# Resolution enhancement of time-frequency representations

**BACKGROUND** : Time-frequency (TF) and time-scale analysis are fundamental tools for the extraction of relevant information from a non stationary signal f(t). In applications involving strong oscillating multicomponent signals, for instance radar, audio, speech signals (chirp-like signals ), modelled as

$$\boxed{f(t) = \sum_{k=1}^{N} f_k(t) = \sum_{k=1}^{N} a_k(t) e^{i\phi_k(t)}}$$

one main objective is to estimate the instantaneous frequencies (IFs) $\boxed{\phi_k'(t), \quad k = 1, ..., N}$

and recover each single mode.

For this purpose, increasing readability of the TF representations is crucial for applications.

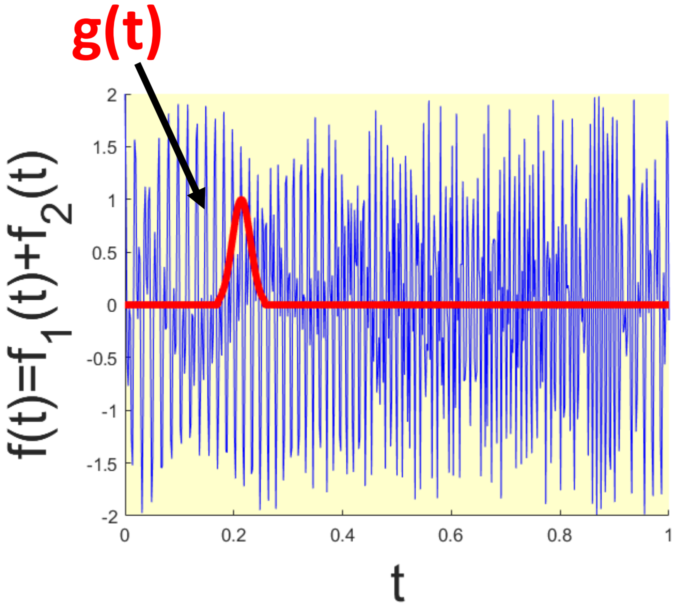
The existing approaches are essentially aimed at concentrating the TF distribution on the ridge curves, i.e. the curve where a single and isolated mode would reach its maximum, making the IF estimation more feasible and the multicomponent decomposition achievable .

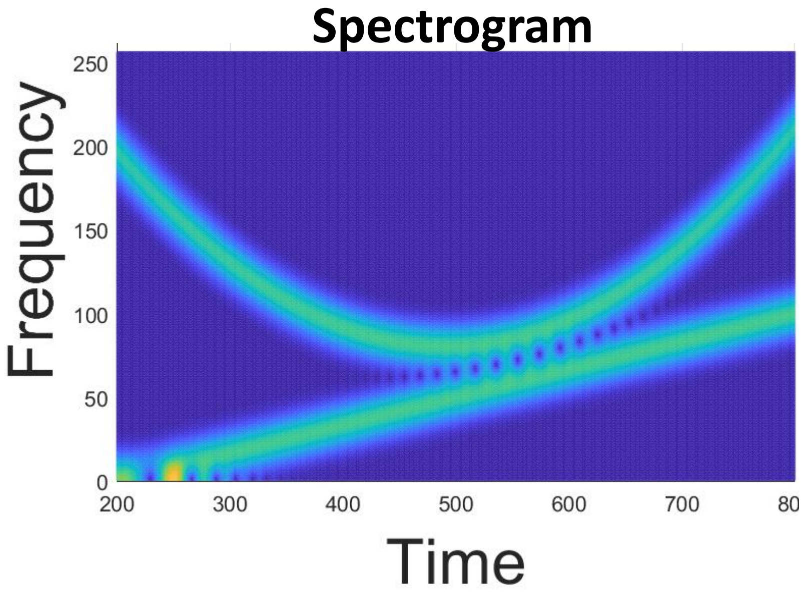
# The spectrogram

Among the various time-frequency distributions, the spectrogram (i.e. the modulus square of the **S**hort **T**ime **F**ourier **T**ransform) is probably the most widely used, due to its simplicity and computational gain.

