Department of Basic and Applied Sciences for Engineering
Sapienza University of Rome

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SAPIENZA
UNIVERSITÀ DI ROMA

Analysis and decomposition of frequency modulated multicomponent signals

PhD Thesis in Mathematical Models for Engineering, Electromagnetics and Nanoscience

Curriculum Mathematics

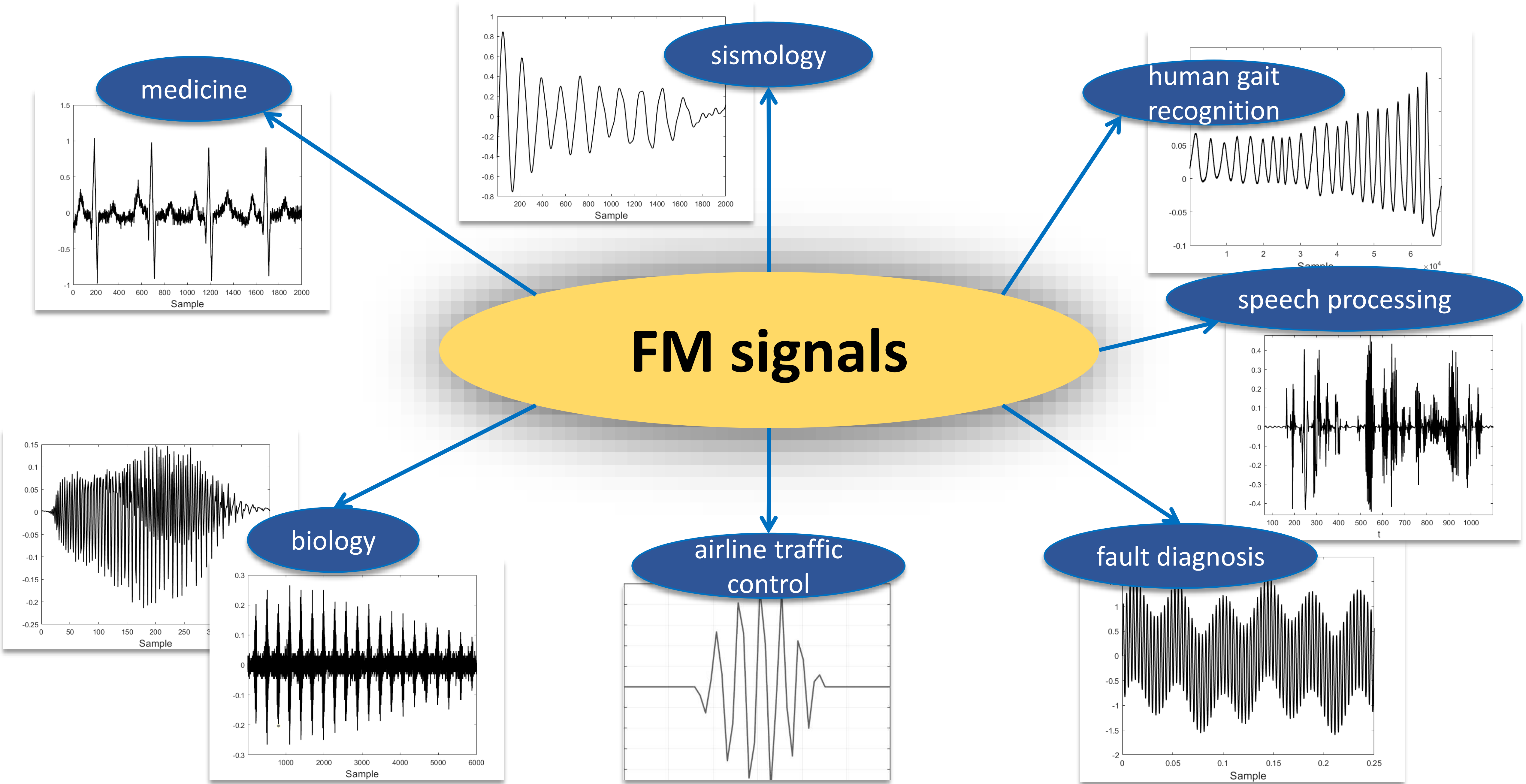
XXXIII Cycle

Michela Tartaglione

Advisor: Prof. Vittoria Bruni

michela.tartaglione@uniroma1.it

Frequency modulated signals everywhere



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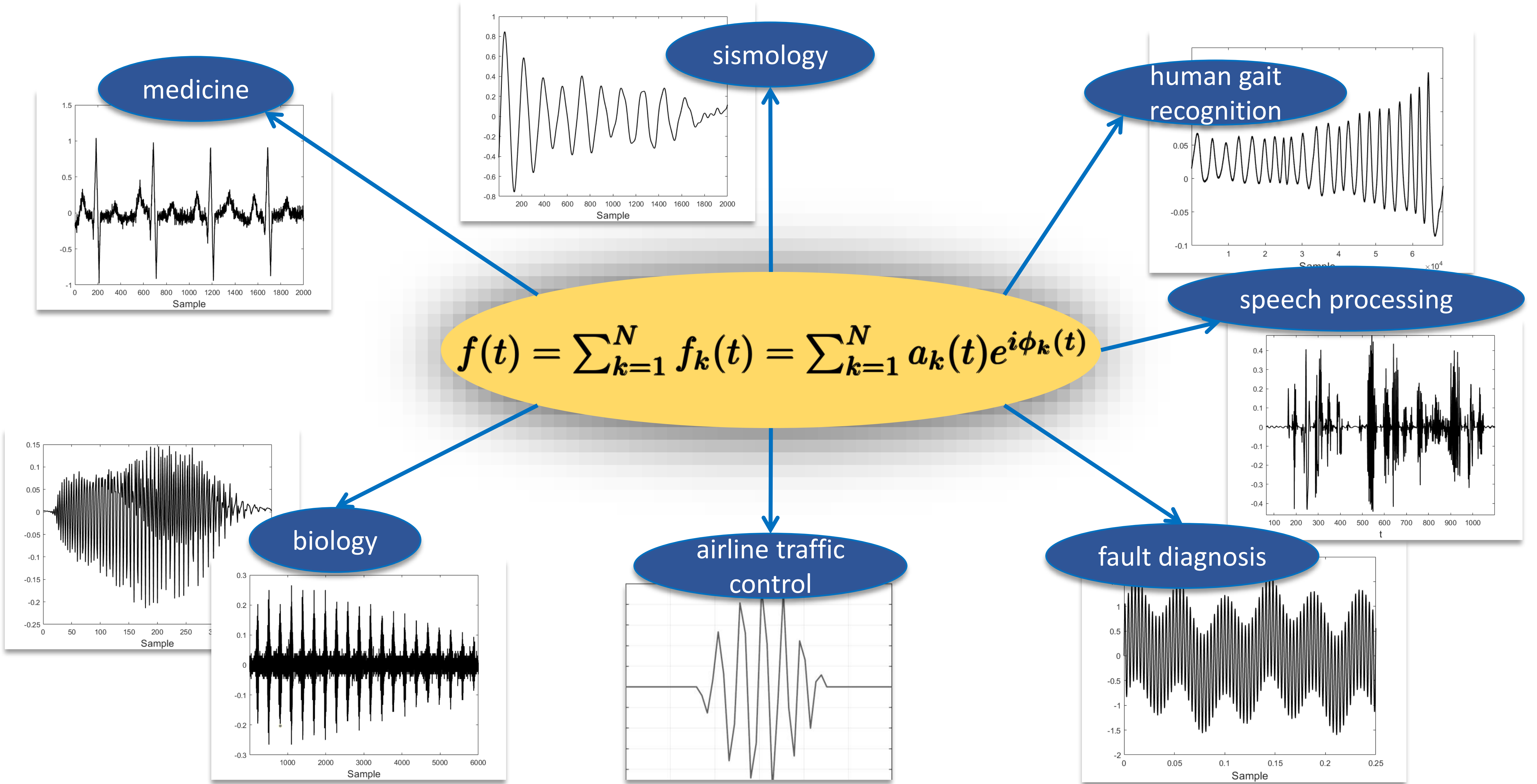
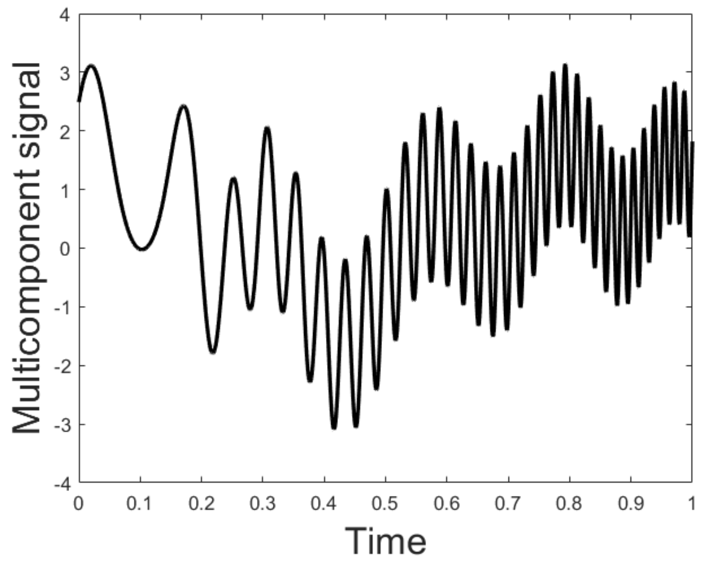
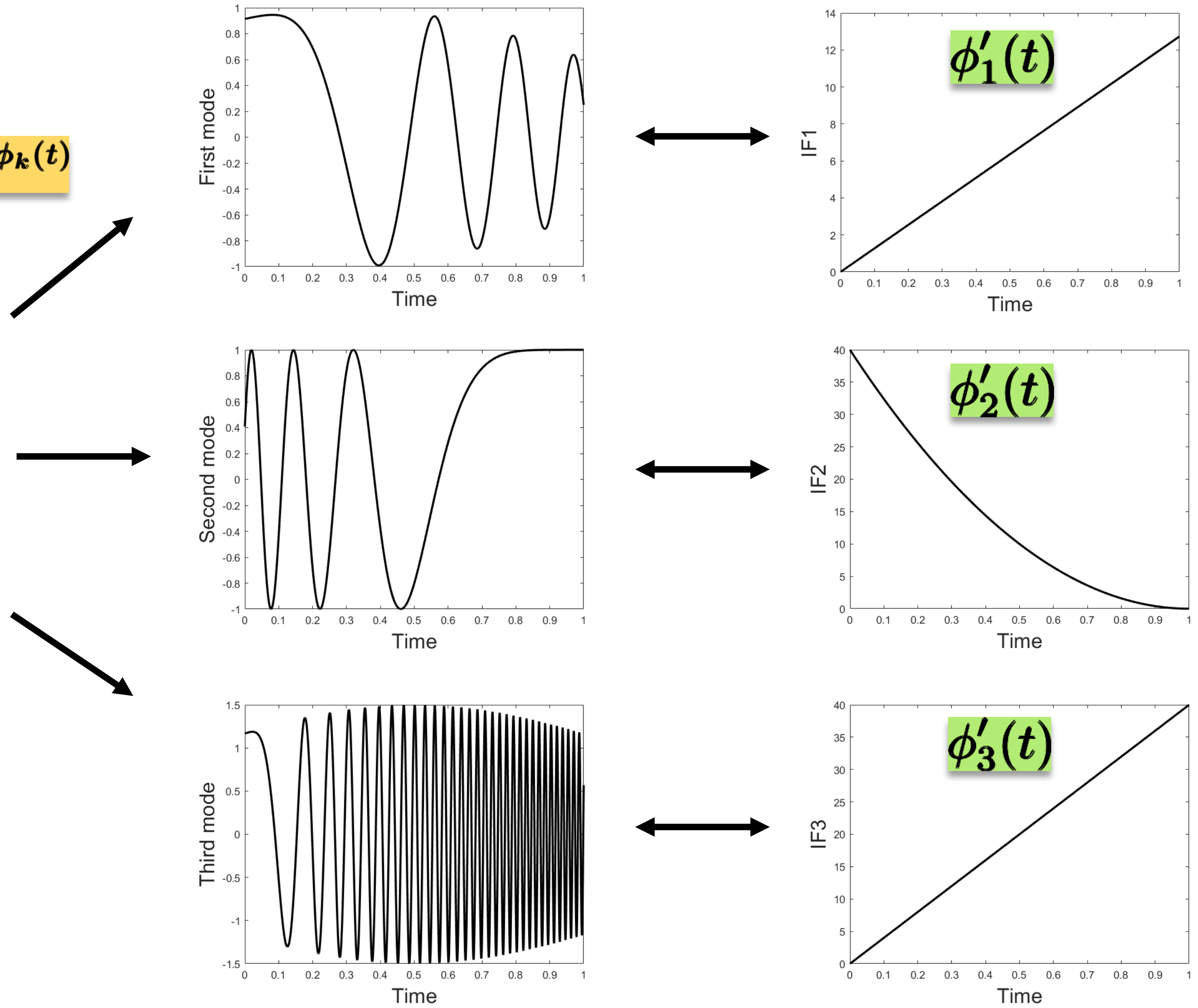


Decomposition of FM signals/IFs separation

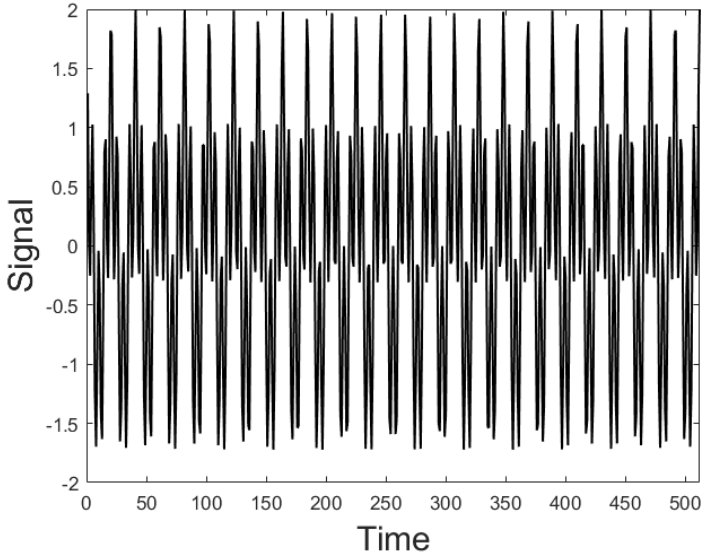
$$f(t) = \sum_{k=1}^{N} f_k(t) = \sum_{k=1}^{N} a_k(t)e^{i\phi_k(t)}$$

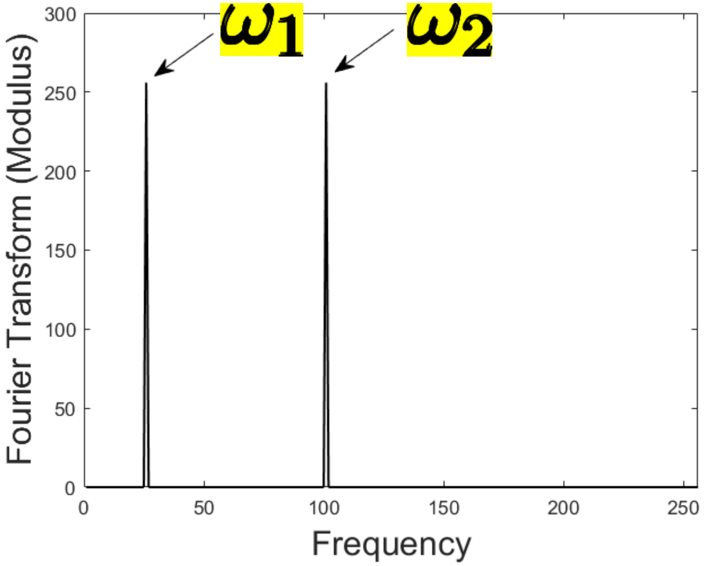




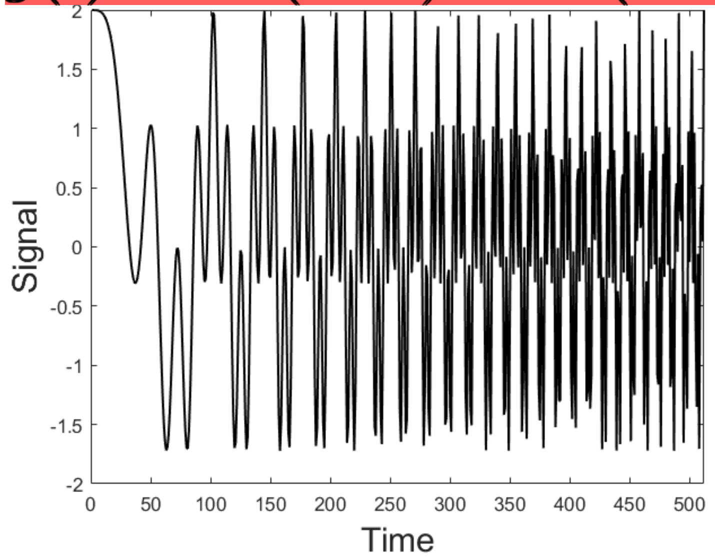
Pure tones vs FM signals

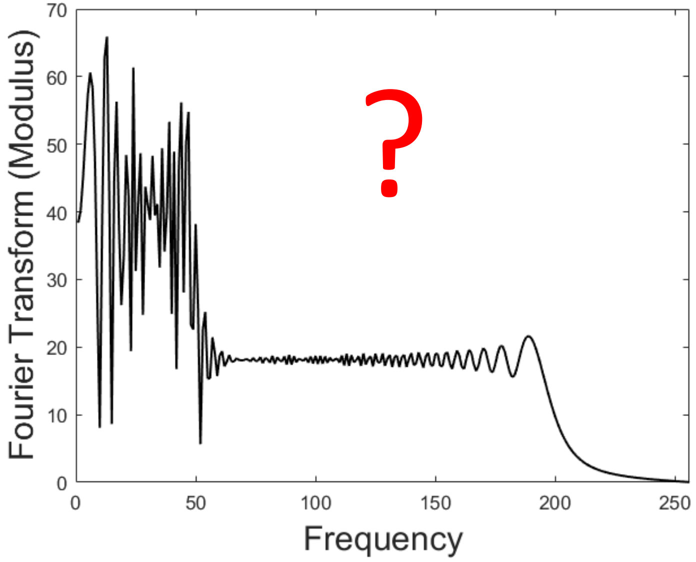
$$f(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$$



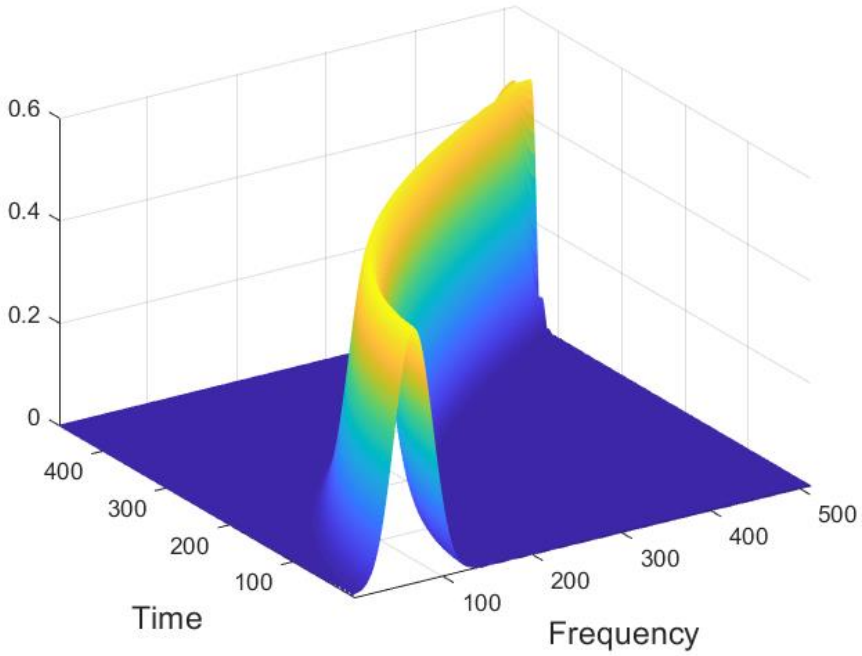


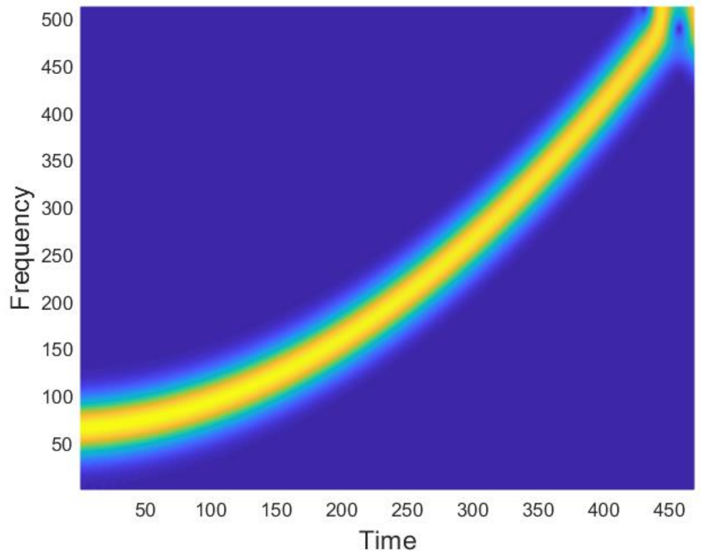
$$g(t) = \cos(\omega_1 t^2) + \cos(\omega_2 t^2)$$





