Formulario di Analisi Matematica I[1]

Università degli Studi "La Sapienza" di Roma

Ing. per l'Ambiente ed il Territorio - Ing. Civile - Ing. dei Trasporti

(Canale M - Z)

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Richiami di matematica elementare:

• Proprietà delle potenze ad esponente reale ($x, y \in \mathbb{R}^+$);

$$
\begin{aligned}
&1)\, x^0 = 1, \quad \forall\, x \in \mathbb{R} \setminus \{0\}; \quad 1^\alpha = 1, \quad \forall\, \alpha \in \mathbb{R}\\
&2)\, x^\alpha \cdot x^\beta = x^{\alpha+\beta}, \quad \forall\, \alpha, \beta \in \mathbb{R};\\
&3)\, x^\alpha \cdot y^\alpha = (xy)^\alpha, \quad \forall\, \alpha \in \mathbb{R};\\
&4)\, \frac{x^\alpha}{x^\beta} = x^{\alpha-\beta}, \quad \forall\, \alpha, \beta \in \mathbb{R};\\
&5)\, \frac{x^\alpha}{y^\alpha} = \left(\frac{x}{y}\right)^\alpha = \left(\frac{y}{x}\right)^{-\alpha}, \quad \forall\, \alpha \in \mathbb{R};\\
&6)\, (x^\alpha)^\beta = x^{\alpha\beta}, \quad \forall\, \alpha, \beta \in \mathbb{R};\\
&7)\, x^{\frac{1}{n}} = \sqrt[n]{x}, \quad \forall n \in \mathbb{N}, \quad \forall x \in \mathbb{R}_0^+;\\
&8)\, x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m, \quad \forall n, m \in \mathbb{N}, \quad \forall x \in \mathbb{R}_0^+;
\end{aligned}
$$

• Proprietà degli esponenziali ($a, b \in \mathbb{R}^+, \quad a, b \neq 1$);

$$
\begin{aligned}
&1)\, a^0 = 1; \quad a^1 = a;\\
&2)\, a^x > 0, \quad \forall\, x \in \mathbb{R}; \quad a^x \lesseqgtr 1 \text{ se } a \lesseqgtr 1, \quad \forall\, x \in \mathbb{R}^+;\\
&3)\, a^x \cdot a^y = a^{x+y}, \quad \forall\, x, y \in \mathbb{R};\\
&4)\, a^x \cdot b^x = (ab)^x, \quad \forall\, x \in \mathbb{R};\\
&5)\, \frac{a^x}{a^y} = a^{x-y}, \quad \forall\, x, y \in \mathbb{R};\\
&6)\, \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x, \quad \forall\, x \in \mathbb{R};\\
&7)\, a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x, \quad \forall\, x \in \mathbb{R};\\
&8)\, (a^x)^y = a^{xy}, \quad \forall\, x, y \in \mathbb{R};\\
&9)\, \text{se } x < y \Longrightarrow a^x \lesseqgtr a^y \text{ se } a \gtreqless 1;\\
&10)\, a \leq b \Longrightarrow a^x \leq b^x, \quad \forall\, x \in \mathbb{R}^+;
\end{aligned}
$$

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- Proprietà dei logaritmi $(x, y, a, b \in \mathbb{R}^+, \quad a, b \neq 1)$;

$$
\begin{aligned}
&1)\, a^{\log_a x} = x;\\
&2)\, \log_a (a^x) = x;\\
&3)\, \log_a 1 = 0;\\
&4)\, \log_a (xy) = \log_a x + \log_a y;\\
&5)\, \log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y;\\
&6)\, \log_a (x^\alpha) = \alpha \cdot \log_a x, \quad \forall\, \alpha \in \mathbb{R};\\
&7)\, \log_a x = \frac{1}{\log_x a} = -\log_{\frac{1}{a}} x, \quad x \neq 1;\\
&8)\, \log_b x = \frac{\log_a x}{\log_a b};
\end{aligned}
$$