$$S_f^g(u,\xi) = \int_{-\infty}^{+\infty} f(t)g(t-u)e^{-i\xi t}\,dt$$

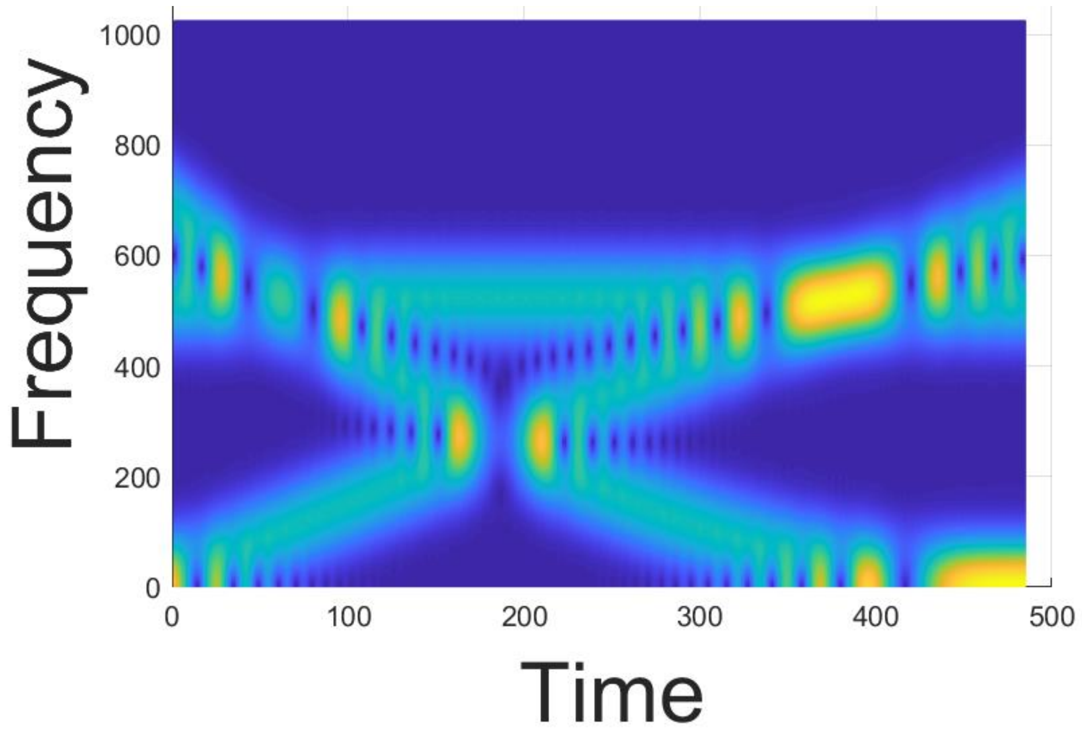
$$P_S f(u,\xi) = |Sf(u,\xi)|^2$$



As in all linear time-frequency transforms, STFT resolution is limited by Heisenberg principle, so the spectrogram is not able to provide a sparse representation of the signal. Furthemore, in the case of multicomponent signal having interfering single modes, the smearing introduced by the spectrogram prevents the modes separation, as in the example showed below.

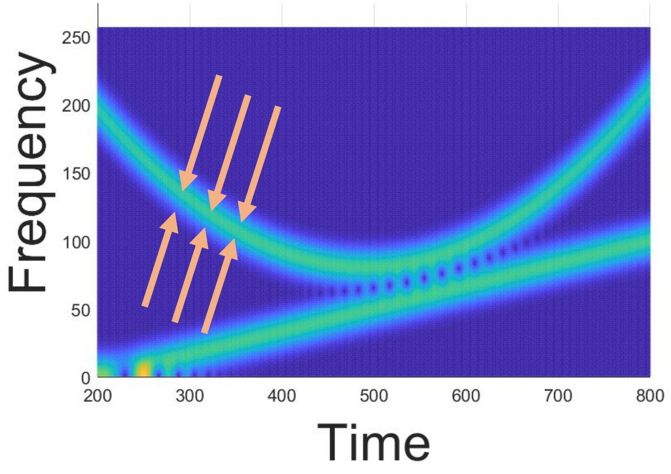
**Spectrogram**
**of 3-components signals**



# The reassigned spectrogram

In order to enhance the readability of the time-frequency distributions, further non linear operators such as Reassignment and Syncrosquezing have been proposed.
Reassignment method reallocates each TF point (t,f) toward the center of mass of energy distribution (ridge curves), giving a more compact representation of a signal in the TF plane.

