describe its properties. They made the following construction (see the first panel in Figure 2). They drew four points and then connected all pairs with segments. We did not explain that each point belongs by default to a fixed plane (z=0). The students were well aware that in order to investigate the properties of a figure they had to change the point of view and that changing the point of view do not change the objects. By moving the point of view, each of them saw images like those in the panels of Figure 2 but none of them was able to conclude that the tetrahedron was degenerate (i.e. contained in a plane) even if this become absolutely evident when we told them it.

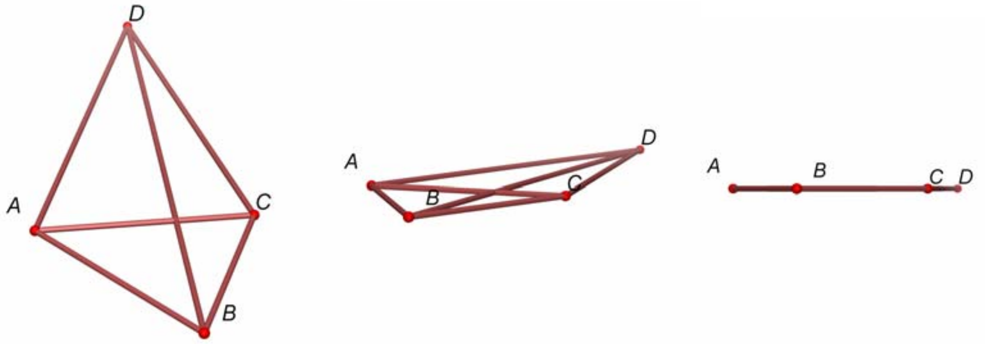

Figure 2 *The tethraedron built by the students under different point of view*

It is hard to believe that even by looking at images like those of the panels of Figure 2 they did not notice that the tethrahedron was contained in a plane, but this is what happened.

After telling them that the four points were on a plane we asked them to try to explain why the software behaved in such way. They seemed to be not aware of the fact that the interaction with a 3D figure through the two dimensional screen of a computer cannot be made by just clicking on the mouse because the information provided by a point on the screen (two Cartesian coordinates) it is not enough to identify a point in space (three Cartesian coordinates). We had to explain in detail that this is not due to bad software design but to the unavoidable fact that the map from space to the plane is not invertible.

CONCLUSIONS

We end this paper by answering some of the questions raised in Section 2.

Our students liked very much using Cabri3D and found it user friendly. Some of them said to have finally found something useful for exploring the geometry of space. Most of them did not feel any need to use 3D models when using Cabri3D. Even when they got confused, like when looking for parallelograms which were not rectangles they did not think they would gain a better understanding with a 3D model (at least the one we provided them with). However we have noticed that Cabri3D may be insidious since one can see things which are not true (perpendicularity of plane sections) and can not see quite evident facts (degeneracy of tetrahedra). Therefore we believe that great care should be put in the preparation of labsheets in order to introduce gradually problematic issues like point placement in space.

We think that Cabri3D is useful for understanding combinatorial topological and affine properties of a 3D construction, but with care. Also simple non metric properties like belonging or not to a plane may not be seen.

We think that Cabri3D may be useful also for understanding Euclidean properties of a 3D construction, but with great care. Euclidean properties cannot be easily detected with Cabri3D but some Euclidean property may be seen dynamically. For example the existence of rectangles whose side ratio is 2 was seen by some of our students by looking at the dynamical change of a rectangular section whose side ratio was $\sqrt{2}$ into a degenerate one whose side ratio was infinite. The search of Euclidean properties in a figure can become more misleading than the search of combinatorial, topological and affine properties. We have noticed for example that wrong ideas about lines in perpendicular planes are not always clarified by Cabri3D, and worse, that some students seemed to have made a step back in their understanding when using Cabri3D.

References

[AMBO] Accascina G., Rogora E. et al., *A.M.B.O. Argomenti di Matematica di Base per l'Orientamento*. CDRom, Università di Roma "La Sapienza", 2004 (available on request)

[AMR] Accascina G., Mastrogiovanni M. & Rogora E. Bridging the gap between high school mathematics and university mathematics, in Andronico, A., Frignani, P., Poletti, G. (eds) *Atti del convegno Didamatica 2004 Ferrara*, 525-533

[AMR05] Accascina G., Margiotta G. & Rogora E. *Making bad conjectures and uncomplete proofs with good drawings within a Dynamic Geometry Environment,* paper submitted to ICTMT 7, Bristol

[Ba] Bako M. Different projecting methods in teaching spatial geometry *Proceedings of CERME III, Thematic group 7*, 2003

[Er] Ehrenfeucht A. *The cube made interesting* New York, McMillan, 1964

[Si] Sinclair M. Some implications of the results of a case study for the design of pre-constructed, dynamic geometry sketches and accompanying materials *Educational Studies in Mathematics*, **52** 289-317, 2003