- Proprietà del modulo o valore assoluto;

$$
\begin{aligned}
&1)\, |x| \geq 0, \quad \forall\, x \in \mathbb{R};\\
&2)\, |x| = 0 \Leftrightarrow x = 0;\\
&3)\, |-x| = |x|, \quad \forall\, x \in \mathbb{R};\\
&4)\, |x| = \sqrt{x^2}, \quad \forall\, x \in \mathbb{R};\\
&5)\, |x \cdot y| = |x| \cdot |y|, \quad \forall\, x, y \in \mathbb{R};\\
&6)\, |x/y| = |x|/|y|, \quad \forall\, x, y \in \mathbb{R},\, y \neq 0;\\
&7)\, |x + y| \leq |x| + |y|, \quad \forall\, x, y \in \mathbb{R};\\
&8)\, ||x| - |y|| \leq |x - y|, \quad \forall\, x, y \in \mathbb{R};
\end{aligned}
$$

- Somma di progressione aritmetica;

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2};$$

- Somma di progressione geometrica;

$$\sum_{k=0}^{n} q^k = \frac{1 - q^{n+1}}{1 - q}, \quad q \neq 1;$$

Numeri complessi:

• Forma algebrica: $z = x + iy, \forall x, y \in \mathbb{R};$ $\overline{z} := x - iy,$ $|z| := \sqrt{x^2 + y^2},$ $\forall z \in \mathbb{C};$

$$
\begin{aligned}
&1)\ \overline{(z \pm w)} = \overline{z} \pm \overline{w}, \quad \forall\, z, w \in \mathbb{C};\\
&2)\ \overline{(zw)} = \overline{z} \cdot \overline{w}, \quad \forall\, z, w \in \mathbb{C};\\
&3)\ \overline{(z/w)} = \overline{z}/\overline{w}, \quad \forall\, z, w \in \mathbb{C};\\
&4)\ z \cdot \overline{z} = |z|^2, \quad \forall z \in \mathbb{C};\\
&4)\ |z| \geq 0, \quad \forall\, z \in \mathbb{C};\\
&5)\ |z| = 0 \Leftrightarrow z = 0;\\
&6)\ |z| = |\overline{z}|, \quad \forall\, z \in \mathbb{C};\\
&4)\ |z \cdot w| = |z| \cdot |w|, \quad \forall\, z, w \in \mathbb{C};\\
&7)\ |z/w| = |z|/|w|, \quad \forall\, z, w \in \mathbb{C},\ w \neq 0;\\
&8)\ |\mathrm{Re}(z)| \leq |z|, \quad |\mathrm{Im}(z)| \leq |z|, \quad |z| \leq |\mathrm{Re}(z)| + |\mathrm{Im}(z)|, \quad \forall\, z \in \mathbb{C};\\
&9)\ |z + w| \leq |z| + |w|, \quad \forall\, z, w \in \mathbb{C};\\
&10)\ ||z| - |w|| \leq |z + w|, \quad \forall\, z, w \in \mathbb{C};
\end{aligned}
$$

• Forma trigonometrica: $z = \rho(\cos\theta + i\sin\theta),\ \rho \in \mathbb{R}^+,\ \theta \in [0, 2\pi),$

.

$$\text{dove } \rho := \sqrt{x^2 + y^2}\,, \quad \cos\theta := \frac{x}{\sqrt{x^2 + y^2}}\,, \quad \sin\theta := \frac{y}{\sqrt{x^2 + y^2}}\,.$$

. Se $w = \eta(\cos\phi + i\sin\phi),\ \eta \in \mathbb{R}^+,\ \phi \in [0, 2\pi)$ allora:

$$
\begin{aligned}
&1)\ z\,w = \rho\,\eta[\cos(\theta + \phi) + i\sin(\theta + \phi)];\\
&2)\ \frac{z}{w} = \frac{\rho}{\eta}\Big[\cos(\theta - \phi) + i\sin(\theta - \phi)\Big];\\
&3)\ z^n = \rho^n[\cos(n\,\theta) + i\sin(n\,\theta)], \quad \text{“Formula di Moivre”};\\
&4)\ \sqrt[n]{z} = \sqrt[n]{\rho}\Big[\cos\Big(\frac{\theta + 2k\pi}{n}\Big) + i \cdot \sin\Big(\frac{\theta + 2k\pi}{n}\Big)\Big], \quad k = 0, 1, 2, \cdots, (n-1);
\end{aligned}
$$

• Forma esponenziale: $z = \rho\, e^{i\theta},\ \rho \in \mathbb{R}^+,\ \theta \in [0, 2\pi),$