### Spectrogram and idea of reassigment operators



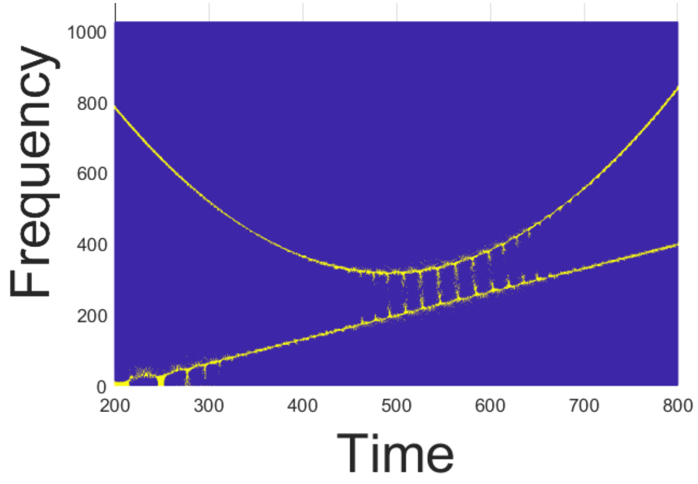

### Local centroids

$$\hat{u}_f(u,\xi) = u + \Re\left(\frac{Sf^{tg}(u,\xi)}{Sf^{g}(u,\xi)}\right)$$

$$\hat{\xi}_f(u,\xi) = \xi - \Im\left(\frac{Sf^{g'}(u,\xi)}{Sf^{g}(u,\xi)}\right)$$

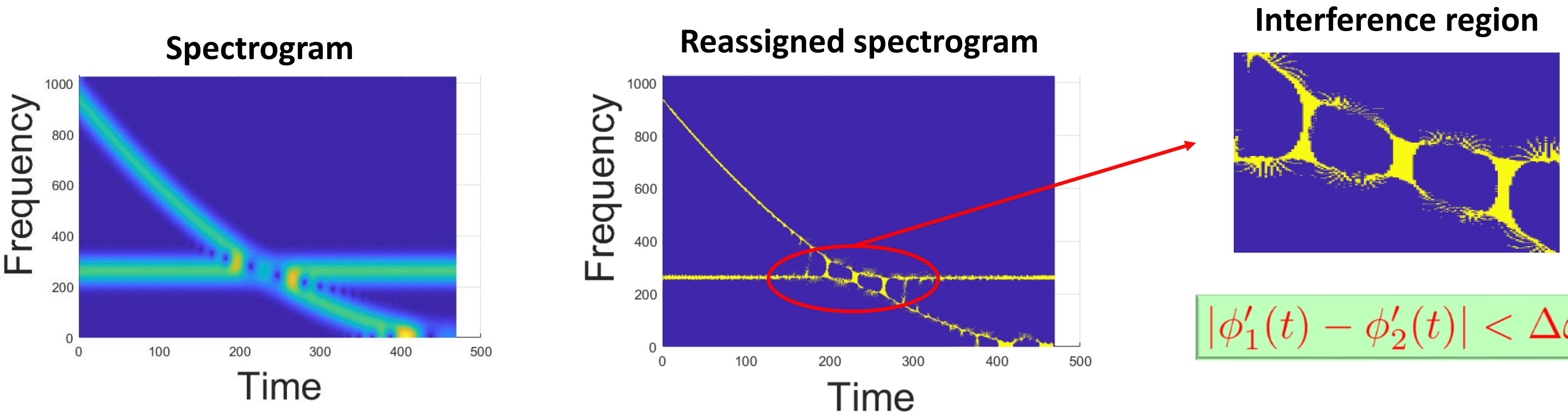
$$(u,\xi) \mapsto (\hat{u}_f(u,\xi), \hat{\xi}_f(u,\xi))$$