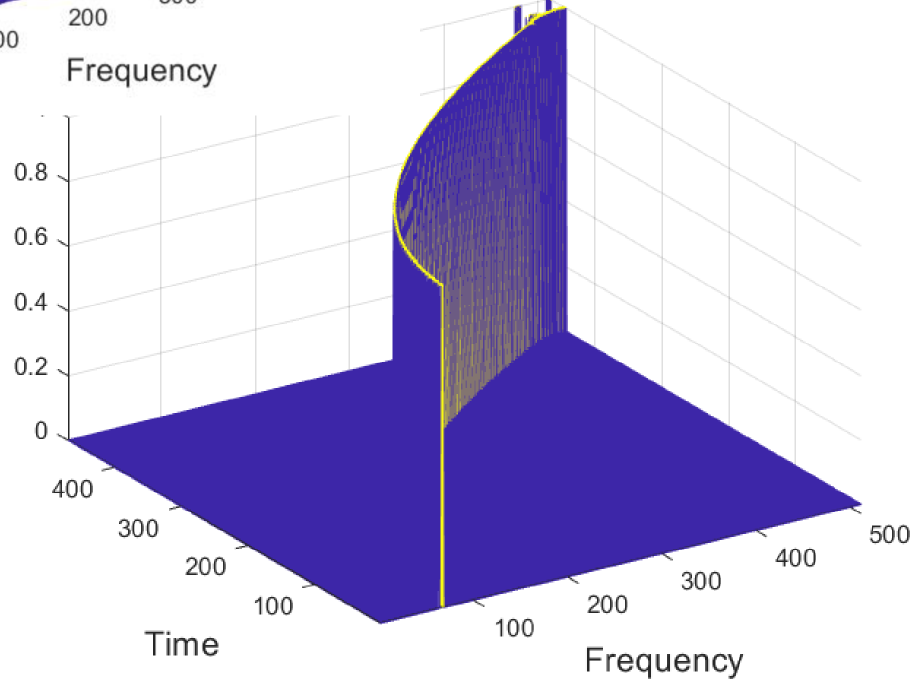
Time-Frequency Analysis

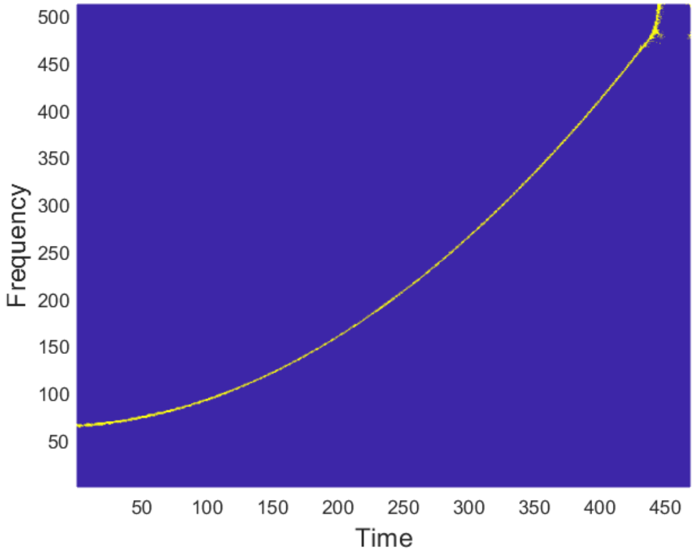


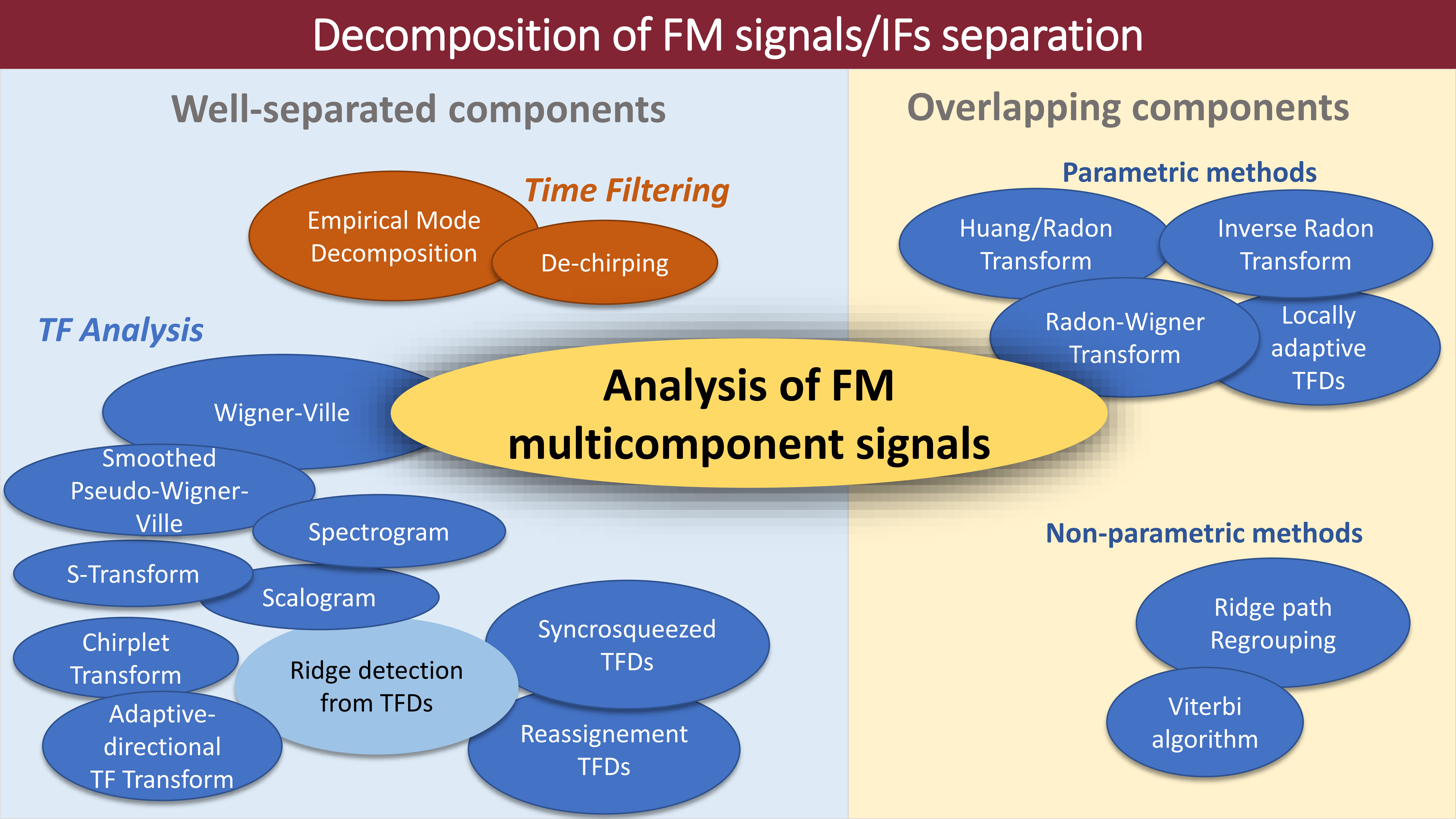


Ridge curve

$$(u, \phi'(u))$$

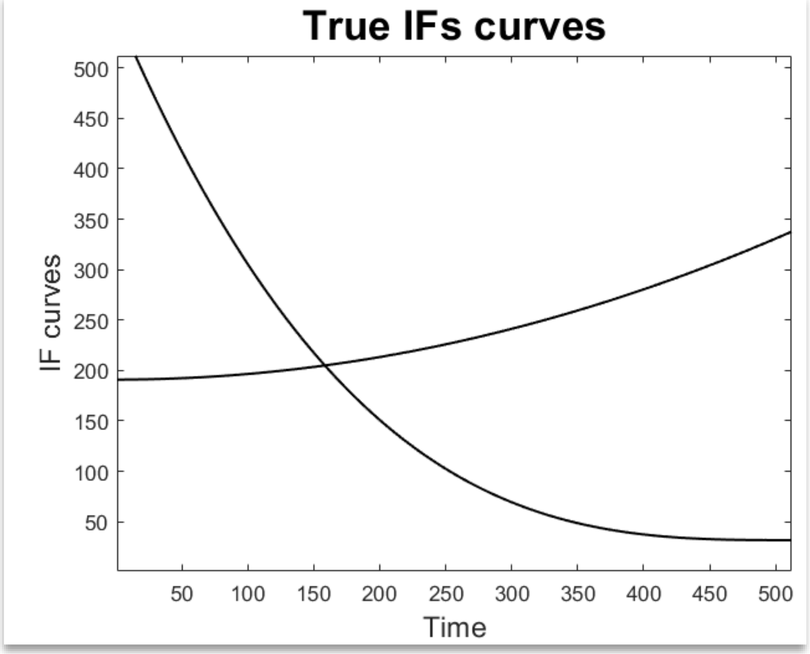


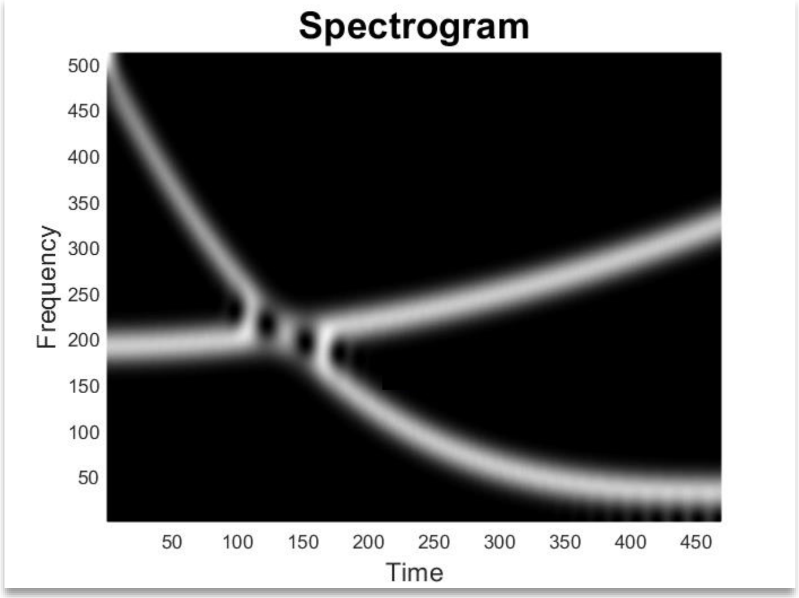


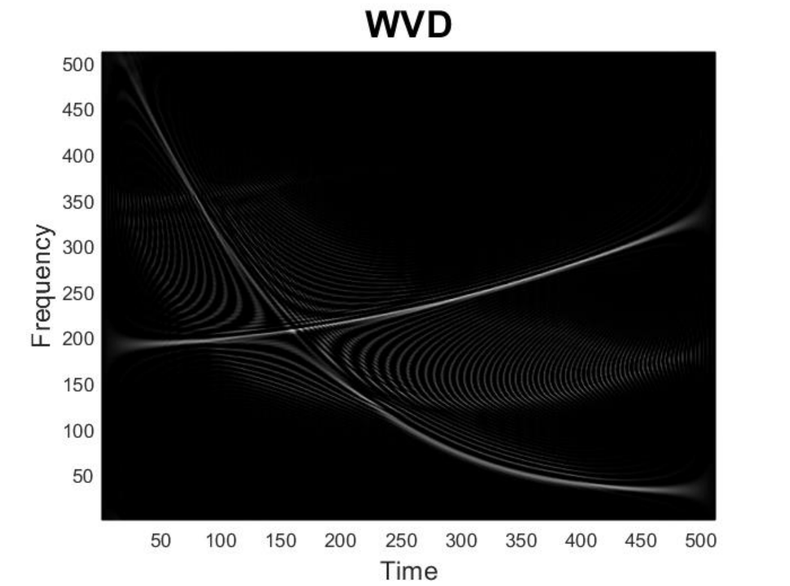

Decomposition of FM signals/IFs separation
Well-separated components
Overlapping components
Time Filtering
Empirical Mode Decomposition
De-chirping
TF Analysis
Wigner-Ville
Smoothed Pseudo-Wigner-Ville
Spectrogram
S-Transform
Scalogram
Chirplet Transform
Ridge detection from TFDs
Syncrosqueezed TFDs
Adaptive-directional TF Transform
Reassignement TFDs
Analysis of FM multicomponent signals
Parametric methods
Huang/Radon Transform
Inverse Radon Transform
Radon-Wigner Transform
Locally adaptive TFDs
Non-parametric methods
Ridge path Regrouping
Viterbi algorithm