. Se $w = \eta\, e^{i\phi},\ \eta \in \mathbb{R}^+,\ \phi \in [0, 2\pi)$ allora:

$$
\begin{aligned}
&1)\ z\,w = \rho\,\eta\, e^{i(\theta+\phi)};\\
&2)\ \frac{z}{w} = \frac{\rho}{\eta}\, e^{i(\theta-\phi)};\\
&3)\ z^n = \rho^n\, e^{i(n\,\theta)};\\
&4)\ \sqrt[n]{z} = \sqrt[n]{\rho}\, e^{\frac{i\,(\theta+2k\pi)}{n}}, \quad k = 0, 1, 2, \cdots, (n-1);
\end{aligned}
$$

CAPITOLO 3

SUCCESSIONI E SERIE NUMERICHE

3.1. Successioni

• **Limiti notevoli**

1) $\left(1+\frac{1}{n}\right)^n \to \mathrm{e}$ per $n \to +\infty$;

2) $\left(1+\frac{1}{a_n}\right)^{a_n} \to \mathrm{e}$ $\quad \left(1+\frac{\vartheta}{a_n}\right)^{a_n} \to \mathrm{e}^{\vartheta}$ per $a_n \to +\infty$, $\forall \vartheta \in \mathbb{R}$;

3) $(1+\varepsilon_n)^{\frac{1}{\varepsilon_n}} \to \mathrm{e}$ per $\varepsilon_n \to 0$;

4) $\frac{\log(1+\varepsilon_n)}{\varepsilon_n} \to 1$ $\quad \frac{\log_a(1+\varepsilon_n)}{\varepsilon_n} \to \frac{1}{\log a}$ per $\varepsilon_n \to 0$, $\forall a > 0$, $a \neq 1$;

5) $\frac{a^{\varepsilon_n} - 1}{\varepsilon_n} \to \log a$ per $\varepsilon_n \to 0$, $\forall a > 0$;

6) $\frac{(1+\varepsilon_n)^{\vartheta} - 1}{\varepsilon_n} \to \vartheta$ per $\varepsilon_n \to 0$, $\forall \vartheta \in \mathbb{R}$;

7) $\frac{(a_n)^{\vartheta}}{\mathrm{e}^{a_n}} \to 0$ $\quad \frac{(a_n)^{\vartheta}}{a^{a_n}} \to 0$ per $a_n \to +\infty$, $\forall \vartheta \in \mathbb{R}$, $\forall a > 1$;

8) $\frac{(\log a_n)^{\vartheta}}{a_n} \to 0$ per $a_n \to +\infty$, $\forall \vartheta \in \mathbb{R}$;

9) $\sqrt[n]{n} \to 1$ per $n \to +\infty$;

10) $\frac{\sin \varepsilon_n}{\varepsilon_n} \to 1$ per $\varepsilon_n \to 0$;

11) $\frac{1 - \cos \varepsilon_n}{\varepsilon_n^2} \to \frac{1}{2}$ per $\varepsilon_n \to 0$;

12) $\frac{\tan \varepsilon_n}{\varepsilon_n} \to 1$ per $\varepsilon_n \to 0$;

13) $\frac{\arcsin \varepsilon_n}{\varepsilon_n} \to 1$ per $\varepsilon_n \to 0$;

14) $\frac{\arctan \varepsilon_n}{\varepsilon_n} \to 1$ per $\varepsilon_n \to 0$;

15) $\frac{\sinh \varepsilon_n}{\varepsilon_n} \to 1$ per $\varepsilon_n \to 0$;

16) $\frac{\tanh \varepsilon_n}{\varepsilon_n} \to 1$ per $\varepsilon_n \to 0$;

17) $\frac{\cosh \varepsilon_n - 1}{\varepsilon_n^2} \to \frac{1}{2}$ per $\varepsilon_n \to 0$;

18) $\varepsilon_n \Big| \log |\varepsilon_n| \Big|^{\beta} \to 0$ per $\varepsilon_n \to 0$, $\forall \beta \in \mathbb{R}$;

CAPITOLO 4

FUNZIONI DI UNA VARIABILE REALE

4.1. Limiti, continuità e derivabilità

Ricordiamo che, pur di sostituire ad n, a_n ed ε_n una variabile continua che abbia lo stesso limite, la tabella dei limiti notevoli, così come gli ordini di infinito ($n!$ escluso), richiamati nel precedente capitolo, continuano ad essere validi. Riportiamo qui, avendoli adattati al caso delle funzioni, i risultati precedentemente enunciati per le successioni.