### Reassigned spectrogram




$$|\phi_1'(t) - \phi_2'(t)| \geq \Delta\omega$$

**PROBLEM** : In case of multicomponent signal with single modes interfer each other, Reassignment allows to reduce the interference region, but in correspondance of the region where the separability condition is not met, it is not possible to achieve a not ambiguous representation. Specifically, if the instantaneous frequencies of a pair of modes are not sufficiently apart in a region of the TF plane (interference region), reassignment method is not able to computed the local centroids correctly, then the TF points are not correctly reallocated and the single components separation is not achieved.




$$|\phi_1'(t) - \phi_2'(t)| < \Delta\omega$$

Other techniques aimed at enhancing resolution for multicomponent separation in the TF plane have been proposed. Unfortunately, their effectiveness is subjected again to the separability condition.

## QUESTION:

How to deal with the non-separability condition and recover information in complete interference?

**PROPOSED MODEL:** Mathematically describing what happens at the interference region allows us to partially recover the ridge curves at the interference region and overcome the classical limits of reassignment, i.e. the windows resolution and the number of STFTs to compute the local centroids. The experimental results show that the proposed method is able to better concentrate information on the ridge curves and separate the individual modes.

Reference: **V. Bruni, M. Tartaglione, D. Vitulano, On the time-frequency reassignment of interfering modes in multicomponent FM signals, Proceedings of EUSIPCO 2018.**

**Other challenging questions under the non-separability condition:**

How to define a suitable TF transformation for resolution enhancement in the whole time domain?

It is possible to automatically detect the interference region?

It is possible to automatically detect the number of components?

Some references on this topic are:


L. Angrisani, M. D'Arco, A measurement method based on a modified version of the chirplet transform for IF estimation,IEEE Transactions on Instruments and Measurements, vol. 51, pp: 704-711, 2002.

F. Auger and P. Flandrin, Improving the readability of time-frequency and time-scale representations by the reassignment method, IEEE Transactions on Signal Processing, vol. 43, pp. 1068-1089, May 1995.

F. Auger, P. Flandrin, Y.T. Lin, S. McLaughlin, S. Meignen, T.S Oberlin, H.T. Wu, Time-Frequency Reassignment and Synchrosqueezing, IEEE Signal Processing Magazine, vol. 32, 2013.

V. Bruni, S.Marconi, B.Piccoli, D.Vitulano, Instantaneous frequency estimation of interfering FM signals through time-scale isolevel curves, Signal Processing, vol. 93, pp: 882-896, 2013.

R. A. Carmona, W. L. Hwang, B. Torresani, Multiridge detection and time-frequency reconstruction, IEEE Trans. Signal Processing, vol. 47, no. 2, pp. 480-492, 1999.

I. Daubechies, J. Lu, H. T. Wu, Synchrosqueezed wavelet transforms: An Empirical Mode Decomposition-like tool, Appl. Comput. Harmon. Anal., vol. 30, no. 2, pp. 243-261, 2011.