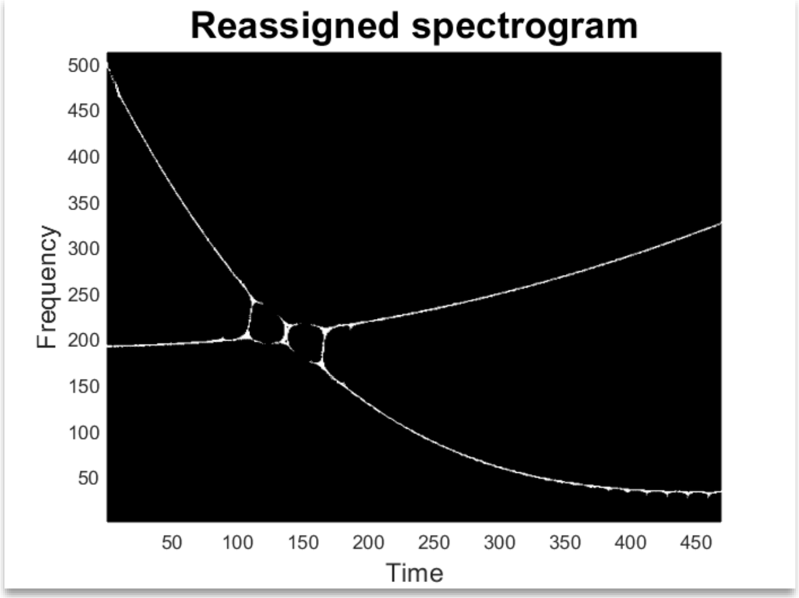
Motivations

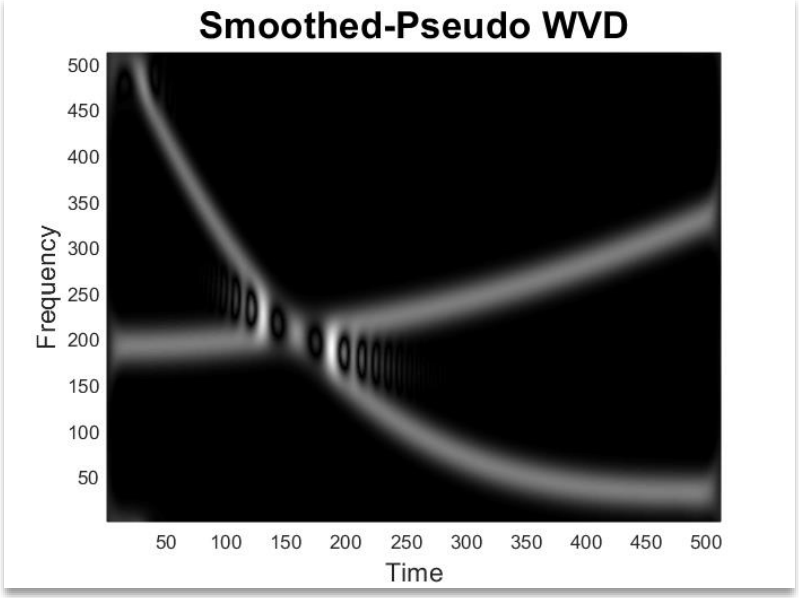
Overlapping components











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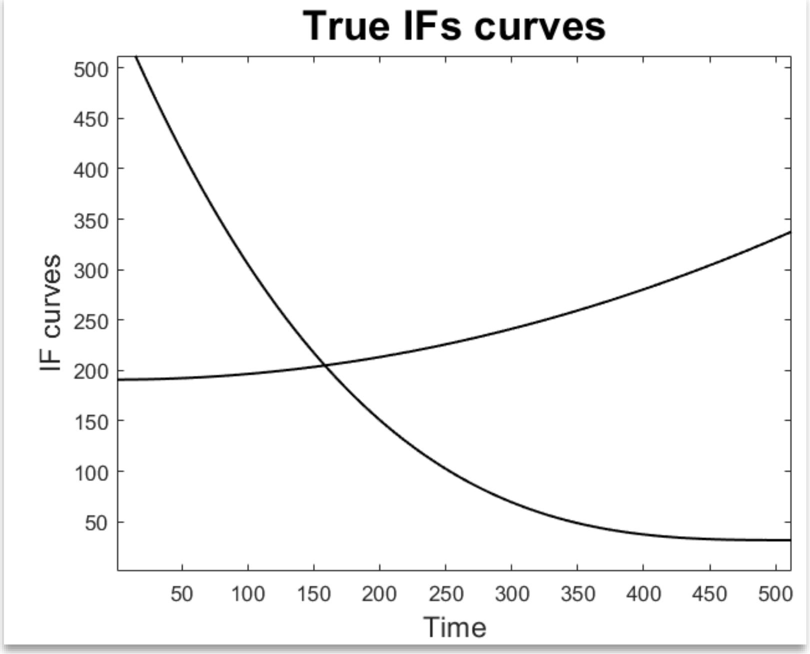
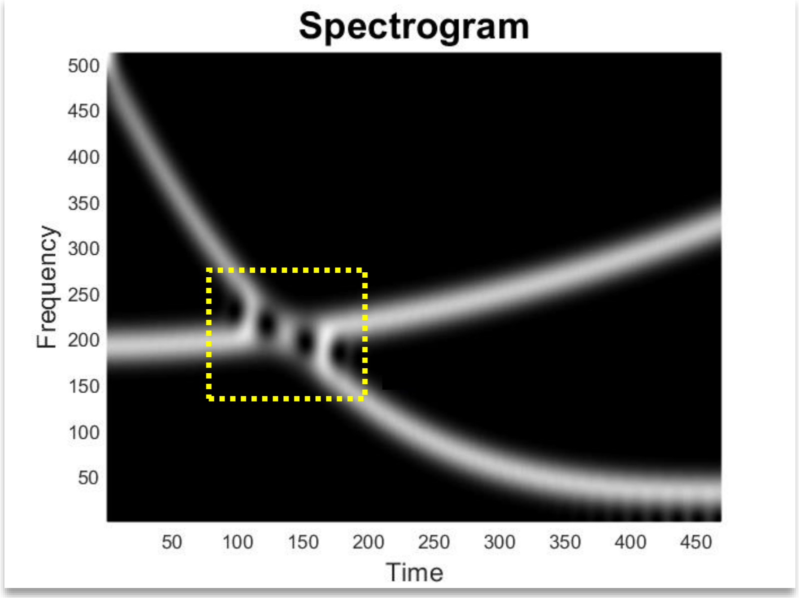
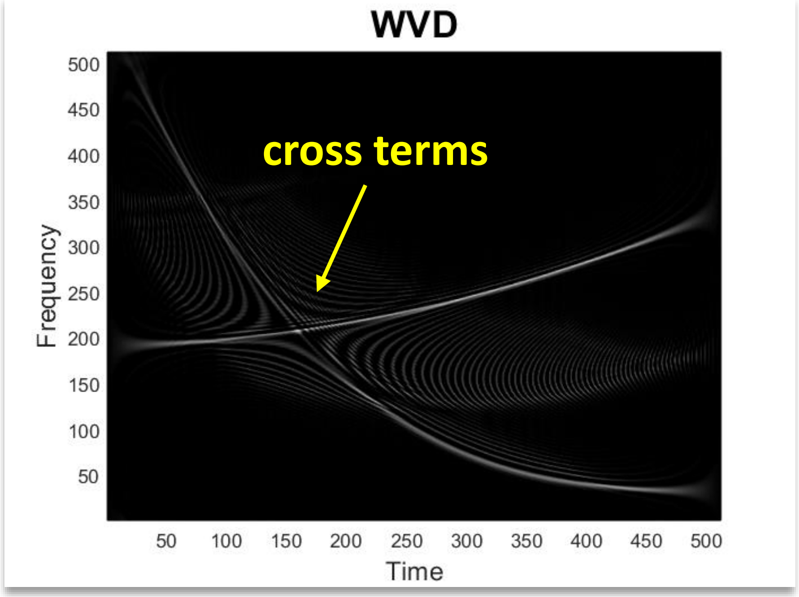
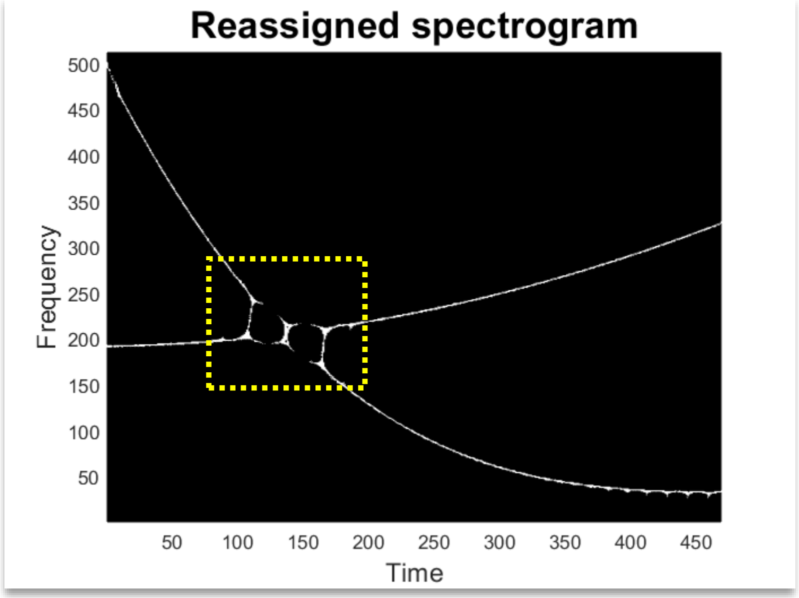
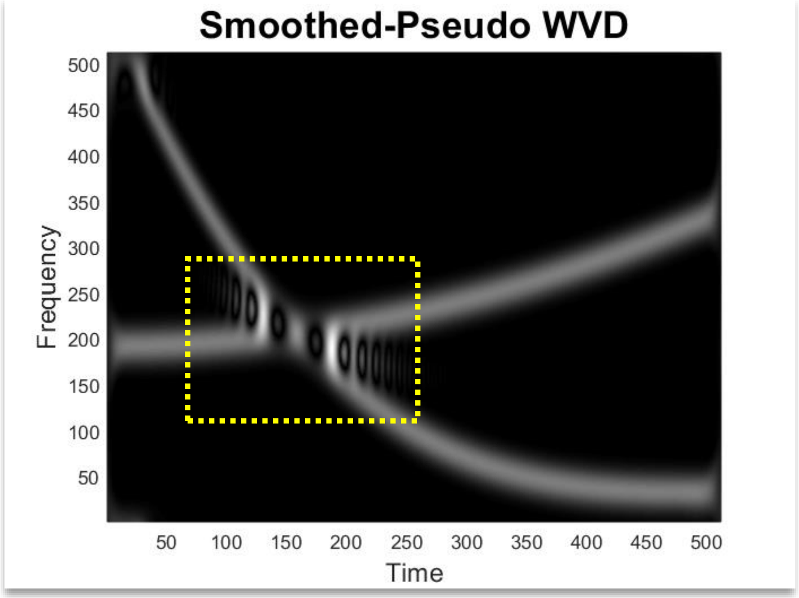




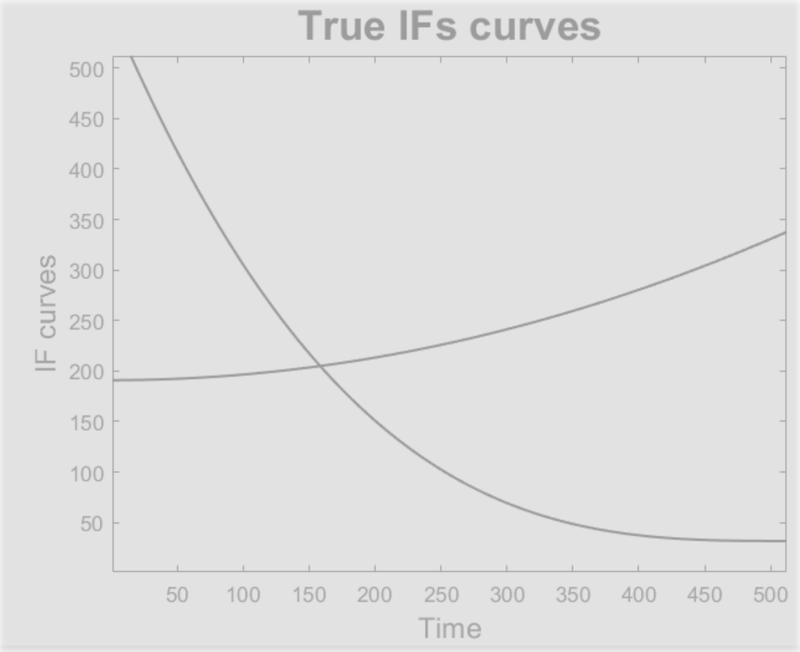


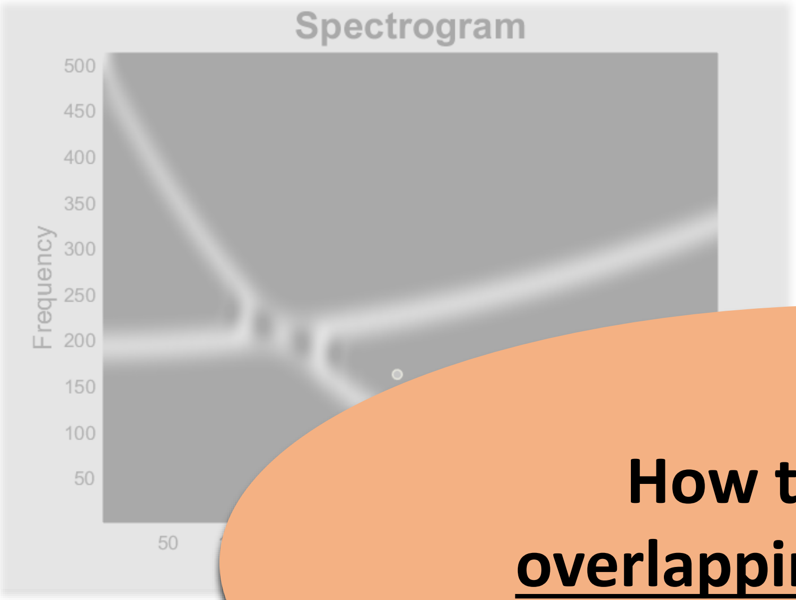


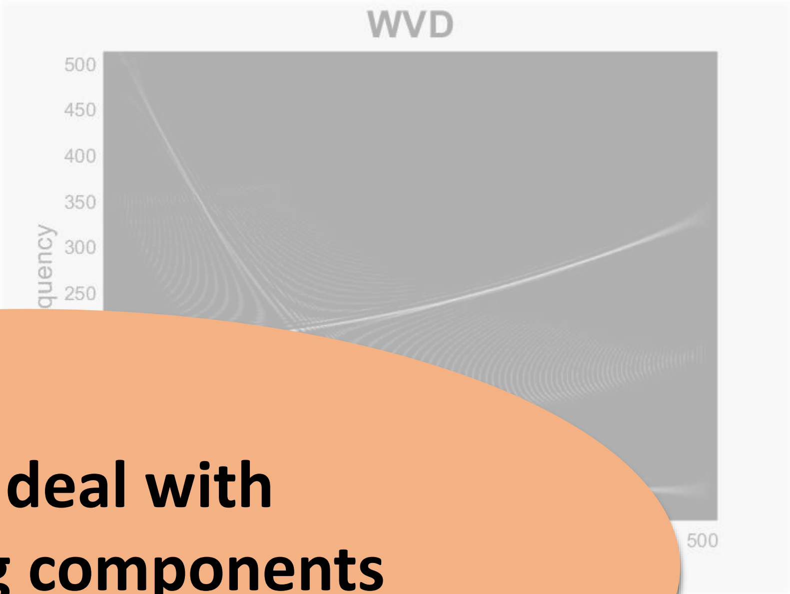


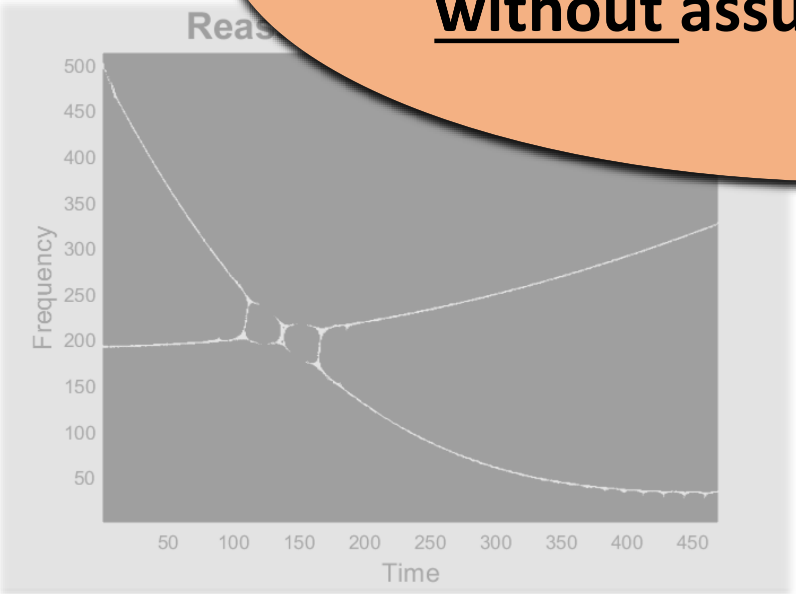
Problem

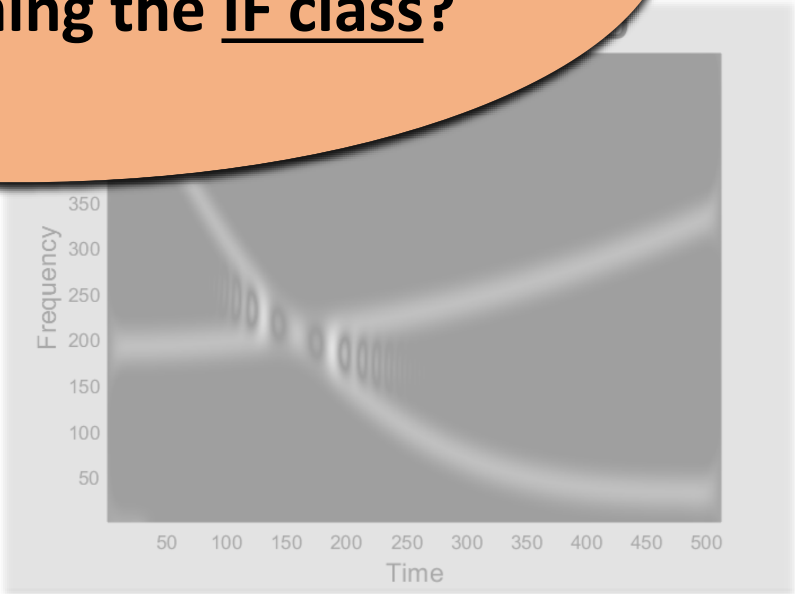
Overlapping components











How to deal with <u>overlapping components</u> <u>without</u> assuming the <u>IF class</u>?

Proposal

Spectrogram **model** in TF domain

Iterative reassignment for **<u>IFs curves resolution enhancement</u>**

Two non-parametric approaches proposed

Energy transform-based method for **<u>IFs curves separation</u>**

Spectrogram model

Definition. The Short-Time Fourier Transform (STFT) of a function $f \in L^2(\mathbb{R})$, with respect to a real and symmetric analysis window $g \in L^2(\mathbb{R})$, is

$$S_f^g(u,\xi) = \int_{-\infty}^{+\infty} f(t)g(t-u)e^{-i\xi t}\,dt,\ \forall\,(u,\xi) \in \mathbb{R} \times \mathbb{R}^+.$$

Spectrogram is defined as STFT squared modulus, i.e. $P(u,\xi) = |S_f^g(u,\xi)|^2$.

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Proposition 1. Let $f(t) = a_1(t)\cos\phi_1(t) + a_2(t)\cos\phi_2(t)$ be a two-components signal and let us set $\hat{g}_k = \hat{g}(s(\xi - \phi_k'(u)))$, where g is the analysis window with length $s > 0$, and $\hat{g}$ denotes its Fourier Transform; $a_k = a_k(u)$ and $\phi_k = \phi_k(u)$, $k = 1, 2$, $\Delta\phi = \phi_1 - \phi_2$, and $(*)'$ denotes the time derivative of $(*)$. Then, the spectrogram $P(u,\xi)$ satisfies the following evolution law

$$\frac{\partial P(u,\xi)}{\partial u} + \phi_1''\frac{\partial P(u,\xi)}{\partial \xi} - \frac{s}{2}a_1a_1'\hat{g}_1^2 - \frac{s}{2}\left[a_2a_2'\hat{g}_2^2 + \hat{g}_1\hat{g}_2(a_1'a_2 + a_2'a_1)\cos\Delta\phi\right]$$
$$+ \frac{s}{2}a_1a_2\hat{g}_1\hat{g}_2\Delta\phi'\sin\Delta\phi + \frac{s^2}{2}\Delta\phi''\hat{g}_2'\left[a_2\hat{g}_2^2 - a_1a_2\hat{g}_1\cos\Delta\phi\right] = 0. \qquad (1)$$

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Spectrogram model

Proposition 2. The spectrogram $P(u,\xi)$ of a monocomponent signal $f(t) = a(t)\cos\phi(t)$ satisfies the following advection equation

$$\frac{\partial P(u,\xi)}{\partial u} + \phi''(u)\frac{\partial P(u,\xi)}{\partial \xi} - \frac{2a'(u)}{a(u)}P(u,\xi) = 0 \quad \forall\, u \in supp\{f\}, \tag{2}$$

whose characteristic curves are

$$\mathcal{C}_{c,\phi} \,:\, \xi(u) = \phi'(u) + c,$$

with $c = \xi_0 - \phi'(u_0)$ and (u_0, ξ_0) is a point in the TF plane.

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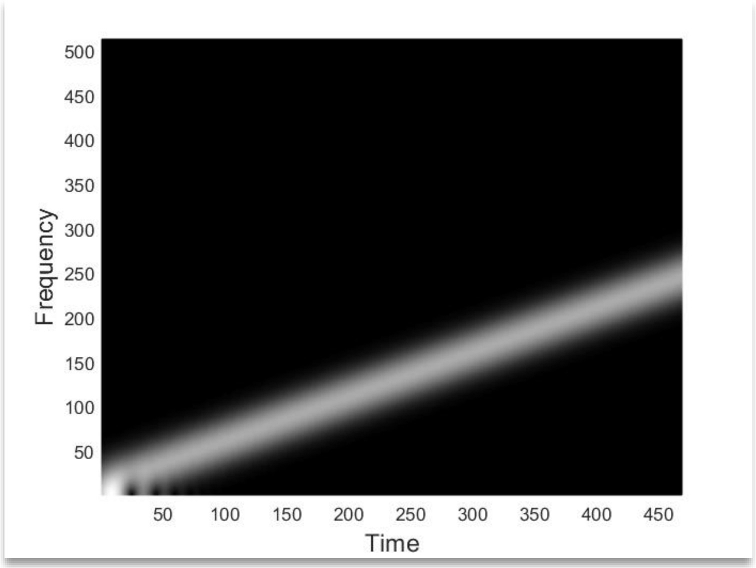

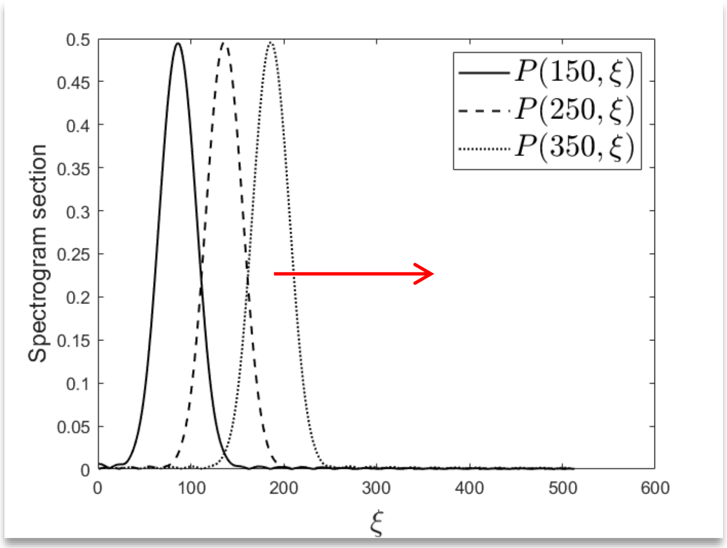

ADVECTION EQUATION

Spectrogram model

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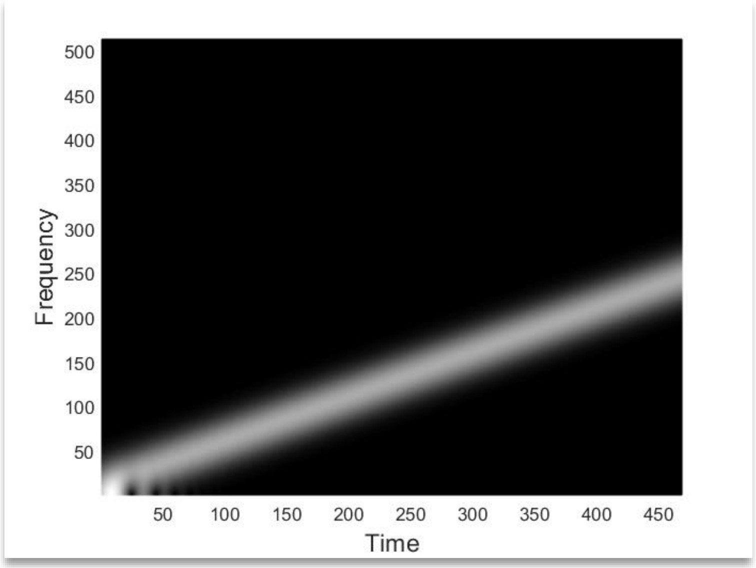