• Limiti notevoli

1) $\left(1+\frac{1}{x}\right)^x \to \mathrm{e}$ per $x \to \pm\infty$;

2) $\left(1+\frac{\vartheta}{x}\right)^{\alpha x} \to \mathrm{e}^{\alpha\vartheta}$ per $x \to +\infty$, $\forall\vartheta \in \mathbb{R}$, $\forall\alpha \in \mathbb{R}$;

3) $(1+x)^{\frac{1}{x}} \to \mathrm{e}$ per $x \to 0$;

4) $\frac{\log(1+x)}{x} \to 1$ $\quad \frac{\log_a(1+x)}{x} \to \frac{1}{\log a}$ per $x \to 0$, $\forall a > 0$, $a \neq 1$;

5) $\frac{a^x - 1}{x} \to \log a$ per $x \to 0$, $\forall a > 0$;

6) $\frac{(1+x)^\vartheta - 1}{x} \to \vartheta$ per $x \to 0$, $\forall\vartheta \in \mathbb{R}$;

7) $\frac{x^\vartheta}{\mathrm{e}^x} \to 0$ $\quad \frac{x^\vartheta}{a^x} \to 0$ per $x \to +\infty$, $\forall\vartheta \in \mathbb{R}$, $\forall a > 1$;

8) $\frac{(\log x)^\vartheta}{x} \to 0$ per $x \to +\infty$, $\forall\vartheta \in \mathbb{R}$;

9) $x^{1/x} \to 1$ per $x \to +\infty$;

10) $\frac{\sin x}{x} \to 1$ per $x \to 0$;

11) $\frac{1-\cos x}{x^2} \to \frac{1}{2}$ per $x \to 0$;

12) $\frac{\tan x}{x} \to 1$ per $x \to 0$;

13) $\frac{\arcsin x}{x} \to 1$ per $x \to 0$;

14) $\frac{\arctan x}{x} \to 1$ per $x \to 0$;

15) $\frac{\sinh x}{x} \to 1$ per $x \to 0$;

16) $\frac{\tanh x}{x} \to 1$ per $x \to 0$;

A Formule utili

1. COSTANTI MATEMATICHE

e	$2.7182818285\ldots$	
π	$3.1415926536\ldots$	
$\log_{10} 2$	$0.3010299957\ldots$	
$\log_{10} e$	$0.4342944819\ldots$	
$\log_{10} \pi$	$0.4971498727\ldots$	
$\log_e 2$	$0.6931471806\ldots$	
$\log_e \pi$	$1.1447298858\ldots$	
$\log_e 10$	$2.3025850930\ldots$	
$\sqrt{2}$	$1.4142135624\ldots$	
$\sqrt{e}$	$1.6487212707\ldots$	
$\sqrt{3}$	$1.7320508076\ldots$	
$\sqrt{\pi}$	$1.7724538509\ldots$	
$\sqrt{5}$	$2.2360679775\ldots$	
$\sqrt{10}$	$3.1622776602\ldots$	
1°	$0.0174532925\ldots$	radianti
1 radiante	$57^\circ 17' 44''.806\ldots$	

2. FUNZIONI TRIGONOMETRICHE

$$\sin x \qquad \cos x \qquad \operatorname{tg} x = \frac{\sin x}{\cos x} \qquad \operatorname{cotg} x = \frac{\cos x}{\sin x}$$

$$(\sin x)^2 + (\cos x)^2 = 1$$

- *Angoli notevoli*

x	$\cos x$	$\sin x$	$\operatorname{tg} x$	$\operatorname{cotg} x$
0	1	0	0	$\pm\infty$
$\frac{\pi}{10} = 18°$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$
$\frac{\pi}{6} = 30°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{5} = 36°$	$\frac{\sqrt{5}+1}{4}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\sqrt{5-2\sqrt{5}}$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$
$\frac{\pi}{4} = 45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{3\pi}{10} = 54°$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$	$\sqrt{5-2\sqrt{5}}$
$\frac{\pi}{3} = 60°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{2\pi}{5} = 72°$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$
$\frac{\pi}{2} = 90°$	0	1	$\pm\infty$	0