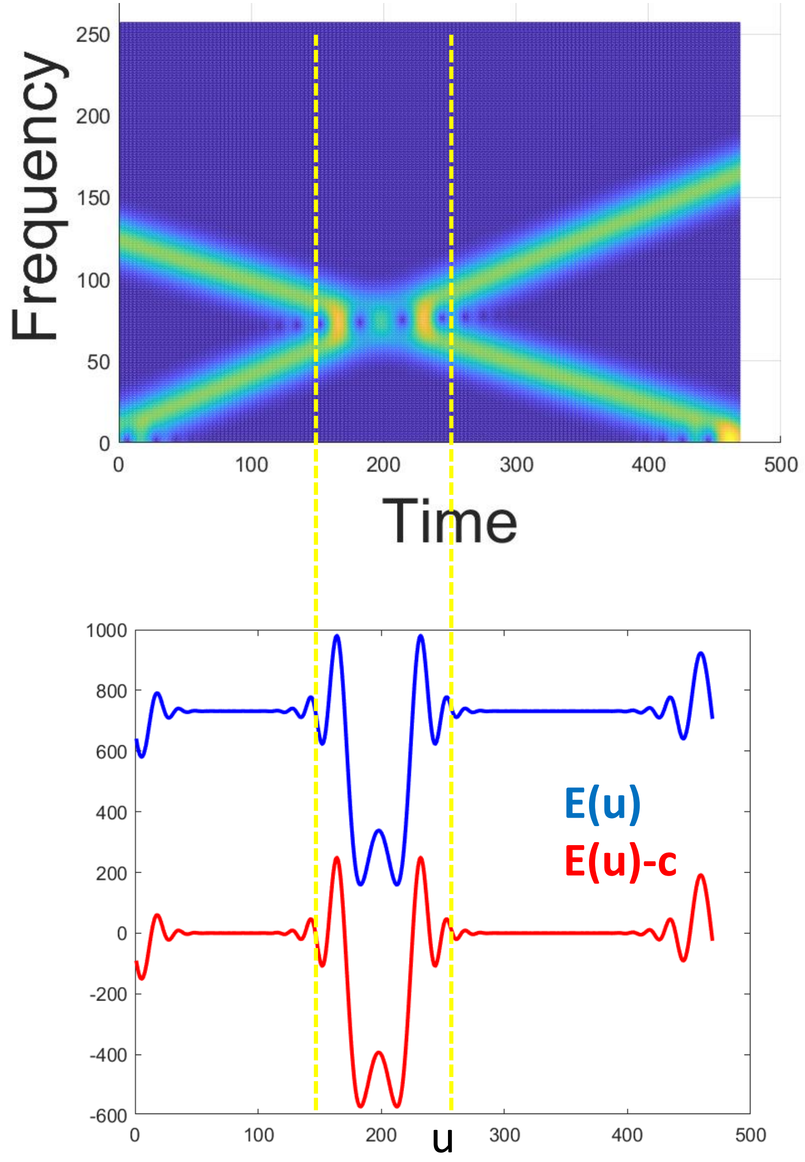
I. Djurovic, L. Stankovic, Robust Wigner distribution with application to the instantaneous frequency estimation, IEEE Transactions on Signal Processing, vol. 49, pp: 2985-2993, 2001.

F. Flandrin, F. Auger, E. Chassandre-Mottin-Time-frequency reassignment: from principles to algorithms, 2003.
S. A. Fulop and K. Fitz, Algorithms for computing the time corrected instantaneous frequency (reassigned) spectrogram, with applications, J. Acoust. Soc. Amer., vol. 119, no. 1, pp. 360-371, 2006.
H.K. Kwok, D.L. Jones,Improved instantaneous frequency estimation using an adaptive short-time Fourier transform, IEEE Transactions on Signal Processing vol. 48, 2964-2972, 2000.
X. Li, G.Bi, S. Stankovic, A.M. Zoubir, Local polynomial Fourier transform: a review on recent developments and applications, Signal Processing, vol. 91. pp: 1370-1393, 2011.
S. Mallat, A wavelet tour of signal processing, Academic Press, 1998.
S. Meignen, T. Oberlin, and S. Mclaughlin, A new algorithm for multicomponent signal analysis based on synchrosqueezing: with an application to signal sampling and denoising, IEEE Trans. Signal Processing, vol. 60, no. 11, pp. 5787-5798, 2012.
I. Sarkar, A.T. Fam, The interlaced chirp Z transform, Signal Processing, vol. 86, pp: 2221-2232, 2006.
E. Sejdic, L. Stankovic, M. Dakovic, J. Jiang, Instantaneous frequency estimation using the S-transform, IEEE Signal Processing Letters, vol. 15, pp: 309-312, 2008.
L. Weruaga, M. Kepesi, The fan-chirp transform for non stationary harmonic signals, Signal Processing, vol. 87, pp: 1504-1522, 2007.