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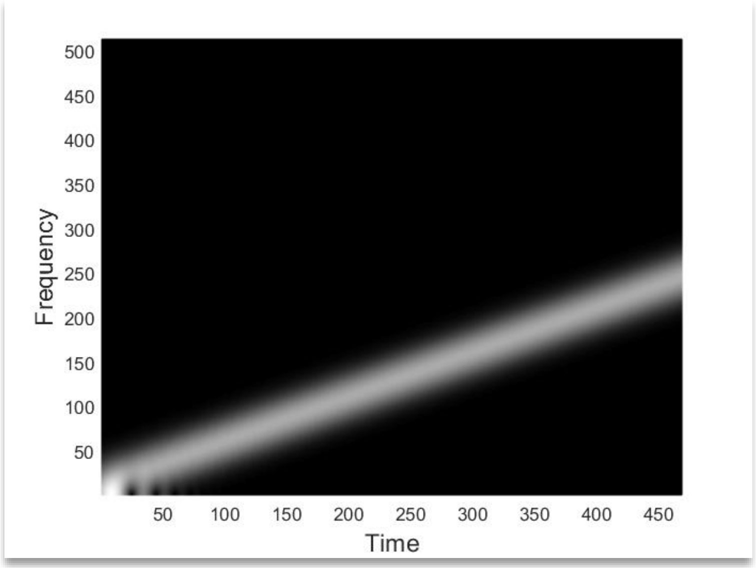
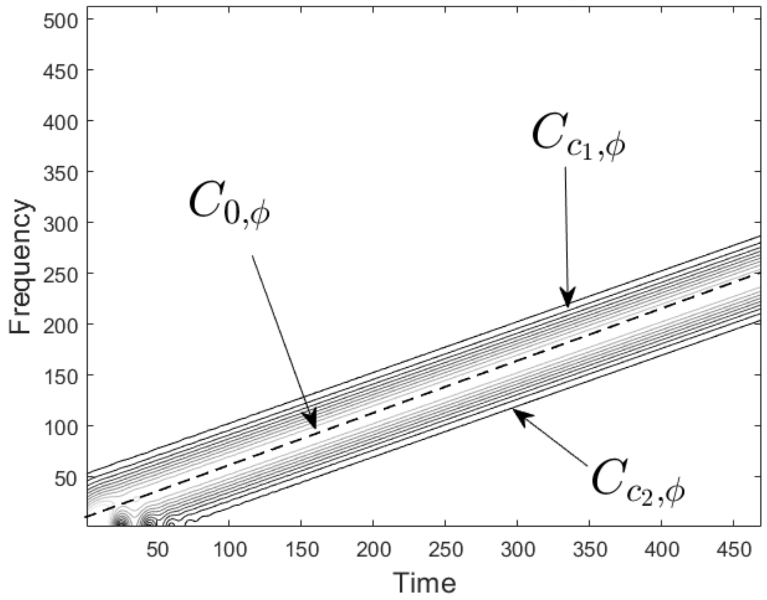


ISOLEVEL CURVES

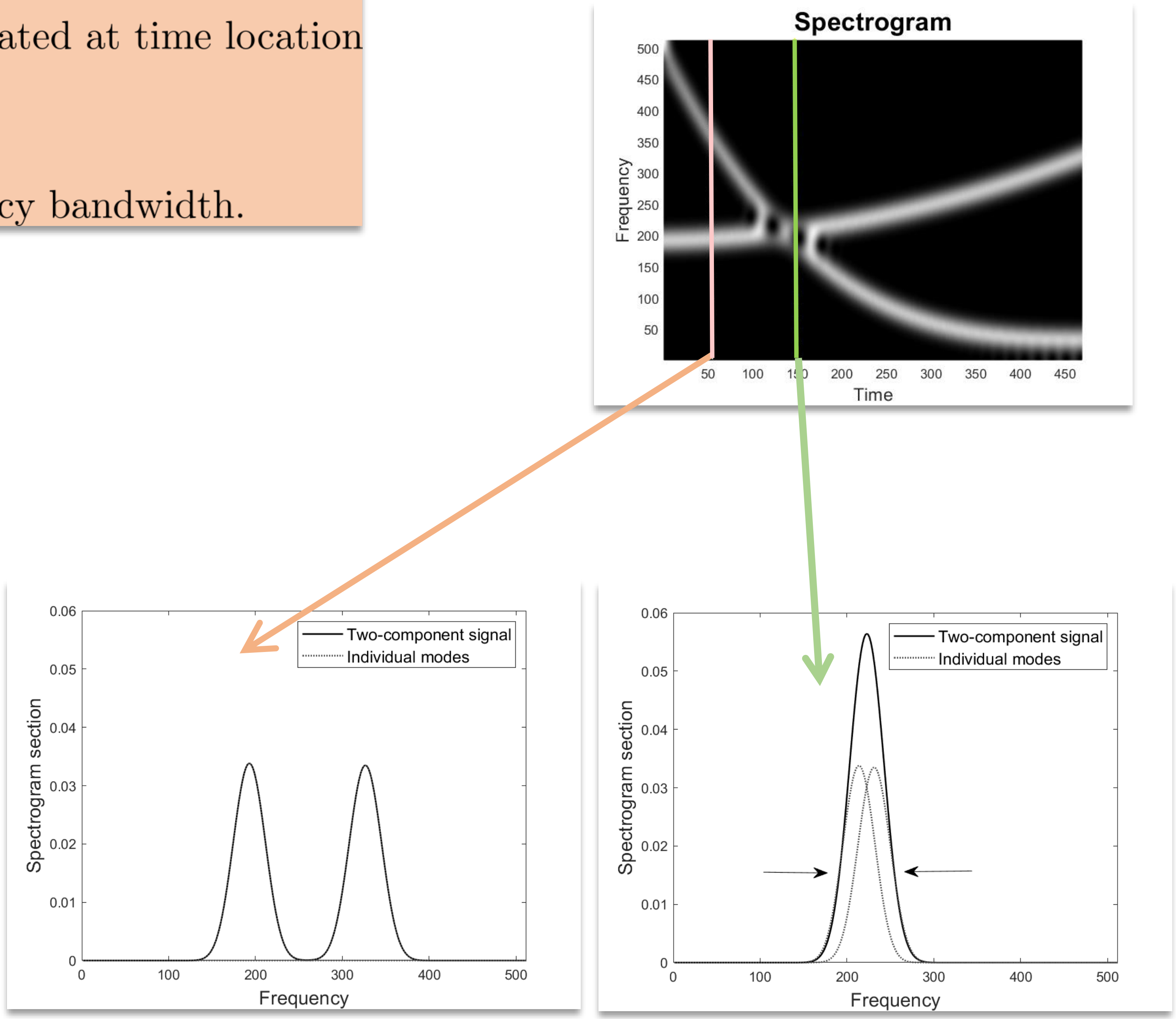


Weakened separability

Definition 1[*Separability condition*]
Two modes with IFs $\phi_1'(u)$ and $\phi_2'(u)$ are separated at time location u if

$$|\phi_1'(u) - \phi_2'(u)| \geq \Delta\omega,$$

where $\Delta\omega$ denotes the analysis window frequency bandwidth.



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Definition 2 [*Weakened separability condition*]
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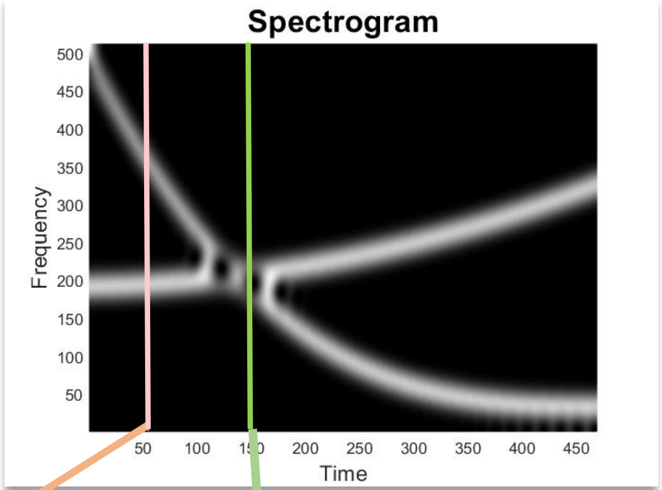
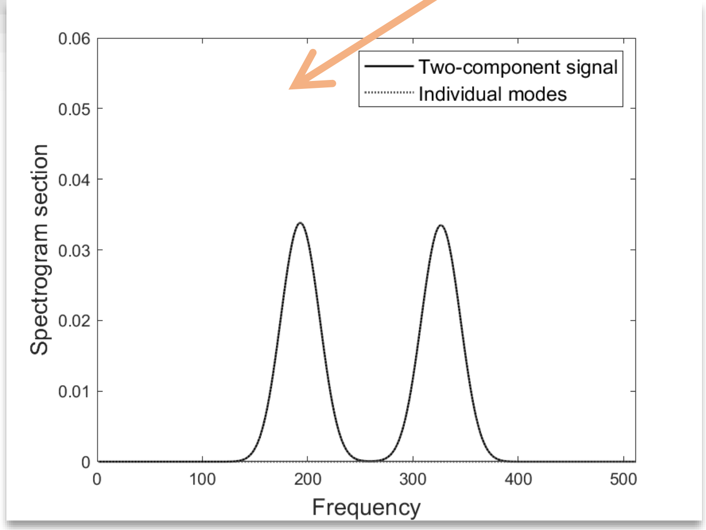
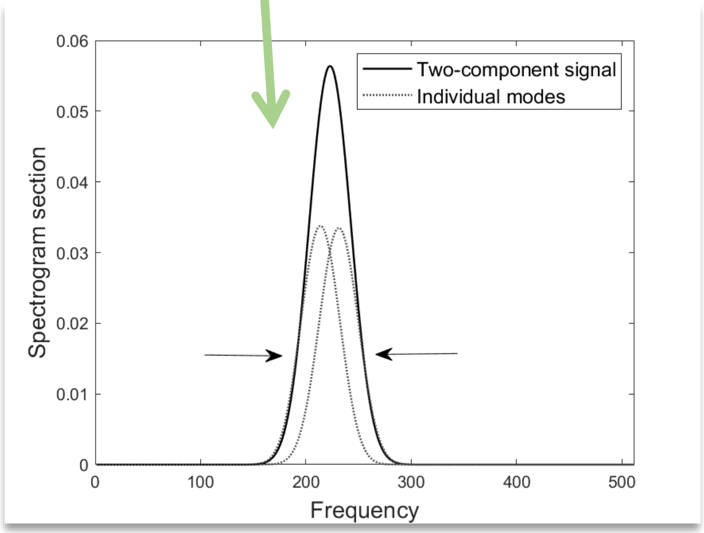
or viceversa.







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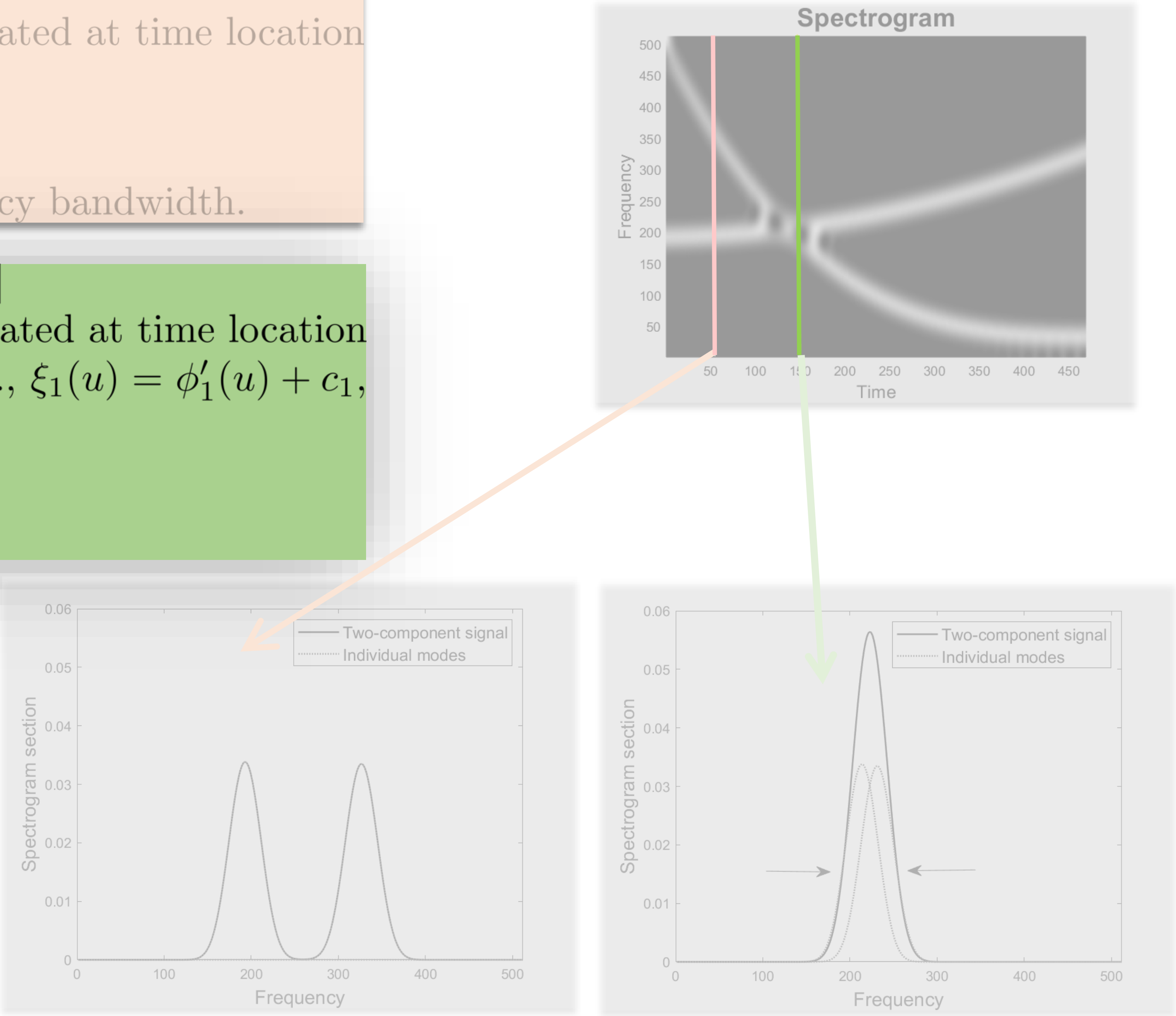

Eq. (1) reduces to

$$\frac{\partial P(u,\xi)}{\partial u} + \phi''(u)\frac{\partial P(u,\xi)}{\partial \xi} - \frac{2a'(u)}{a(u)}P(u,\xi) = 0$$

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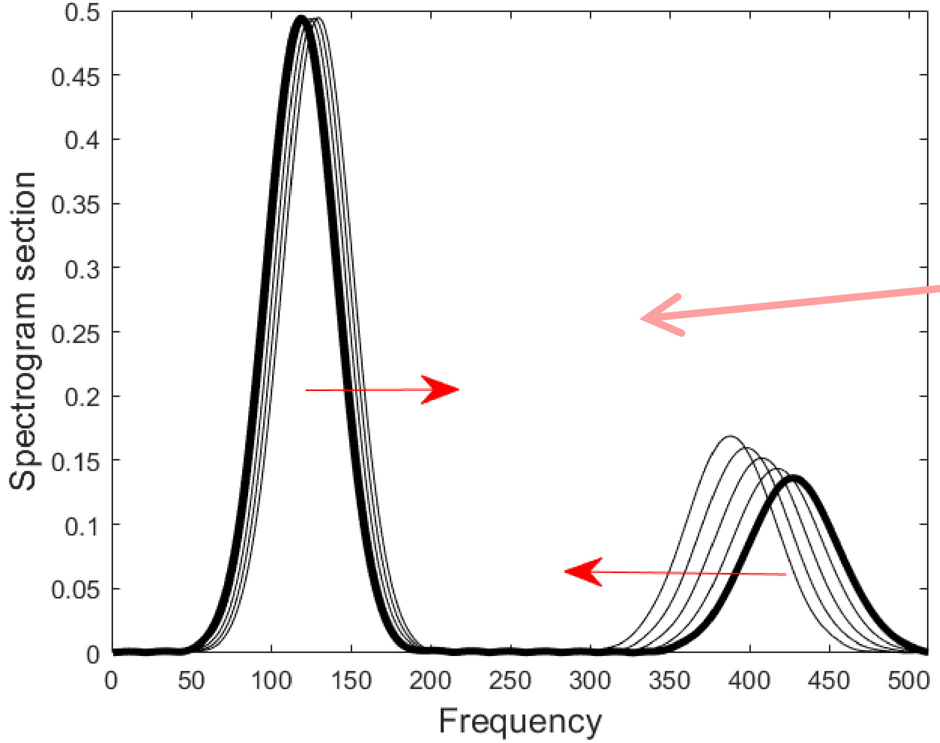