- *Simmetrie, archi complementari e supplementari*

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \operatorname{tg}(-x) = -\operatorname{tg} x$$

$$\sin\left(\frac{\pi}{2} \pm x\right) = \mp\cos x \qquad \cos\left(\frac{\pi}{2} \pm x\right) = \mp\sin x \qquad \operatorname{tg}\left(\frac{\pi}{2} \pm x\right) = \mp\operatorname{cotg} x$$

$$\sin(\pi \pm x) = \pm\sin x \qquad \cos(\pi \pm x) = -\cos x \qquad \operatorname{tg}(\pi \pm x) = \pm\operatorname{tg} x$$

- *Formule di addizione*

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \operatorname{tg} y}$$

- *Formule di duplicazione*

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = (\cos x)^2 - (\sin x)^2 = 2(\cos x)^2 - 1 = 1 - 2(\sin x)^2$$

$$\operatorname{tg} 2x = \frac{2\operatorname{tg} x}{1-(\operatorname{tg} x)^2}$$

- *Formule di bisezione* (scegliere il segno corretto)

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\operatorname{tg}\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$$

- *Formule di prostaferesi*

$$\sin u + \sin v = 2\sin\frac{u+v}{2}\cos\frac{u-v}{2}$$

$$\sin u - \sin v = 2\cos\frac{u+v}{2}\sin\frac{u-v}{2}$$

$$\cos u + \cos v = 2\cos\frac{u+v}{2}\cos\frac{u-v}{2}$$

$$\cos u - \cos v = -2\sin\frac{u+v}{2}\sin\frac{u-v}{2}$$

- *Formule parametriche*

Posto $t = \operatorname{tg}(x/2)$:

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad \operatorname{tg} x = \frac{2t}{1-t^2}$$

- *Teorema di Carnot*

$$a^2 = b^2 + c^2 - 2bc\cdot\cos\alpha$$

3. FUNZIONI IPERBOLICHE

$$\operatorname{Sh} x \qquad \operatorname{Ch} x \qquad \operatorname{Th} x = \frac{\operatorname{Sh} x}{\operatorname{Ch} x}$$

$$(\operatorname{Ch} x)^2 - (\operatorname{Sh} x)^2 = 1$$

$$\operatorname{Sh}(-x) = -\operatorname{Sh} x \qquad \operatorname{Ch}(-x) = \operatorname{Ch}(x) \qquad \operatorname{Th}(-x) = -\operatorname{Th} x$$

- *Formule di addizione*

$$\operatorname{Sh}(x \pm y) = \operatorname{Sh} x \operatorname{Ch} y \pm \operatorname{Ch} x \operatorname{Sh} y$$

$$\operatorname{Ch}(x \pm y) = \operatorname{Ch} x \operatorname{Ch} y \pm \operatorname{Sh} x \operatorname{Sh} y$$

$$\operatorname{Th}(x \pm y) = \frac{\operatorname{Th} x \pm \operatorname{Th} y}{1 \pm \operatorname{Th} x \operatorname{Th} y}$$

- *Formule di duplicazione*

$$\operatorname{Sh} 2x = 2 \operatorname{Sh} x \operatorname{Ch} x$$

$$\operatorname{Ch} 2x = (\operatorname{Ch} x)^2 + (\operatorname{Sh} x)^2 = 2(\operatorname{Ch} x)^2 - 1 = 1 + 2(\operatorname{Sh} x)^2$$

$$\operatorname{Th} 2x = \frac{2 \operatorname{Th} x}{1 + (\operatorname{Th} x)^2}$$

- *Formule di bisezione* (scegliere il segno corretto)

$$\operatorname{Sh} \frac{x}{2} = \pm\sqrt{\frac{\operatorname{Ch} x - 1}{2}}$$

$$\operatorname{Ch} \frac{x}{2} = \sqrt{\frac{\operatorname{Ch} x + 1}{2}}$$

$$\operatorname{Th} \frac{x}{2} = \frac{\operatorname{Ch} x - 1}{\operatorname{Sh} x} = \frac{\operatorname{Sh} x}{\operatorname{Ch} x + 1}$$