# <u>PROPOSED MODEL</u>: energy along frequencies

$$E(u) = \int_{-\infty}^{+\infty} |Sf(u,\xi)|^2 \, d\xi$$

**Up to a constant, energy E(u) is still a FM signal having significant oscillations at the interference region and a specific IF: $\Delta\phi' = \phi_k' - \phi_j'$**




**Proposition 1**

Let $f(t) = \sum_k a_k cos\phi_k(t)$, then $E(u) = c + \sum_{k \neq j} A_{k,j}(u) \cos(\phi_k(u) - \phi_j(u))$

where $A_{k,j}(u) = \frac{a_k a_j}{2} \int_{-\infty}^{+\infty} \hat{g}(\xi - \phi_k')\hat{g}(\xi - \phi_j')d\xi$,

$c = \frac{\pi}{2} \sum_k a_k^2 \int_{-\infty}^{+\infty} g(t)^2 dt$,

$Sf_k(u,\xi) = \frac{1}{2} a_k e^{i(\phi_k(u) - \xi u)}(\hat{g}(\xi - \phi_k'(u)) + \epsilon_k(u,\xi))$.

V. Bruni, M. Tartaglione, D. Vitulano, On the time-frequency reassignment of interfering modes in multicomponent FM signals, Proceedings of EUSIPCO 2018.

# PROPOSED METHOD FOR RECONSTRUCTION (Gaussian window)

**Proposition 2**
Let $f(t) = \cos\phi_1(t) + \cos\phi_2(t)$, $f_{int}(u) = E(u) - c$ and $\Delta\phi'(u) = \phi_2'(u) - \phi_1'(u)$, then

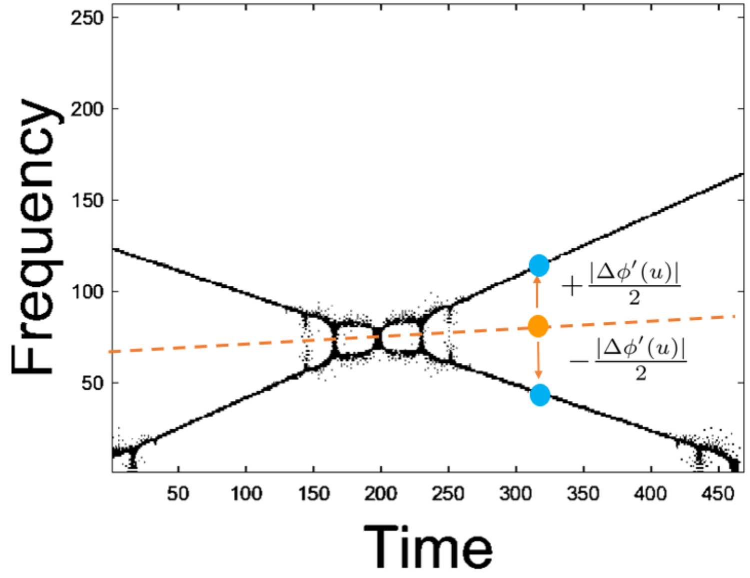
$$|\Delta\phi'(u)| = \frac{2}{\sigma^2}\sqrt{-\ln\left(\frac{2}{\sqrt{2\pi\sigma^2}}\frac{\Re(Sf_{int}(u,0))}{f_{int}(u)}\right)},$$

$\forall u \in \mathrm{supp}(f_{int})$.