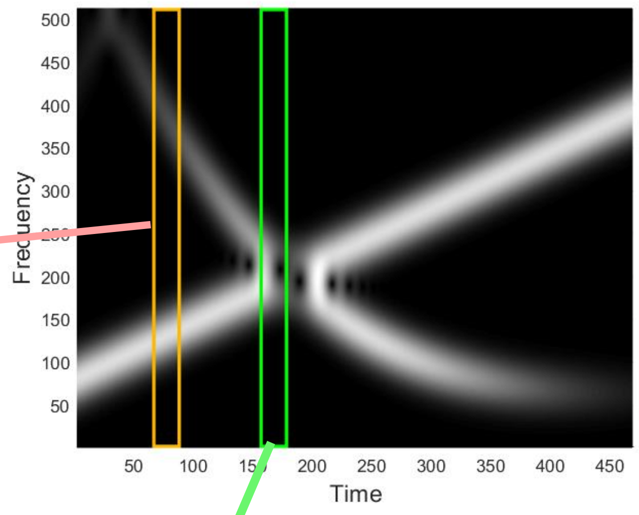

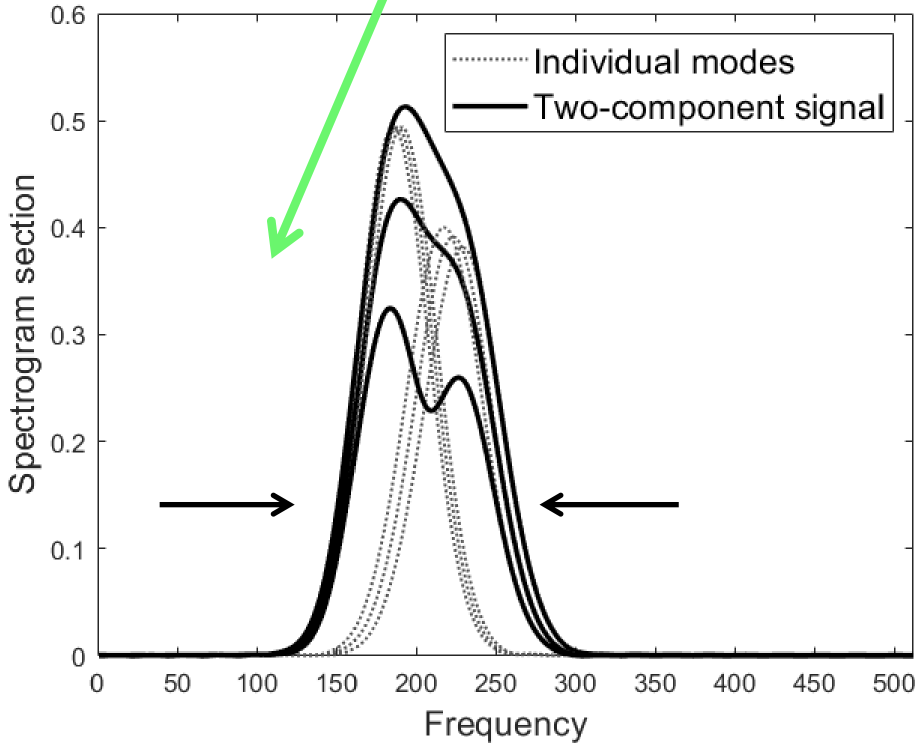

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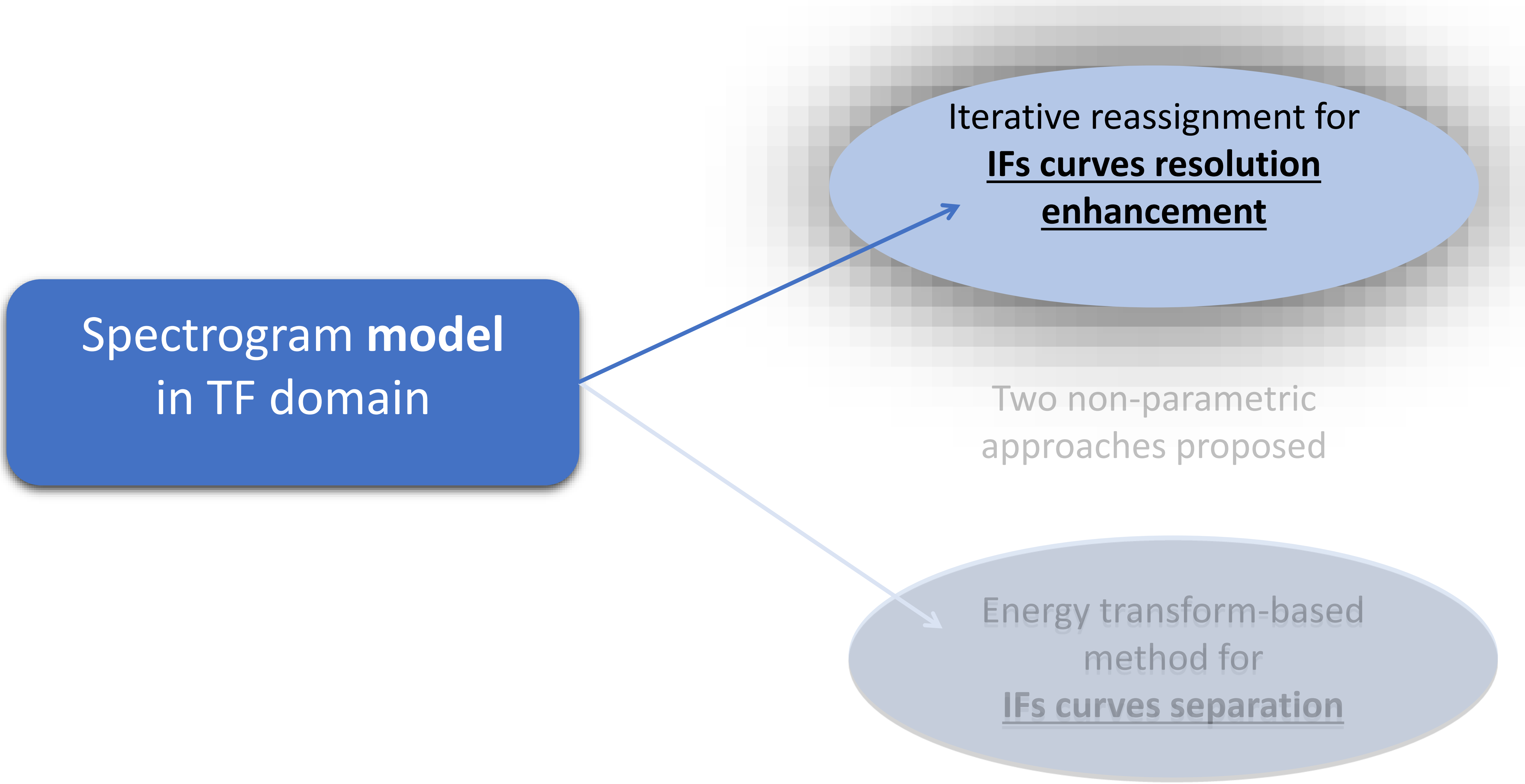
Proposal


Iterative reassignment for
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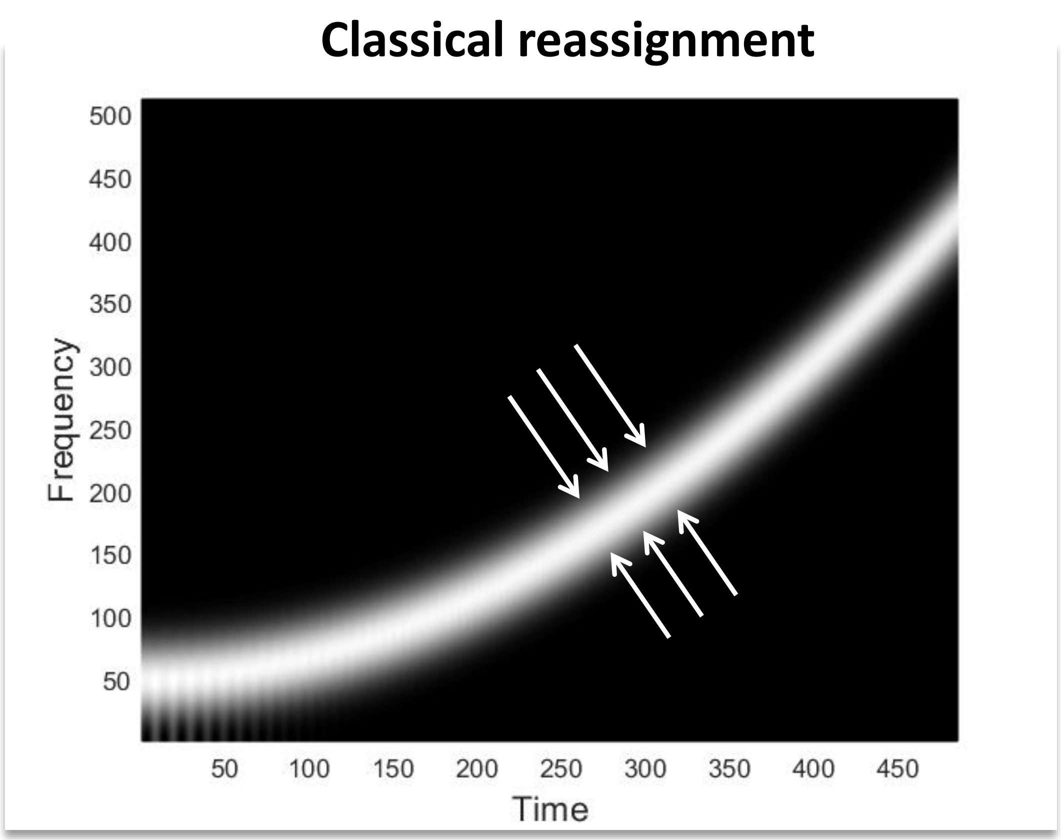
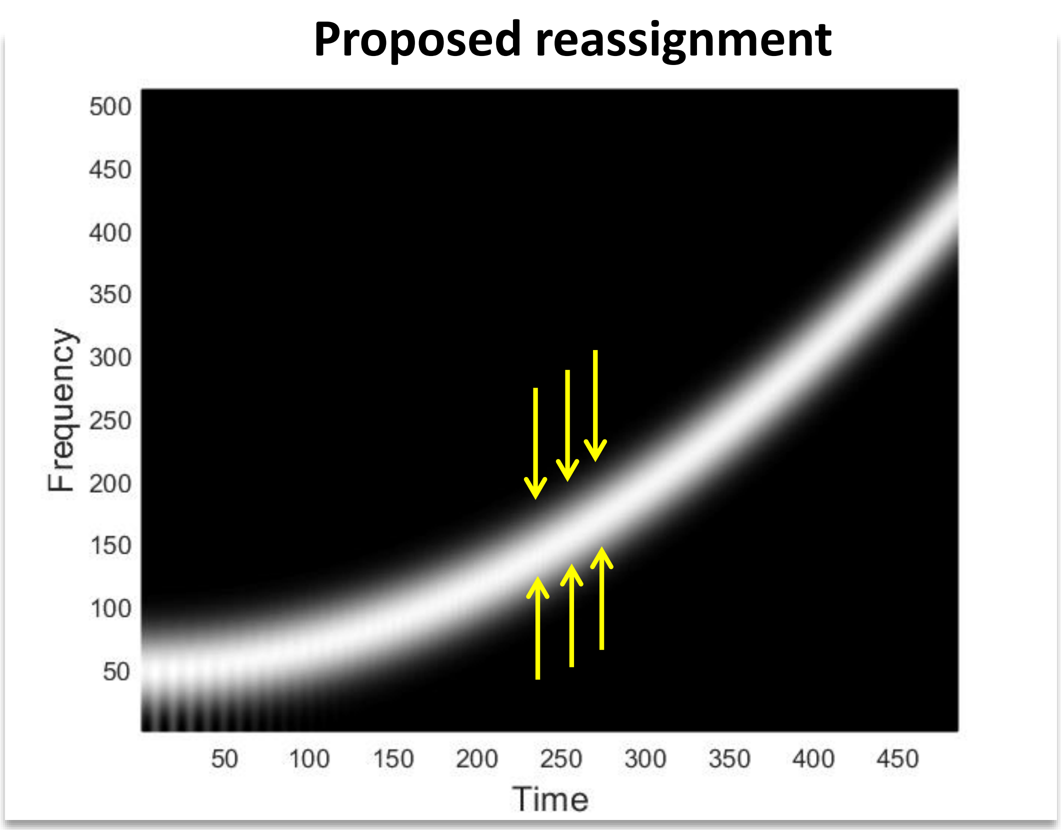
Iterative spectrogram reassignment





$$\hat{u}_f(u,\xi) = u + \Re\left(\frac{S_f^{tg}(u,\xi)}{S_f^{g}(u,\xi)}\right)$$

$$\hat{\xi}_f(u,\xi) = \xi - \Im\left(\frac{S_f^{g'}(u,\xi)}{S_f^{g}(u,\xi)}\right)$$

?

Spectrogram model

Proposition 2. The spectrogram $P(u,\xi)$ of a monocomponent signal $f(t) = a(t)\cos\phi(t)$ satisfies the following advection equation

$$\frac{\partial P(u,\xi)}{\partial u} + \phi''(u)\frac{\partial P(u,\xi)}{\partial \xi} - \cancel{\frac{2a'(u)}{a(u)}P(u,\xi)} = 0 \quad \forall\, u \in supp\{f\}, \tag{2}$$

whose characteristic curves are

$$\mathcal{C}_{c,\phi} \,:\, \xi(u) = \phi'(u) + c,$$

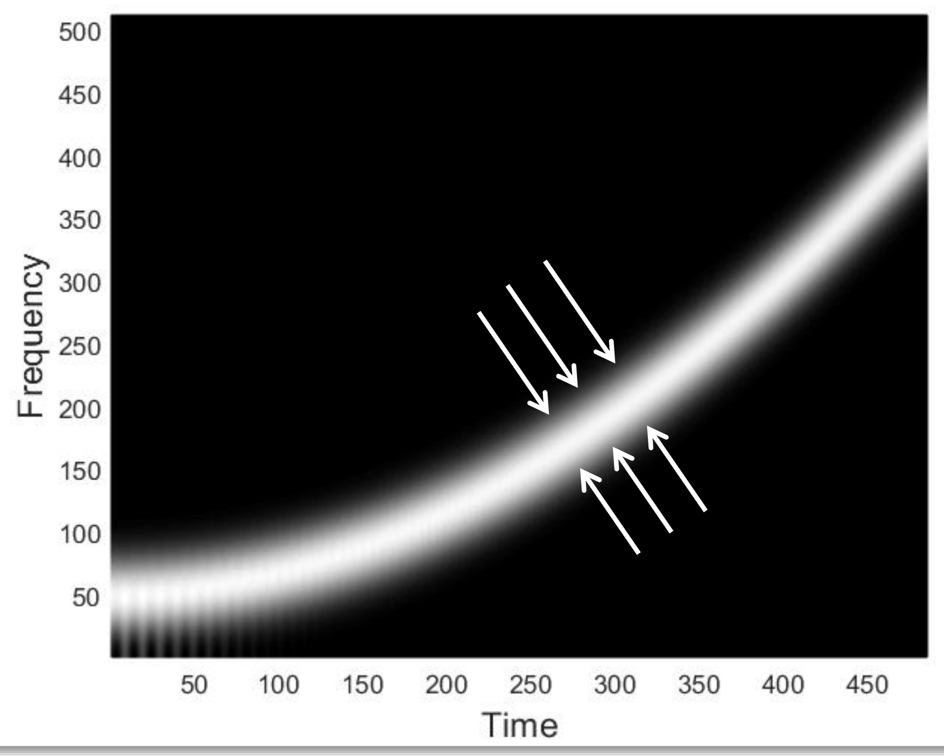
with $c = \xi_0 - \phi'(u_0)$ and (u_0, ξ_0) is a point in the TF plane.

$$\frac{\partial P(u,\xi)}{\partial \xi} = 0$$

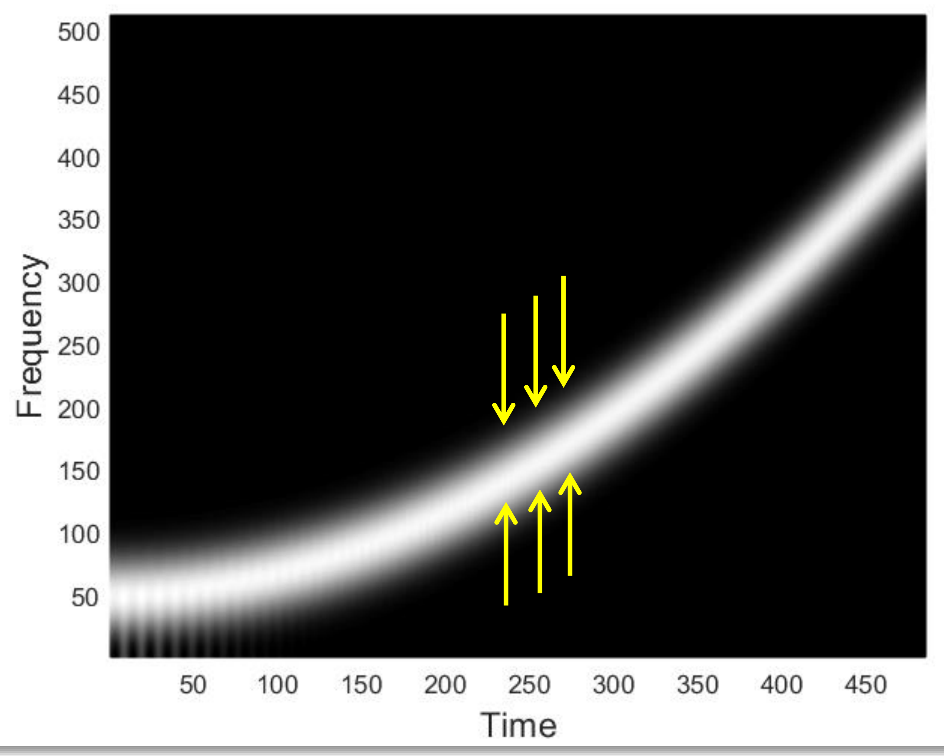
RIDGE POINTS CHARACTERIZATION

Iterative spectrogram reassignment

Classical reassignment



Proposed reassignment



$$\hat{u}_f(u,\xi) = u + \Re\left(\frac{S_f^{tg}(u,\xi)}{S_f^{g}(u,\xi)}\right)$$

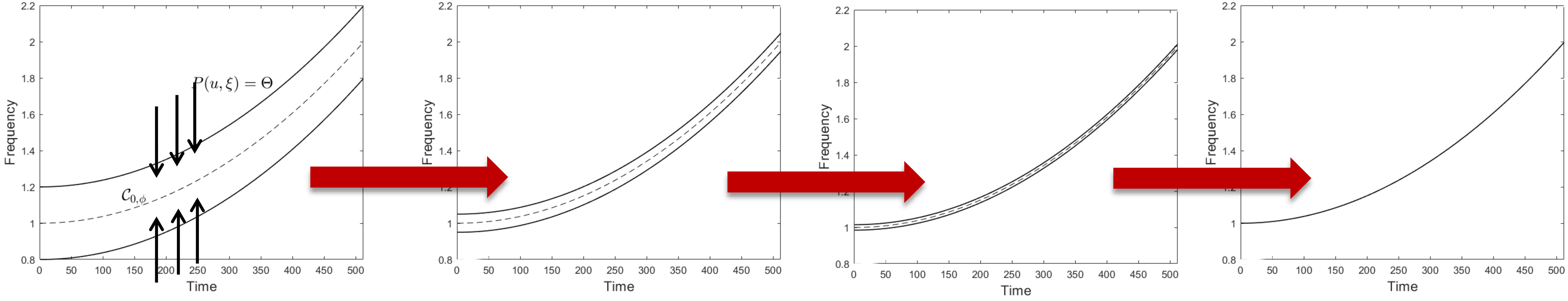
$$\hat{\xi}_f(u,\xi) = \xi - \Im\left(\frac{S_f^{g'}(u,\xi)}{S_f^{g}(u,\xi)}\right)$$

$$\boxed{\frac{\partial P(u,\xi)}{\partial \xi} = 0} \Leftarrow \text{Newton-like method}$$

Iterative spectrogram reassignment

$$\xi_{k+1} = \xi_k \pm \Delta_k(P(u,\xi_k), g),\ k \geq 0,\ \text{fixed } u$$

the shift Δ_k is determined from the data



Iterative spectrogram reassignment

Proposition 3. Let us consider $f(t) = a\cos\phi(t)$ and its normalized spectrogram $p(u,\xi) = \frac{\sqrt{P(u,\xi)}}{\frac{\sqrt{s}}{2}a_i\hat{g}(0)}$. In addition, let us consider ξ_0 s.t. $\hat{g}''(\xi_0 - \xi_R) < 0$, with $\xi_R := \phi'(u)$, and let define $\varphi(\xi) = \xi + sign(p_\xi(\xi))\sqrt{\frac{1-p(\xi)}{|\hat{g}''(0)|}}$ and the sequence

$$\xi_{k+1} = \xi_k + \alpha\, sign(p_\xi(\xi_k))\sqrt{\frac{p(\varphi(\xi_k)) - p(\xi_k)}{|\hat{g}''(0)|}}, \quad k \geq 0, \tag{3}$$

with $\alpha \in (0, 2\sqrt{1-\varphi'^2(\tau)})$, $\tau \in (\xi_0, \varphi(\xi_0))$. Then the sequence $\{\xi_k\}_k$ converges to the ridge point ξ_R.

Moreover, if $\alpha = \left(\frac{1}{\sqrt{2}} - \frac{1}{4}\right)^{-\frac{1}{2}}$ the convergence is at least quadratic.

Iterative spectrogram reassignment

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Newton-like method with «fixed tangent»

Iterative spectrogram reassignment

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Convergence interval

Iterative spectrogram reassignment

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Fast convergence

Iterative spectrogram reassignment

Proposition 3. Let us consider $f(t) = a\cos\phi(t)$ and its normalized spectrogram $p(u,\xi) = \frac{\sqrt{P(u,\xi)}}{\frac{\sqrt{s}}{2} a_i \hat{g}(0)}$. In addition, let us consider ξ_0 s.t. $\hat{g}''(\xi_0 - \xi_R) < 0$, with $\xi_R := \phi'(u)$, and let define $\varphi(\xi) = \xi + sign(p_\xi(\xi))\sqrt{\frac{1-p(\xi)}{|\hat{g}''(0)|}}$ and the sequence

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with $\alpha \in (0, 2\sqrt{1-\varphi'^2(\tau)})$, $\tau \in (\xi_0, \varphi(\xi_0))$. Then the sequence $\{\xi_k\}_k$ converges to the ridge point ξ_R.

Moreover, if $\alpha = \left(\frac{1}{\sqrt{2}} - \frac{1}{4}\right)^{-\frac{1}{2}}$ the convergence is at least quadratic.

APPLICATION to MULTICOMPONENT signals?

By involving the **characteristic curves such that WSC is met**, interference effects are negligible and the problem reduces to the monocomponent case.

Problem: if the modes cross in the TF plane, there exists a region where after one iteration the WSC is no more satisfied.

Iterative spectrogram reassignment

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APPLICATION to MULTICOMPONENT signals?

By involving the **characteristic curves such that WSC is met**, interference effects are negligible and the problem reduces to the monocomponent case.

Problem: if the modes cross in the TF plane, there exists a region where after one iteration the WSC is no more satisfied.

Solution: **Convergence properties are exploited for detecting this region**.