- *Formule di prostaferesi*

$$\operatorname{Sh} u + \operatorname{Sh} v = 2 \operatorname{Sh} \frac{u+v}{2} \operatorname{Ch} \frac{u-v}{2}$$

$$\operatorname{Sh} u - \operatorname{Sh} v = 2 \operatorname{Ch} \frac{u+v}{2} \operatorname{Sh} \frac{u-v}{2}$$

$$\operatorname{Ch} u + \operatorname{Ch} v = 2 \operatorname{Ch} \frac{u+v}{2} \operatorname{Ch} \frac{u-v}{2}$$

$$\operatorname{Ch} u - \operatorname{Ch} v = 2 \operatorname{Sh} \frac{u+v}{2} \operatorname{Sh} \frac{u-v}{2}$$

- *Formule parametriche*

Posto $t = \operatorname{Th}(x/2)$:

$$\operatorname{Sh} x = \frac{2t}{1-t^2} \qquad \operatorname{Ch} x = \frac{1+t^2}{1-t^2} \qquad \operatorname{Th} x = \frac{2t}{1+t^2}$$

4. DERIVATE ELEMENTARI

$f(x)$	$f'(x)$
x^α	$\alpha x^{\alpha-1}$
$\|x\|$	$\operatorname{sgn} x$
$\log\|x\|$	$1/x$
$\log_a\|x\|$	$1/(x\log a)$
e^x	e^x
a^x	$a^x \log a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\operatorname{tg} x$	$1+(\operatorname{tg} x)^2 = 1/(\cos x)^2$
$\operatorname{ctg} x$	$-1-(\operatorname{ctg} x)^2 = -1/(\sin x)^2$
$\operatorname{Sh} x$	$\operatorname{Ch} x$
$\operatorname{Ch} x$	$\operatorname{Sh} x$
$\operatorname{Th} x$	$1-(\operatorname{Th} x)^2 = 1/(\operatorname{Ch} x)^2$
$\operatorname{Cth} x$	$1-(\operatorname{Cth} x)^2 = -1/(\operatorname{Sh} x)^2$
$\log\|\sin x\|$	$-\operatorname{ctg} x$
$\log\|\cos x\|$	$\operatorname{tg} x$
$\log\|\operatorname{Sh} x\|$	$\operatorname{Cth} x$
$\log \operatorname{Ch} x$	$\operatorname{Th} x$
$\arcsin x$	$1/\sqrt{1-x^2}$
$\arccos x$	$-1/\sqrt{1-x^2}$
$\operatorname{arctg} x$	$1/(1+x^2)$
$\operatorname{arccotg} x$	$-1/(1+x^2)$

5. REGOLE DI DERIVAZIONE

$$\mathrm{D}(\lambda f(x)+\mu g(x)) = \lambda f'(x)+\mu g'(x)$$

$$\mathrm{D}(f(x)g(x)) = f'(x)g(x)+f(x)g'(x)$$

$$\mathrm{D}\,\frac{f(x)}{g(x)} = \frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$$

$$\mathrm{D}\,f(g(x)) = f'(g(x))g'(x)$$

$$\mathrm{D}\,f(g(h(x))) = f'(g(h(x)))g'(h(x))h'(x)$$

$$\mathrm{D}\,e^{f(x)} = e^{f(x)}f'(x)$$

$$\mathrm{D}\log|f(x)| = \frac{f'(x)}{f(x)}$$

$$\mathrm{D}[f(x)]^{g(x)} = [f(x)]^{g(x)}\left\{g'(x)\log f(x) + \frac{g(x)f'(x)}{f(x)}\right\}$$

6. SVILUPPI DI MAC LAURIN DELLE PRINCIPALI FUNZIONI

$f(x)$	Sviluppo	
e^x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$	
$\mathrm{Sh}\, x$	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots$	
$\mathrm{Ch}\, x$	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$	
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots$	
$\cos x$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$	
$\log(1+x)$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots$	
$\mathrm{arctg}\, x$	$x - \frac{x^3}{3} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots$	
$(1+x)^\alpha$	$1 + \alpha x + \binom{\alpha}{2}x^2 + \cdots + \binom{\alpha}{n}x^n + \cdots$	$\alpha \in \mathbb{R}$

In particolare:

$\frac{1}{1+x}$	$1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots$	$(\alpha = -1)$
$\sqrt{1+x}$	$1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \cdots + \frac{(-1)^{n+1}(2n-3)!!x^n}{(2n)!!}$	$(\alpha = \frac{1}{2})$
$\frac{1}{\sqrt{1+x}}$	$1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots + \frac{(-1)^n(2n-1)!!x^n}{(2n)!!}$	$(\alpha = -\frac{1}{2})$

ove

$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)}{n!}$ (coefficiente binomiale generalizzato)