**$\Delta\phi' = \phi_k' - \phi_j'$ estimate**

## Resolution enhancement

$$\bar{\xi}(u) = G(u) \pm \frac{|\Delta\phi'(u)|}{2}$$

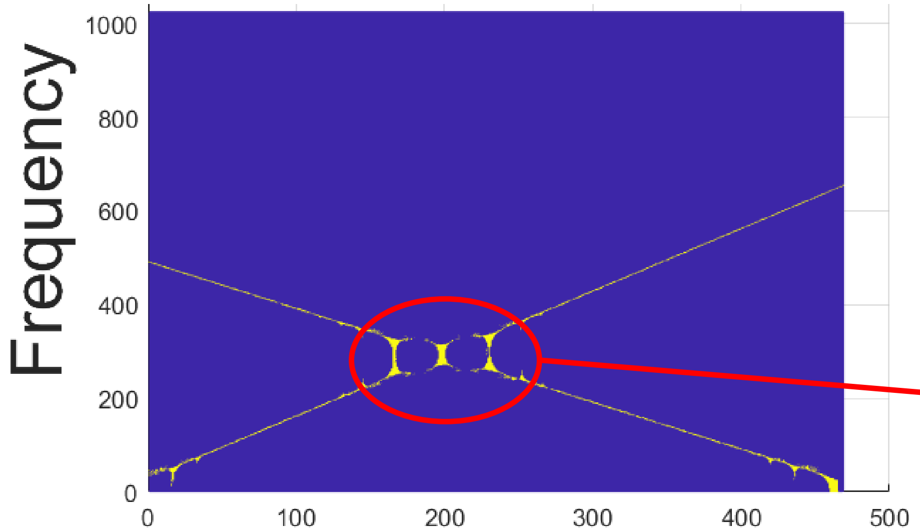



Ridge points recovered starting from the center of mass G(u) of the distribution

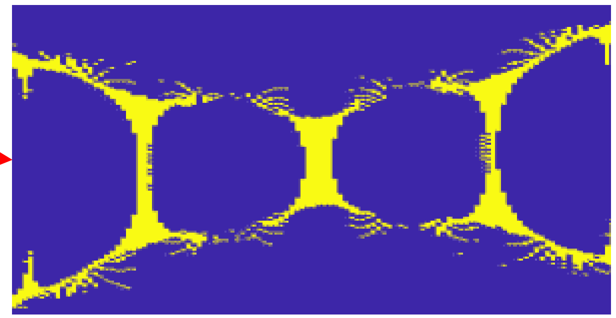
V. Bruni, M. Tartaglione, D. Vitulano, On the time-frequency reassignment of interfering modes in multicomponent FM signals, Proceedings of EUSIPCO 2018.