Relaxed method

$$\xi_{k+1} = \xi_k + sign(p_\xi(\xi_0))\left(1 - \frac{1}{\sqrt{2}}\right)\sqrt{\frac{1-p(\xi_0)}{|\hat{g}''(0)|}}, \quad k = 1, 2. \tag{4}$$

Iterative spectrogram reassignment

Proposition 3. Let us consider $f(t) = a\cos\phi(t)$ and its normalized spectrogram $p(u,\xi) = \frac{\sqrt{P(u,\xi)}}{\frac{\sqrt{s}}{2} a_i \hat{g}(0)}$. In addition, let us consider ξ_0 s.t. $\hat{g}''(\xi_0 - \xi_R) < 0$, with $\xi_R := \phi'(u)$, and let define $\varphi(\xi) = \xi + sign(p_\xi(\xi))\sqrt{\frac{1-p(\xi)}{|\hat{g}''(0)|}}$ and the sequence

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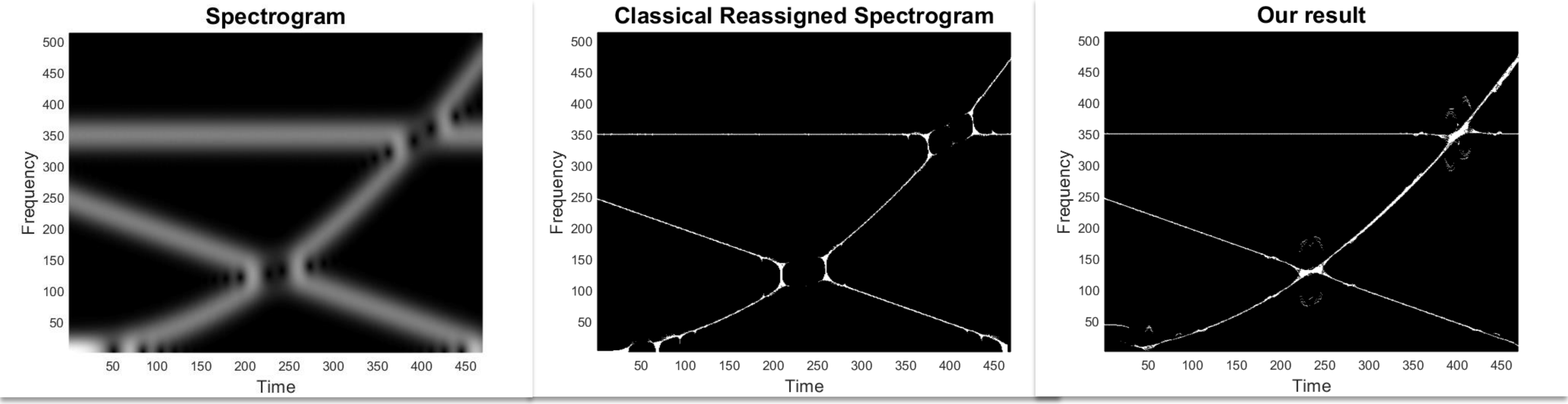
Moreover, if $\alpha = \left(\frac{1}{\sqrt{2}} - \frac{1}{4}\right)^{-\frac{1}{2}}$ the convergence is at least quadratic.

Further details:
[3] Bruni, V., Tartaglione, M., Vitulano, D., *A Fast and Robust Spectrogram Reassignment Method*, Mathematics, 7(4), 358, **2019**
[4] Bruni, V., Tartaglione, M., Vitulano, D., *An iterative approach for spectrogram reassignment of frequency modulated multicomponent signals*, Mathematics and Computers in Simulation, **2019**.

Iterative spectrogram reassignment



benefits: computational effort (convergence reached in 2-3 iterations, only 1 STFT required),
robustness to interference

issues: limited to constant amplitude signals.

[4] Bruni, V., Tartaglione, M., Vitulano, D., *An iterative approach for spectrogram reassignment of frequency modulated multicomponent signals,* Mathematics and Computers in Simulation, **2019**

Proposal

Iterative reassignment for
<u>IFs curves resolution enhancement</u>

Spectrogram **model** in TF domain

Two non-parametric approaches proposed

Energy transform-based method for
<u>IFs curves separation</u>

Radon-Spectrogram Distribution

Definition. Given a function $F(x,y) \in C^\infty(\mathbb{R}^2;\mathbb{R})$, compactly supported or rapidly decreasing to zero, its Radon Transform (RT) is defined as

$$R_F(r,\theta) = \int_{\mathbb{R}} F(x,y)\,\delta(r - x\cos\theta - y\sin\theta)dx\,dy\,, \quad (r,\theta) \in \mathbb{R} \times [0,\pi)$$

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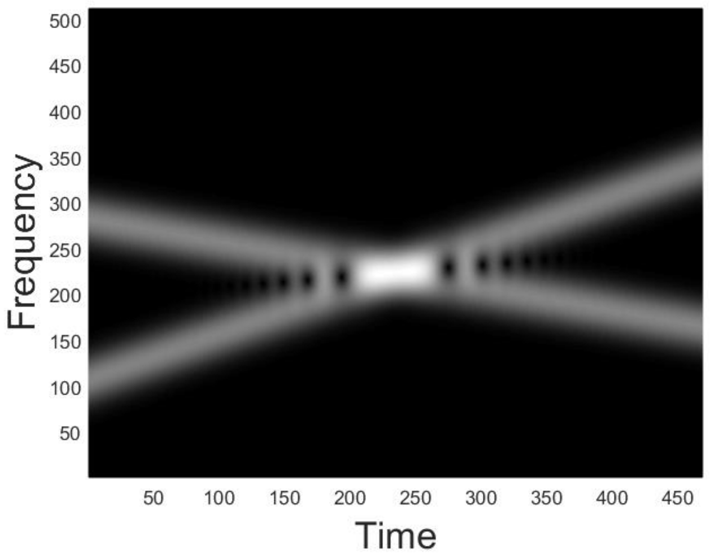
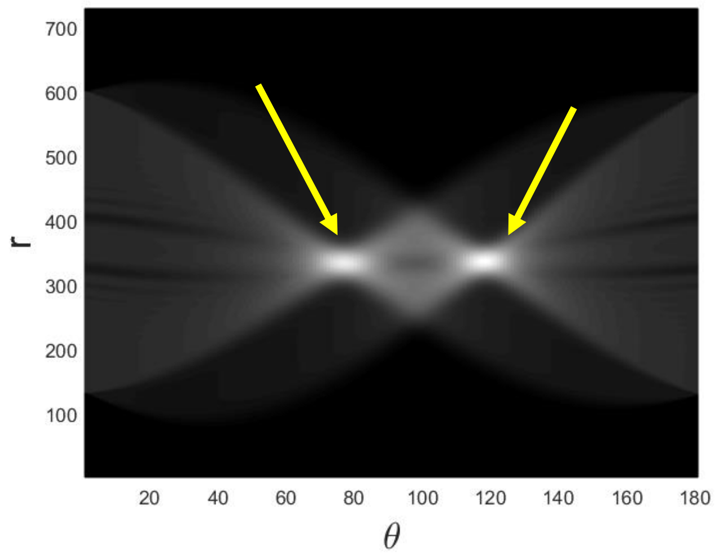
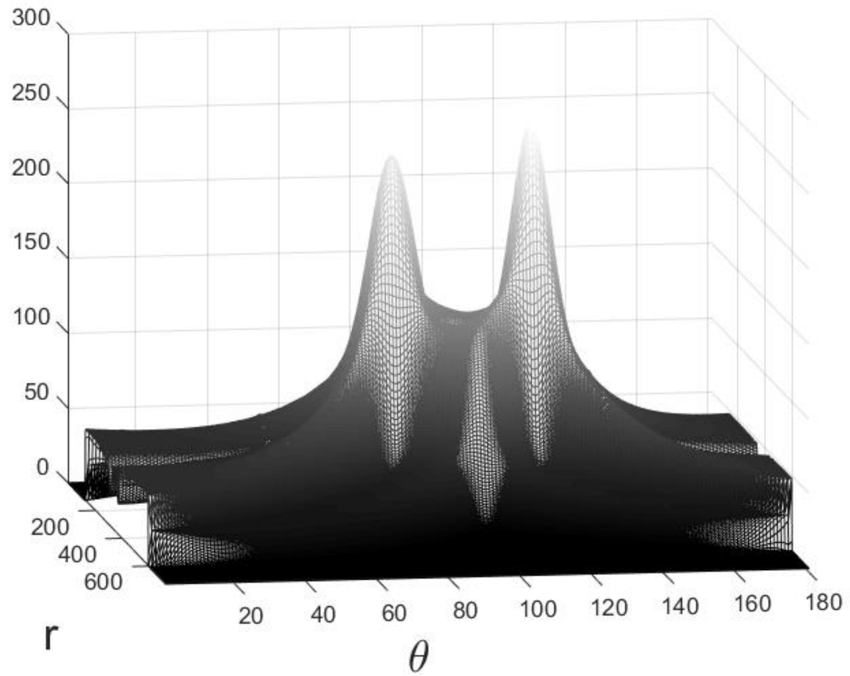
TF DOMAIN → **RADON DOMAIN (view from above)**

RADON DOMAIN (3D view)







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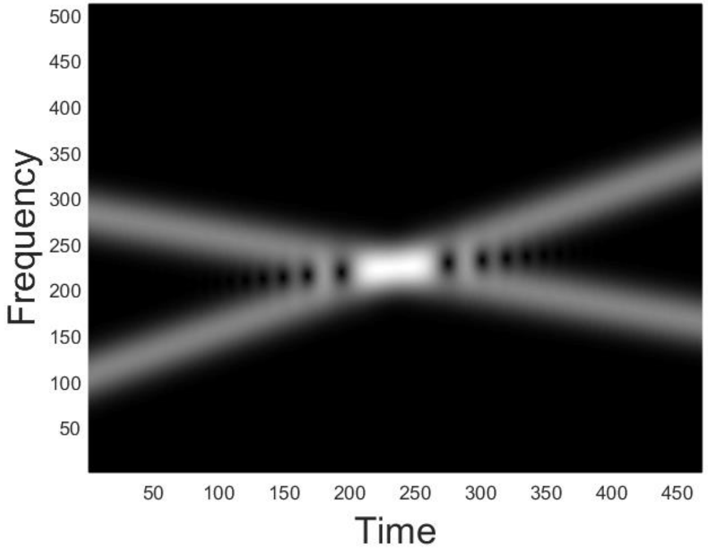
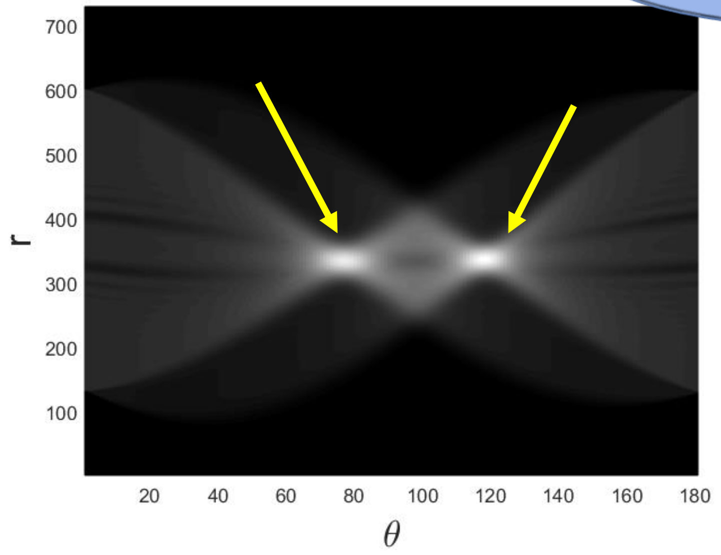
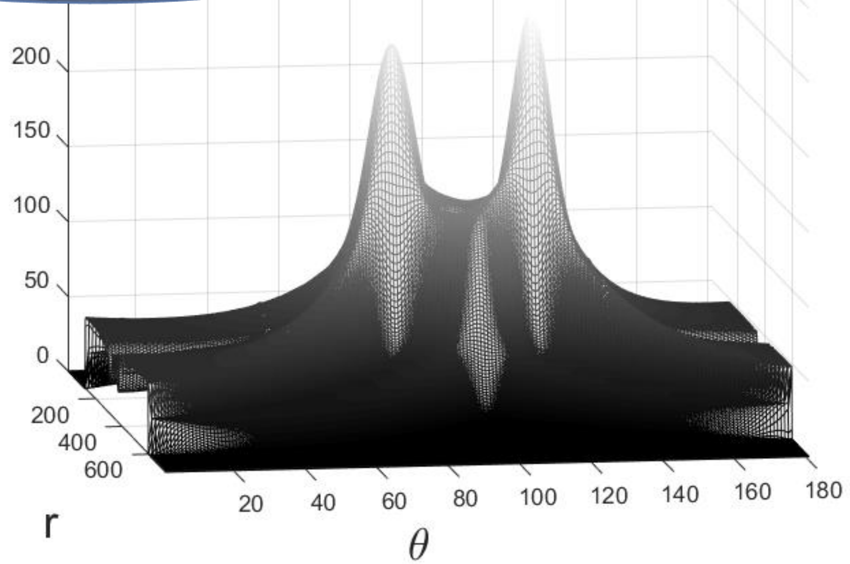
TF DOMAIN → **RADON DOMAIN** (view from above)

RADON DOMAIN (3D view)

Peaks detection

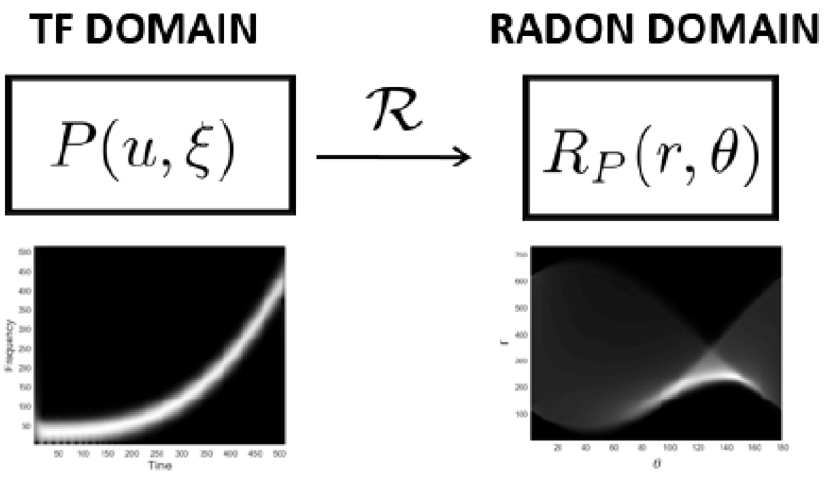






Why Radon-Spectrogram?

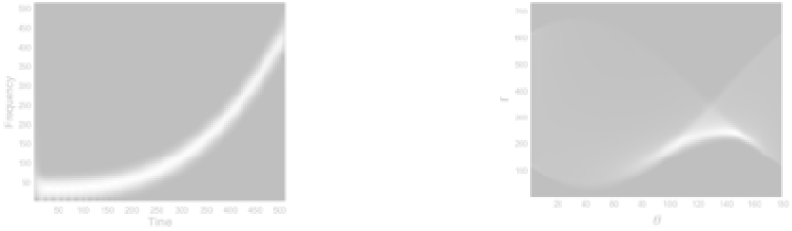
Non-linear IFs?

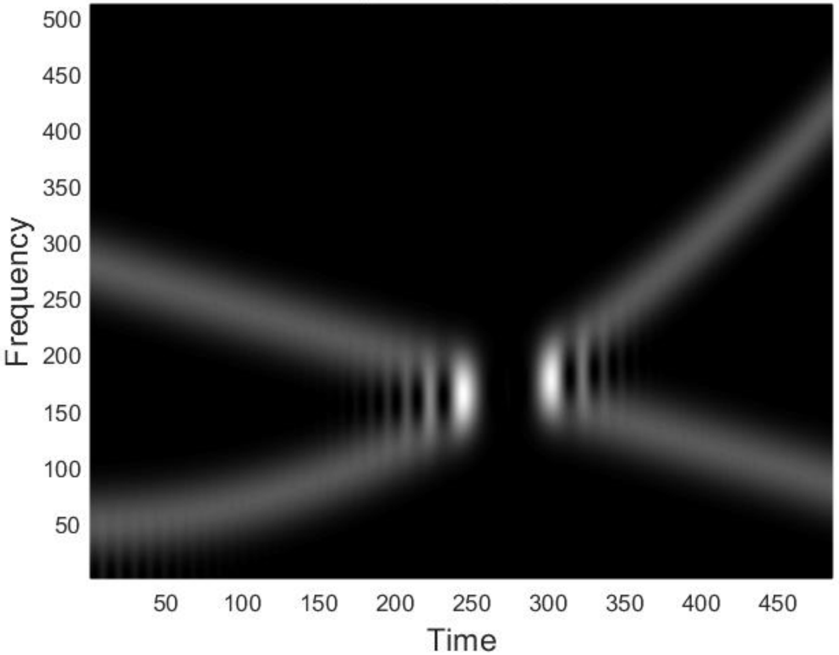


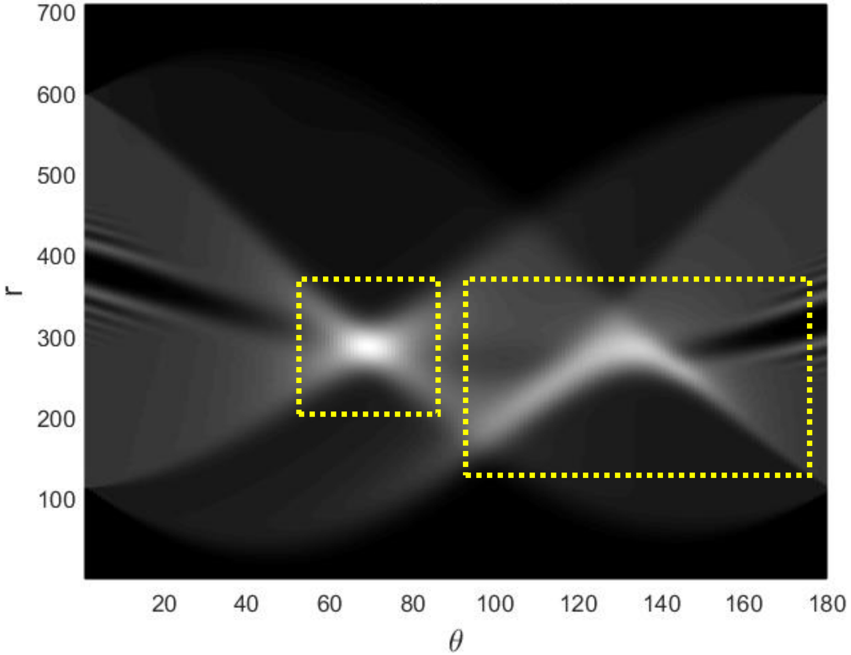
Why Radon-Spectrogram?