$k!! = k(k-2)(k-4)\cdots 2$ (k semifattoriale)

7. TABELLA DI PRIMITIVE

$f(x)$	$F(x)$	
x^α	$\dfrac{x^{\alpha+1}}{(\alpha+1)}$	se $\alpha \neq -1$
$\dfrac{1}{x}$	$\log\lvert x\rvert$	
e^x	e^x	
a^x	$\dfrac{a^x}{\log a}$	$a > 0, \quad a \neq 1$
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\operatorname{tg} x$	$\log\lvert\cos x\rvert$	
$\operatorname{cotg} x$	$-\log\lvert\sin x\rvert$	
$\operatorname{Sh} x$	$\operatorname{Ch} x$	
$\operatorname{Ch} x$	$\operatorname{Sh} x$	
$\operatorname{Th} x$	$\log \operatorname{Ch} x$	
$\operatorname{Cth} x$	$\log\lvert\operatorname{Sh} x\rvert$	
$\dfrac{1}{(\cos x)^2}$	$\operatorname{tg} x$	
$\dfrac{1}{(\sin x)^2}$	$-\operatorname{cotg} x$	
$\dfrac{1}{(\operatorname{Ch} x)^2}$	$\operatorname{Th} x$	
$\dfrac{1}{(\operatorname{Sh} x)^2}$	$-\operatorname{Cth} x$	
$\dfrac{1}{1+x^2}$	$\operatorname{arctg} x$	
$\dfrac{1}{1-x^2}$	$\dfrac{1}{2}\log\dfrac{1+x}{1-x}$	se $\lvert x\rvert < 1$
$\dfrac{1}{1-x^2}$	$\dfrac{1}{2}\log\dfrac{x+1}{x-1}$	se $\lvert x\rvert > 1$
$\dfrac{1}{\sqrt{1-x^2}}$	$\arcsin x$	
$\dfrac{1}{\sqrt{1+x^2}}$	$\log\lvert x+\sqrt{1+x^2}\rvert$	
$\dfrac{1}{\sqrt{x^2-1}}$	$\log\lvert x+\sqrt{x^2-1}\rvert$	se $\lvert x\rvert > 1$

$f(x)$	$F(x)$
$f^\alpha f'$	$\dfrac{f^{\alpha+1}}{(\alpha+1)}$ se $\alpha \neq -1$
$\dfrac{f'}{f}$	$\log\|f\|$
$\dfrac{f'}{1+f^2}$	$\operatorname{arctg} f$
$\dfrac{f'}{\sqrt{1-f^2}}$	$\arcsin f$

8. GEOMETRIA LINEARE NEL PIANO

- *Equazione della retta*

canonica: $y = mx + q \qquad (m = \operatorname{tg}\alpha)$

segmentaria: $\dfrac{x}{p} + \dfrac{y}{q} = 1$

generale: $ax + by + c = 0$

parametrica: $\begin{cases} x = x_0 + ht \\ y = y_0 + kt \end{cases} \quad t \in \mathbb{R}$

- *Retta per un punto $P_0(x_0, y_0)$ di coefficiente angolare m:*

$$y - y_0 = m(x - x_0)$$

- *Retta per due punti $P_1(x_1, y_1)$ e $P_2(x_2, y_2)$:*

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}, \quad x_1 \neq x_2,\ y_1 \neq y_2, \qquad \text{oppure} \qquad \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

- Date due rette r_1 e r_2: $\begin{cases} y = m_1 x + q_1 \\ y = m_2 + q_2 \end{cases}$ oppure $\begin{cases} a_1 x + b_1 y + c_1 = 0 \\ a_2 x + b_2 y + c_2 = 0 \end{cases}$

condizione di parallelismo: $m_1 = m_2$ oppure $a_1 b_2 - a_2 b_1 = 0$

condizione di perpendicolarità: $m_1 m_2 = -1$ oppure $a_1 a_2 + b_1 b_2 = 0$

angolo $\theta = \widehat{r_1 r_2}$: $\operatorname{tg}\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

- *Distanza di un punto $P_0(x_0, y_0)$ dalla retta: $ax + by + c = 0$:*

$$h = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$