# EXPERIMENTAL RESULTS

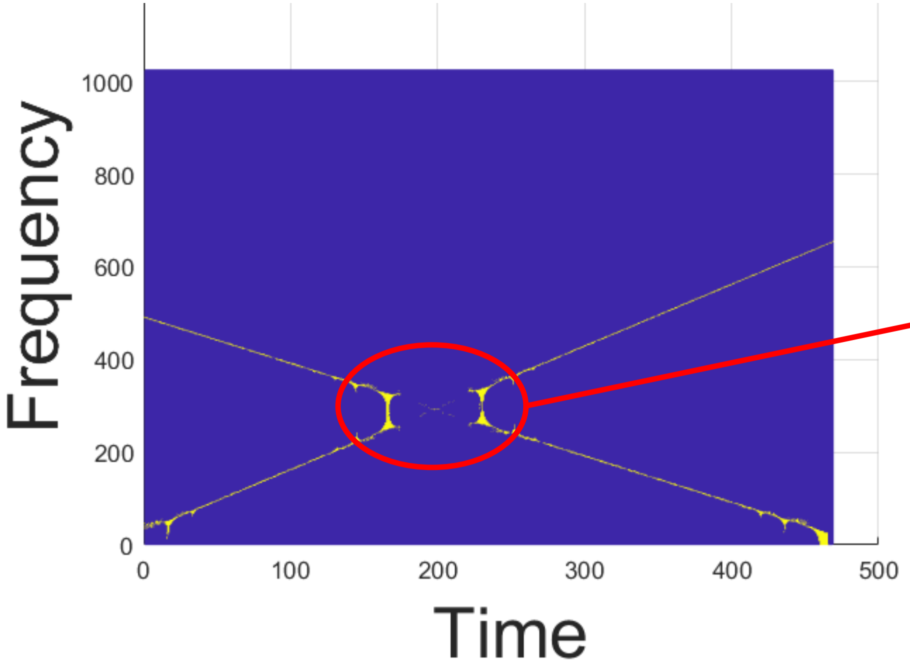
**STANDARD REASSIGNMENT**




$$|\phi_1'(t) - \phi_2'(t)| < \Delta\omega$$



**OUR APPROACH**

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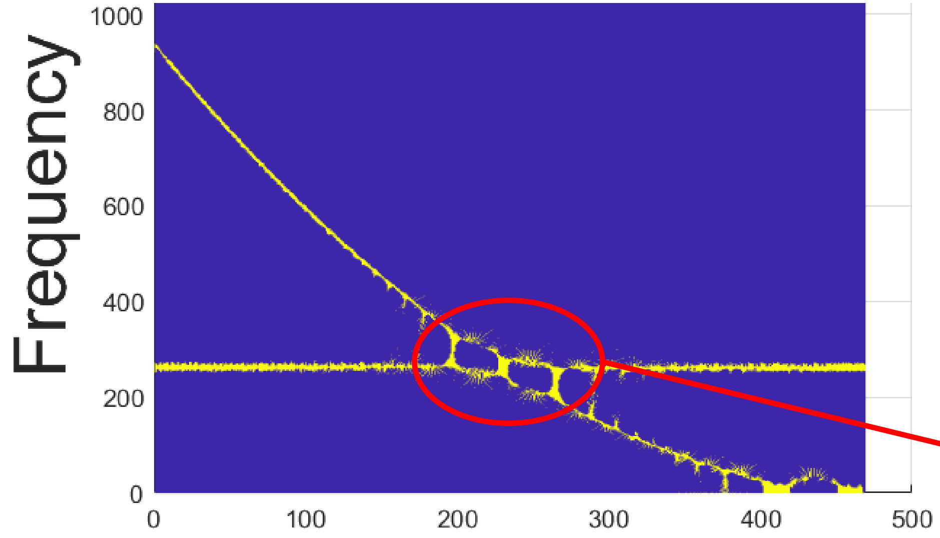
STANDARD REASSIGNMENT

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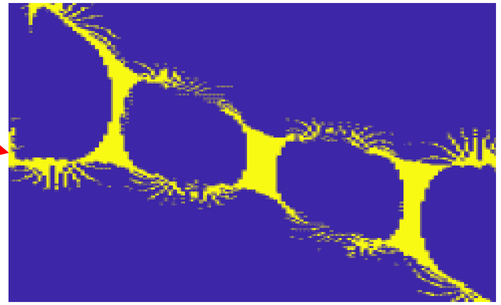

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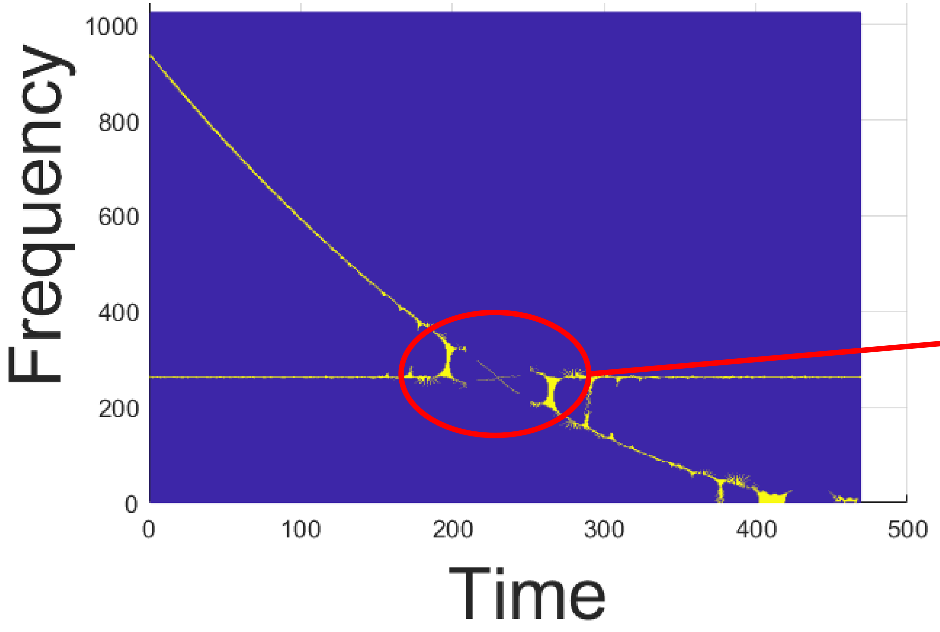


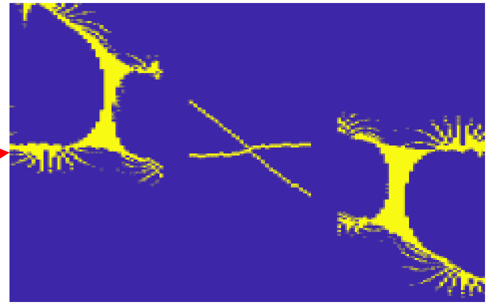

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