TF DOMAIN → **RADON DOMAIN**

$$\boxed{P(u,\xi)} \xrightarrow{\mathcal{R}} \boxed{R_P(r,\theta)}$$

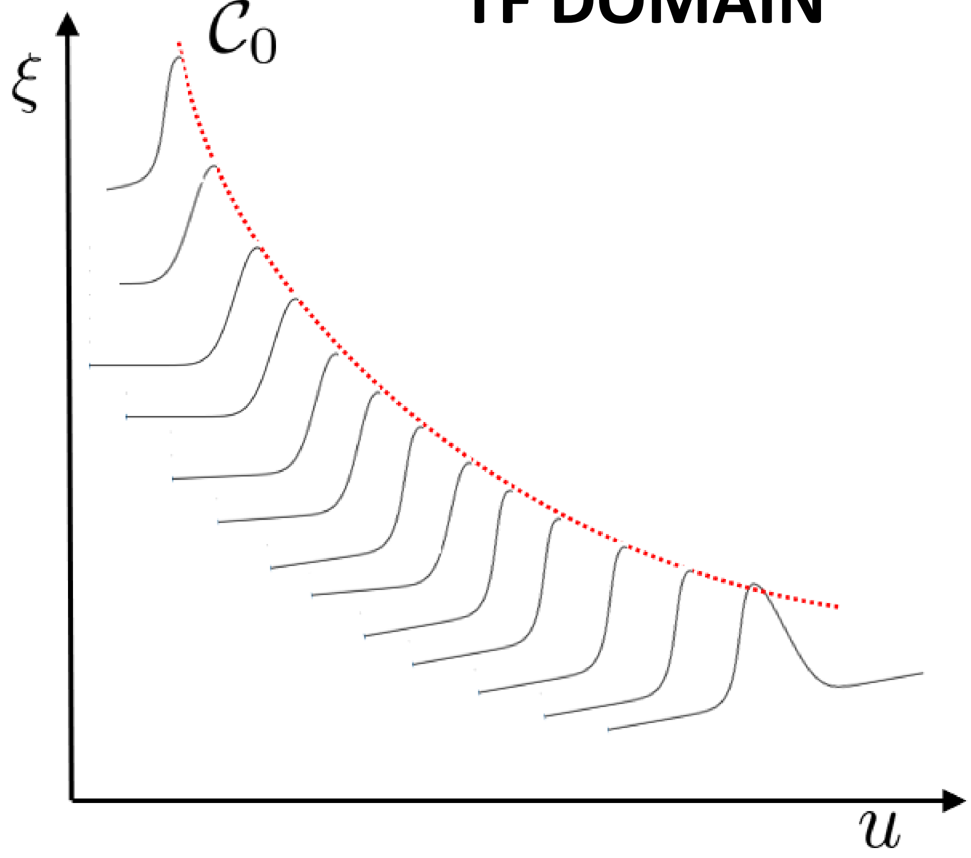






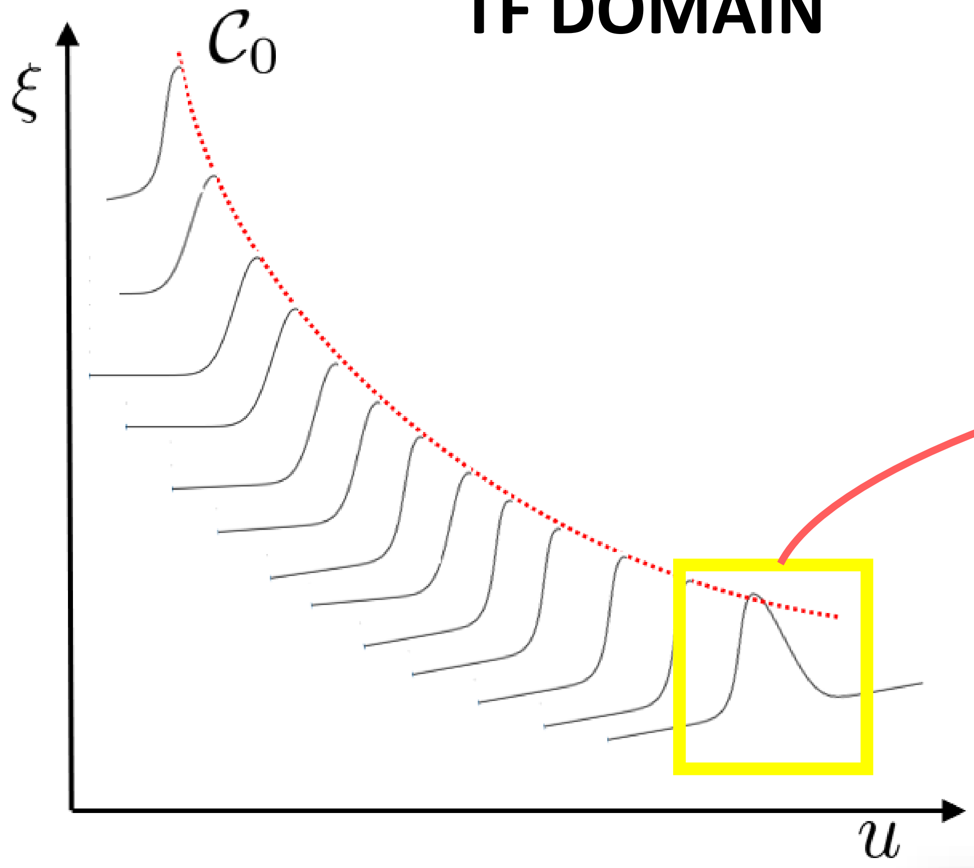
Radon-Spectrogram Model

TF DOMAIN

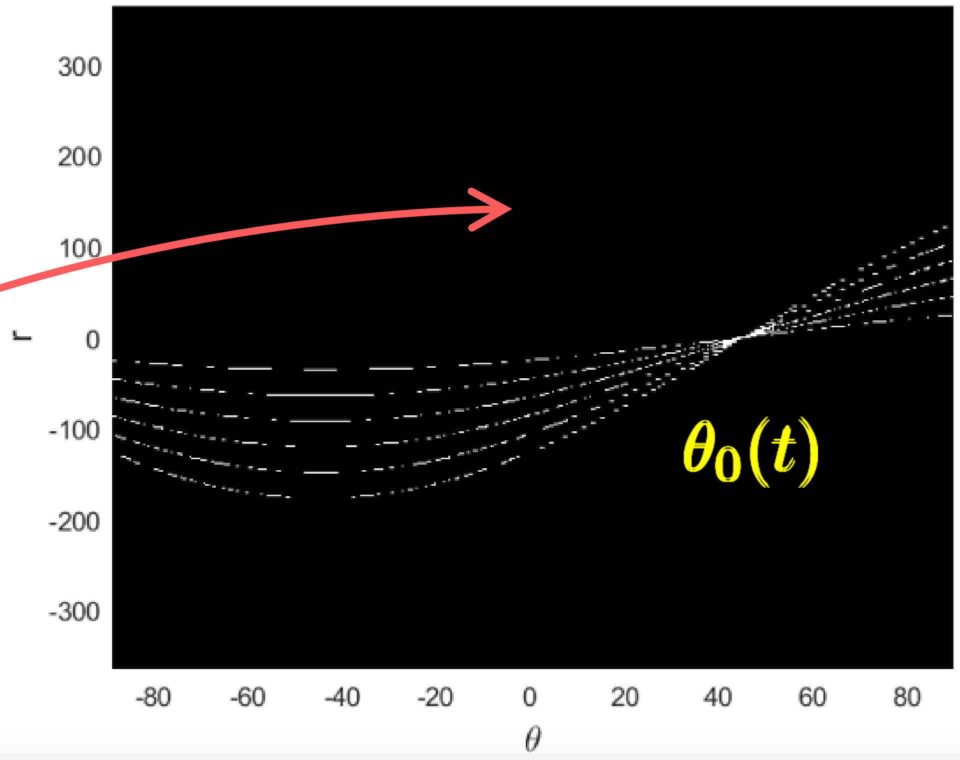


Radon-Spectrogram Model

TF DOMAIN



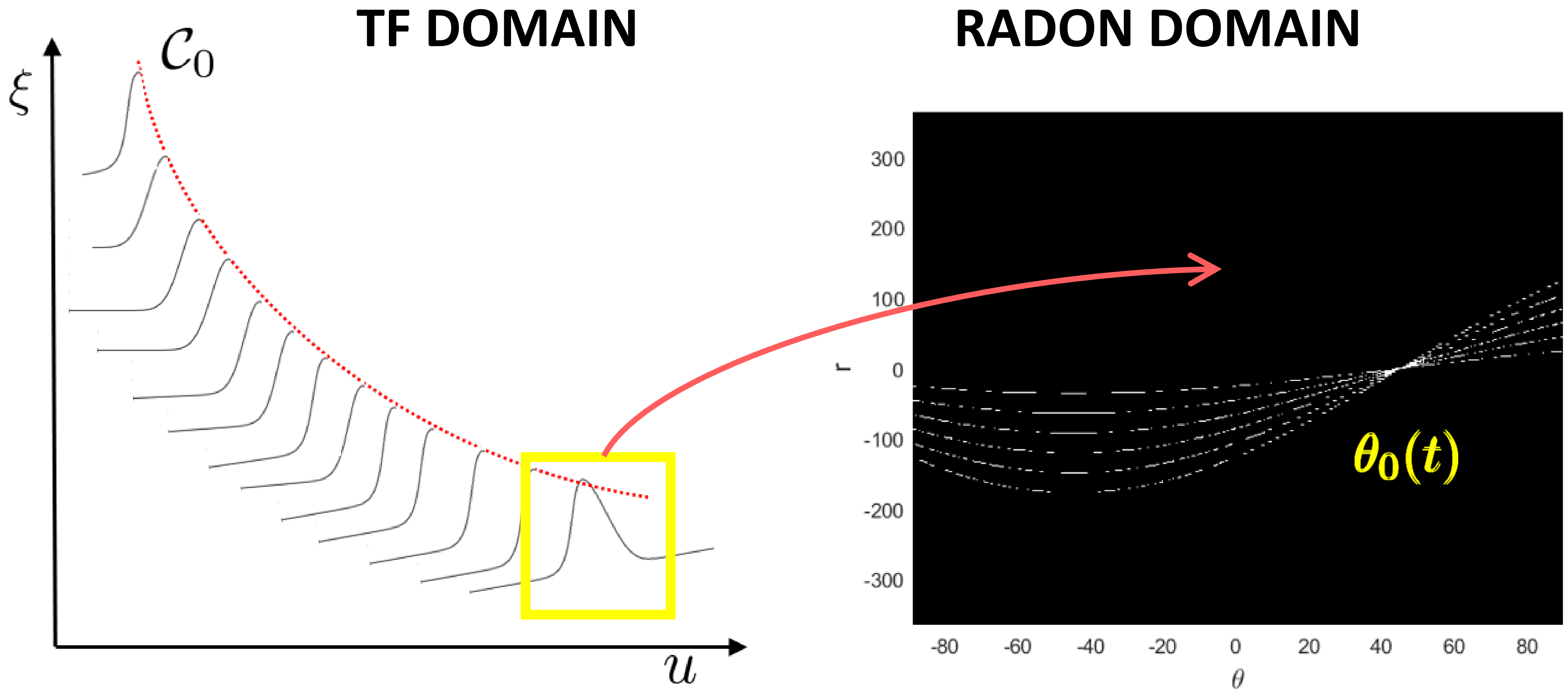
RADON DOMAIN



$$R(r,\theta,t) = \frac{1}{2\pi\cos(\theta-\theta_0)}\,\hat{g}\left(\frac{r - t\cos\theta - \phi'(t)\sin\theta}{\cos(\theta-\theta_0)}\right)$$

$$\theta_0(t) = -\arctan\left(\frac{1}{\phi''(t)}\right)$$

Radon-Spectrogram Model



$$R(r,\theta,t) = \frac{1}{2\pi\cos(\theta-\theta_0)}\,\hat{g}\left(\frac{r-t\cos\theta-\phi'(t)\sin\theta}{\cos(\theta-\theta_0)}\right)$$

$$R(r,\theta) = \int_0^{+\infty} R(r,\theta,t)\,dt - \int_0^{+\infty} \frac{\phi'''(t)\sin^2\theta_0}{\cos(\theta-\theta_0)}(r - t\cos\theta - \phi'(t)\sin\theta)\,R(r,\theta,t)\,dt$$

Radon-Spectrogram Model

Proposition. Let $f(t) = a\cos\phi(t)$ be a constant amplitude FM signal satisfying one of the following assumptions:

(i) $\phi'''(t) = 0$ or $\phi''(t) = 0$, $\forall\, t$;

(ii) $0 < |\phi''(t)| < L_1$, $|\phi'''(t)| \geq L_2 > 0$ and the spectrogram $P(u,\xi)$ is computed with a compactly supported analysis window g whose bandwidth satisfies $\Delta\omega \leq \frac{1+L_1^2}{L_2}$.

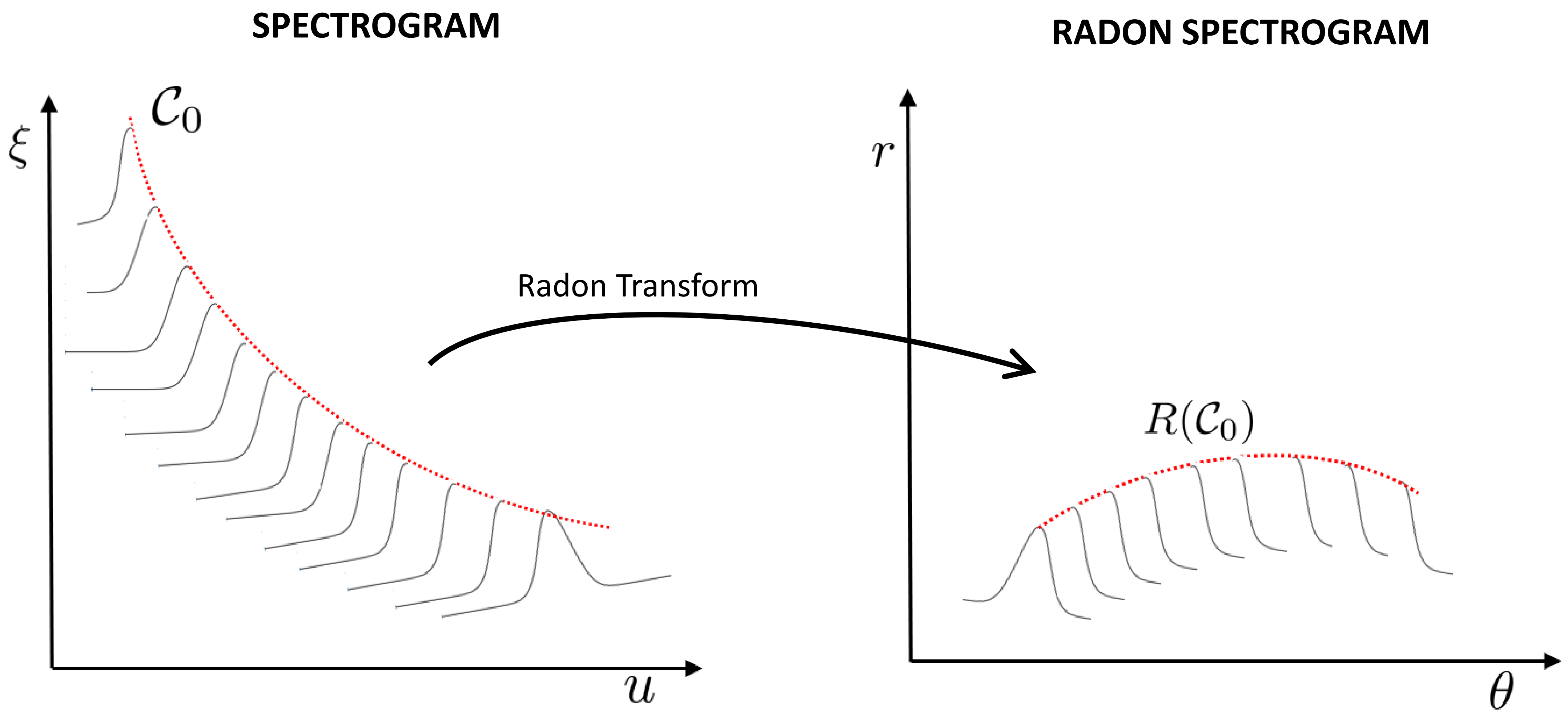
Then, the Radon Spectrogram of $f(t)$ is

$$R(r,\theta) = \int_0^{+\infty} R(r,\theta,t)\,dt - \int_0^{+\infty} \frac{\phi'''(t)\sin^2\theta_0}{\cos(\theta-\theta_0)}(r - t\cos\theta - \phi'(t)\sin\theta)\,R(r,\theta,t)\,dt,\ \forall\,\theta\ :\ |\theta-\theta_0| \in \left[0,\frac{\pi}{2}\right),\forall\, r\in\mathbb{R},$$

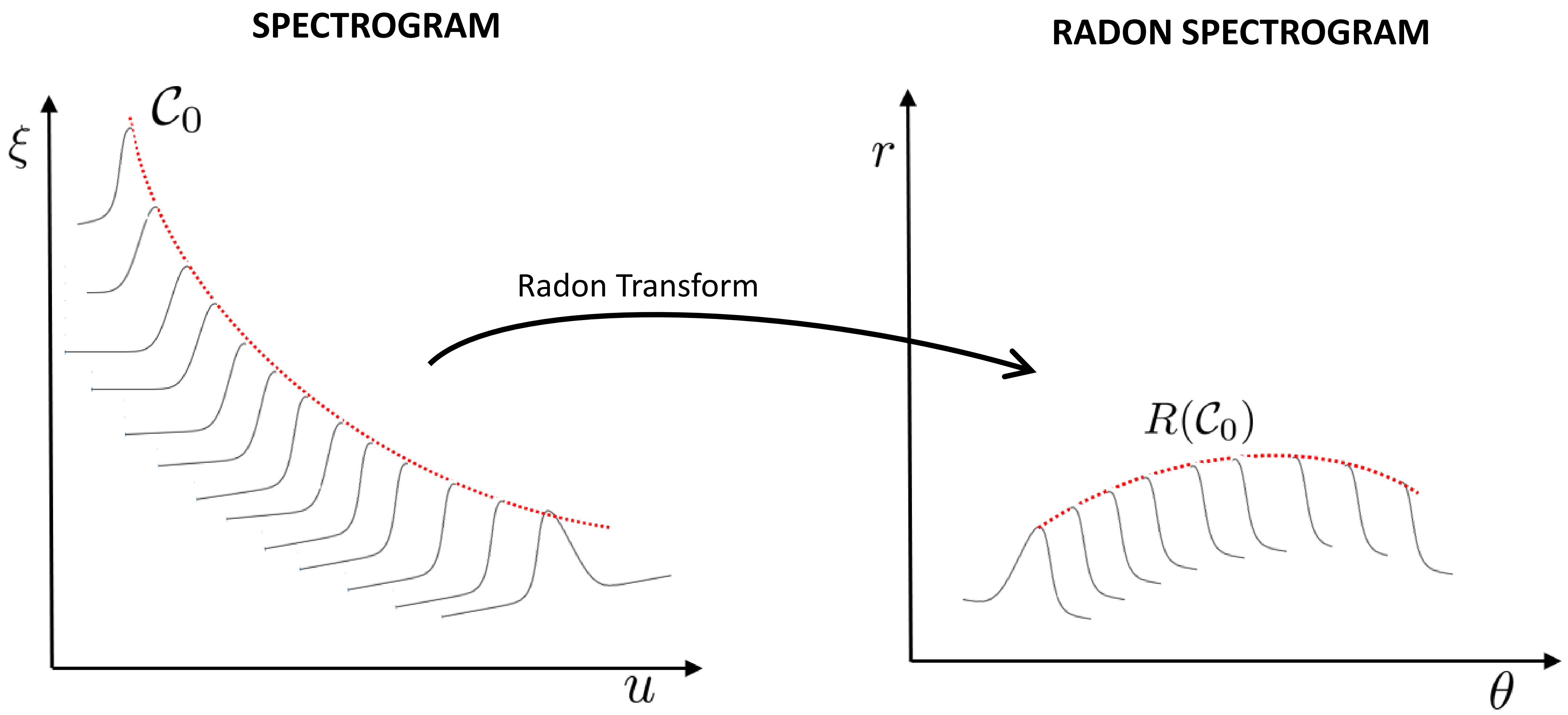
with $\theta_0 = \theta_0(t) = -\arctan\left(\dfrac{1}{\phi''(t)}\right)$,

$$R(r,\theta,t) = \frac{1}{2\pi\cos(\theta-\theta_0)}\,\hat{g}\left(\frac{r - t\cos\theta - \phi'(t)\sin\theta}{\cos(\theta-\theta_0)}\right).$$

Radon-Spectrogram Model

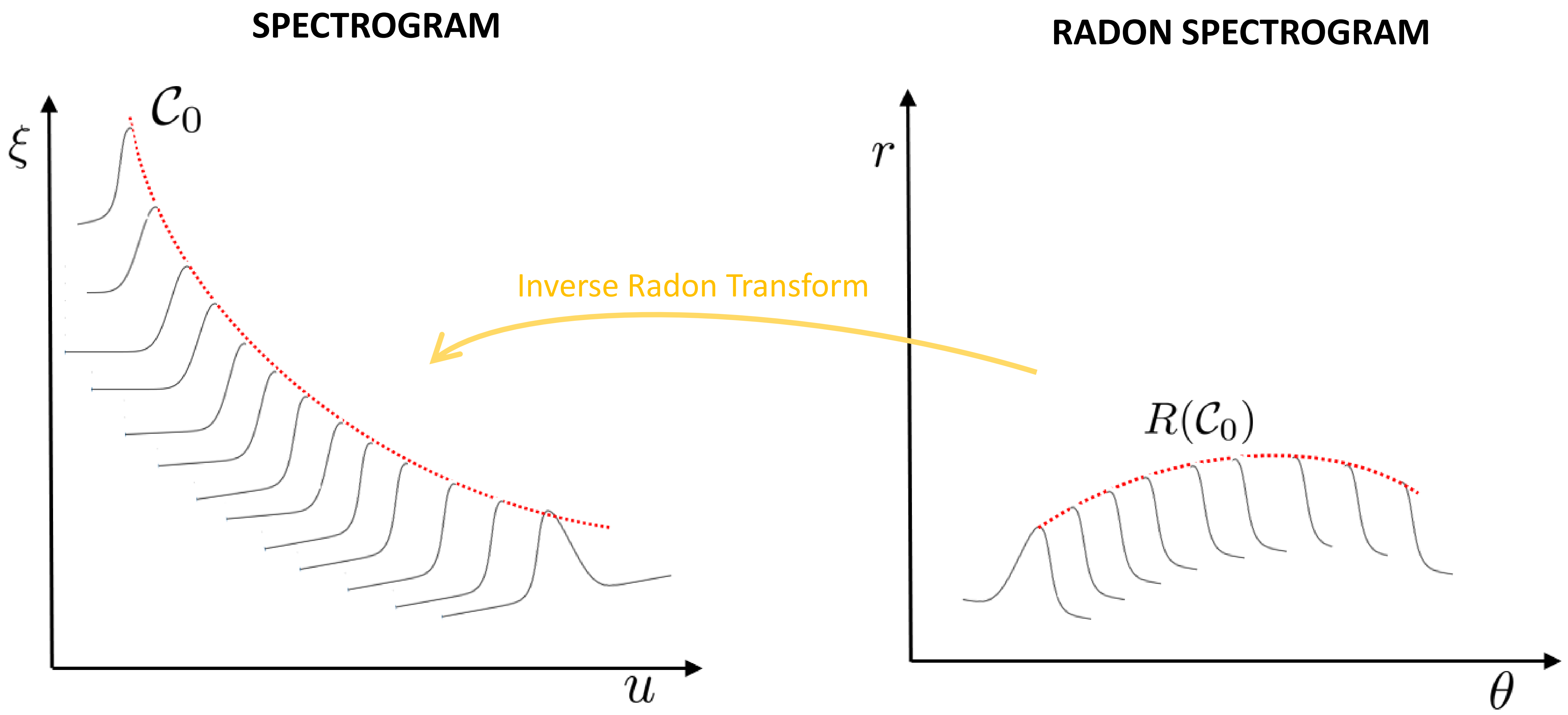


Radon-Spectrogram Model



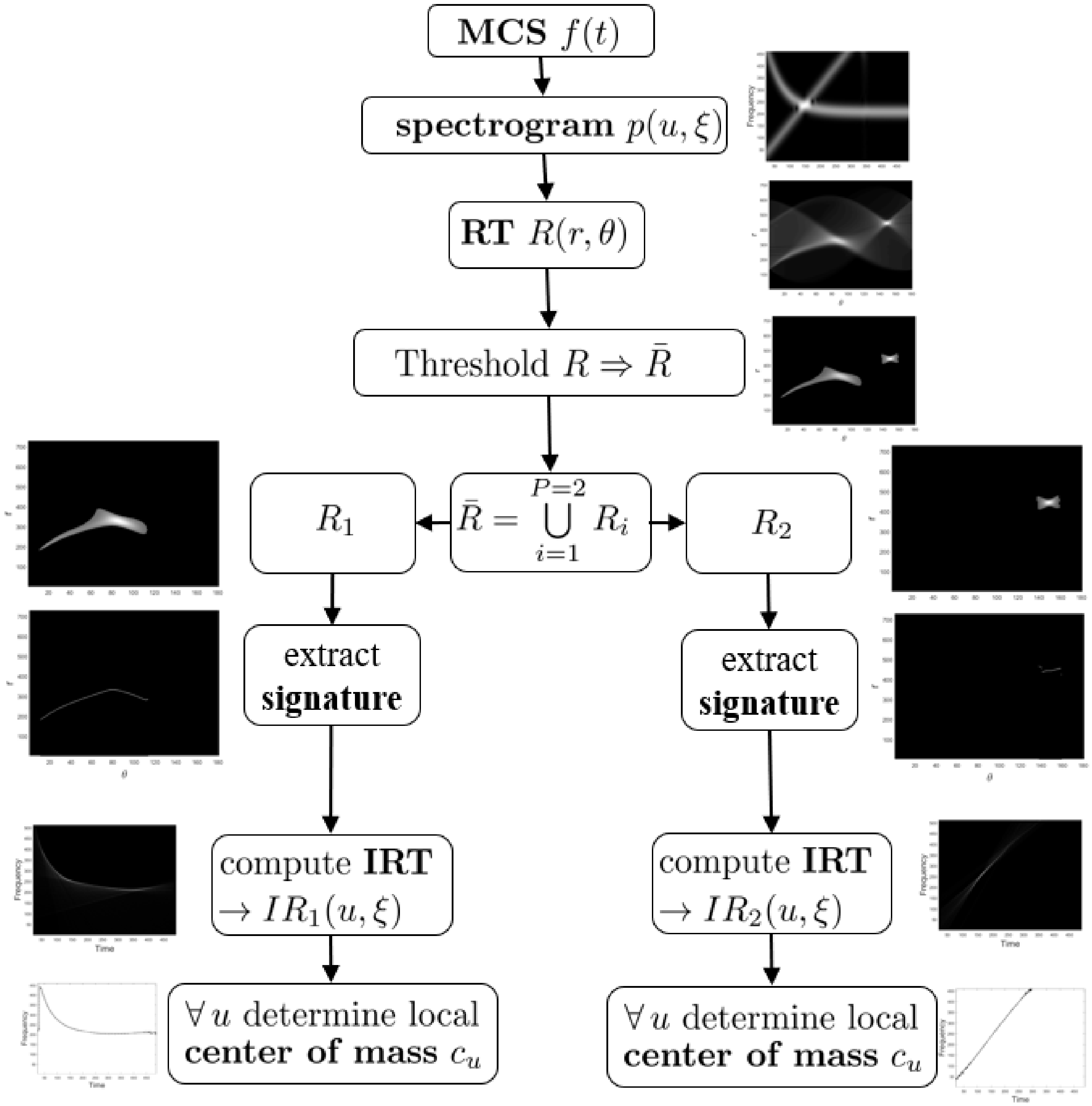
Radon Spectrogram reaches its maximum along the IF curve mapped in the Radon Domain (up to an error depending on $\phi'''(t)$, $t \in supp\{f\}$)

Radon-Spectrogram Model

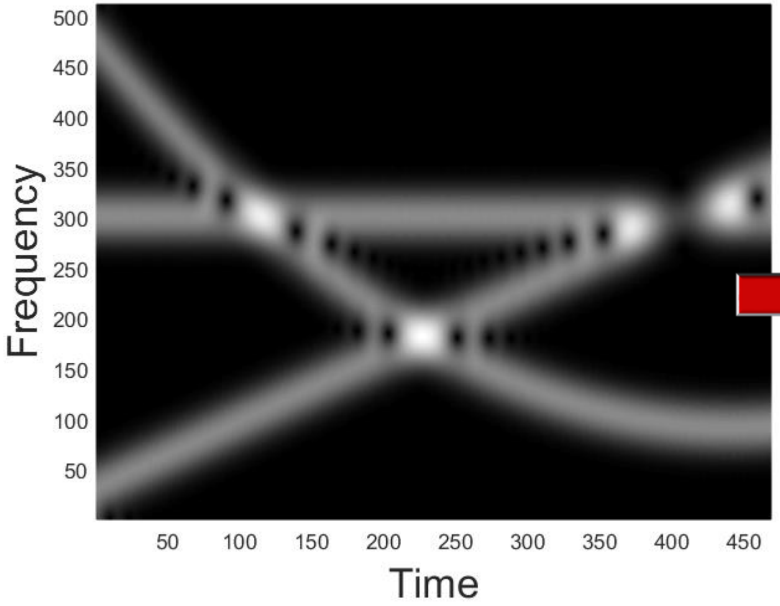


IF curves in TF domain can be **recovered**
by **inverting Radon Transform on Radon maxima**

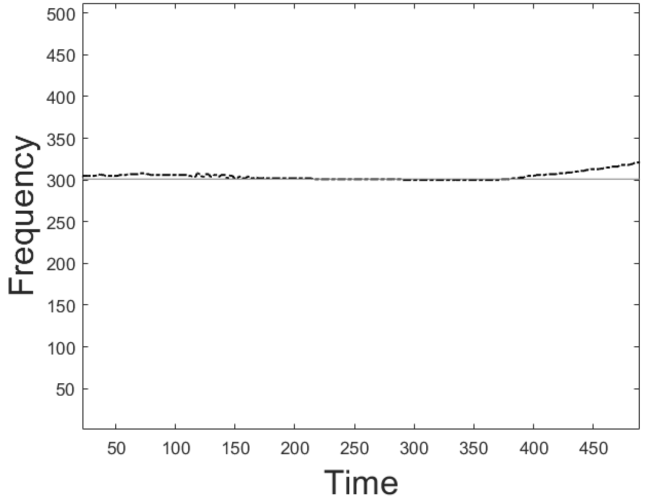
IFs separation



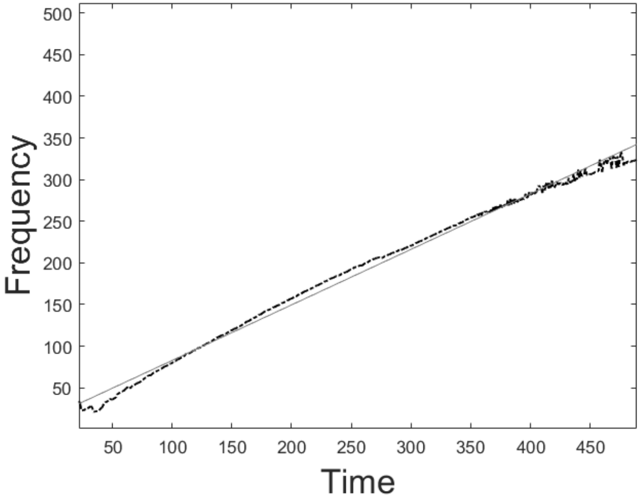
Some results



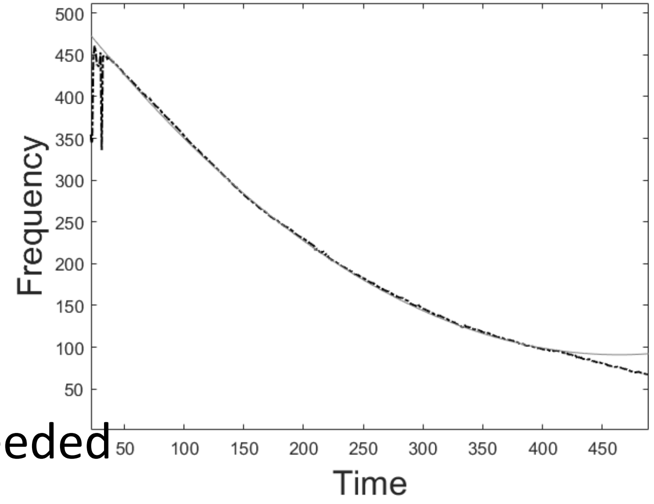
SEPARATED IF1



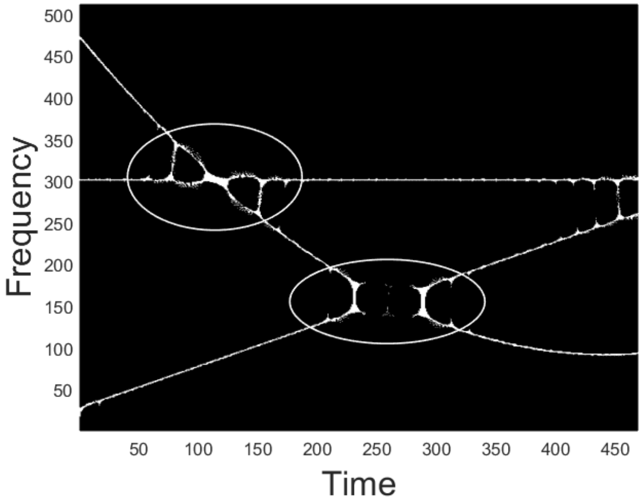
SEPARATED IF2



SEPARATED IF3



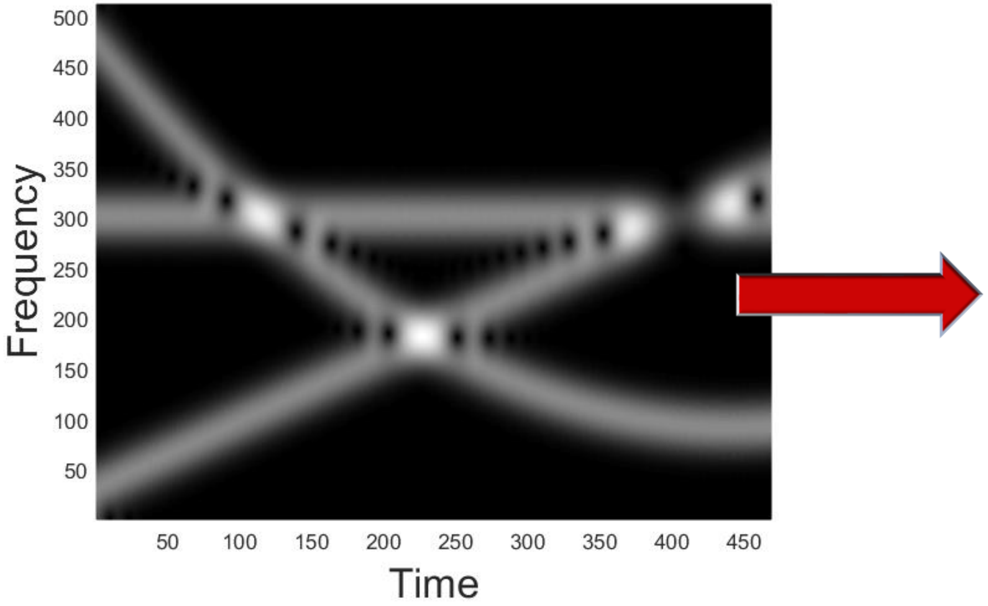
CLASSICAL REASSIGNED SPECTROGRAM



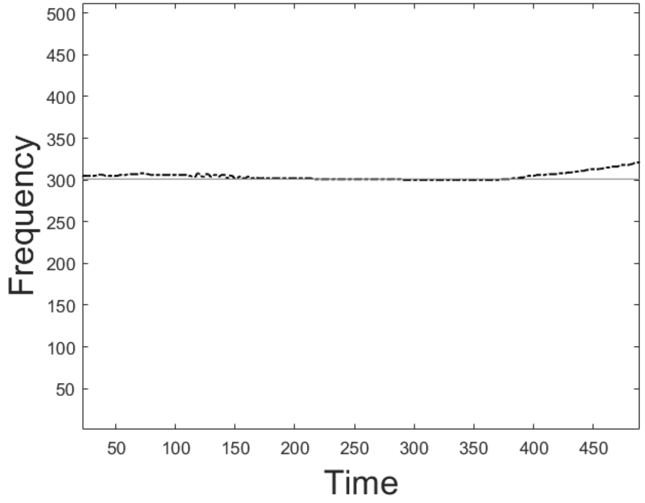
benefits: robustness to interference and noise

issues: more advance thresholding procedure needed

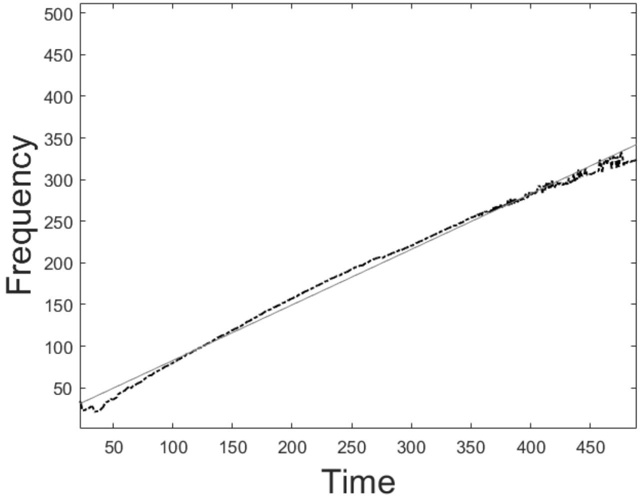
Some results



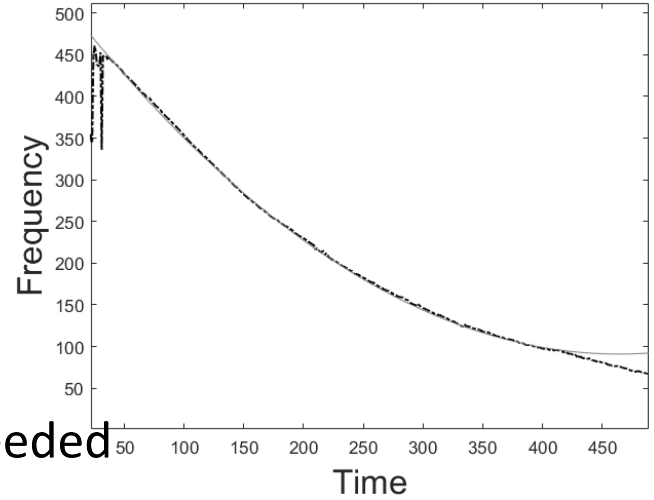
SEPARATED IF1



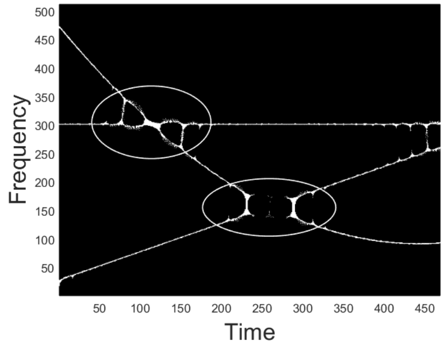
SEPARATED IF2



SEPARATED IF3



CLASSICAL REASSIGNED SPECTROGRAM



benefits: robustness to interference and noise

issues: more advance thresholding procedure needed

[5] Bruni, V., Tartaglione, M., Vitulano, D., *Instantaneous frequency modes separation via a Spectrogram-Radon based approach*. In 2019 11th International Symposium on Image and Signal Processing and Analysis (ISPA), pp. 347-351, IEEE, **2019**

[6] Bruni, V., Tartaglione, M., Vitulano, D., *Radon spectrogram-based approach for automatic IFs separation*, EURASIP Journal on Advances in Signal Proc., **2020**

Conclusions

- The problem of the decomposition / IFs separation of frequency modulated multicomponent signals with interfering modes has been presented
- A spectrogram TF evolution law and a model for Radon-Spectrogram have been proposed
- Advantages : non parametric approach; some limitations in state-of-the-art have been overcome
- **Immediate developments:** time-varying amplitudes

  Bruni, V., Tartaglione, M., Vitulano, D., *A Signal Complexity-Based Approach for AM–FM Signal Modes Counting*. Mathematics, 8, MDPI, 2170, **2020**

  Bruni, V., Tartaglione, M., Vitulano, D., *A pde-Based Analysis of the Spectrogram Image for Instantaneous Frequency Estimation*. Mathematics, 9(3):247, **2021**

  Bruni, V., Tartaglione, M., Vitulano, D., *Automatic interference region detection and ridge curves recovery in AM-FM multicomponent signals*, **in preparation**
- Future perspectives: real-world signals; addressing the open questions appeared

Publications

[1] Bruni, V., Tartaglione, M., Vitulano, D., *On the time-frequency reassignment of interfering modes in multicomponent FM signals.* In 2018 26th European Signal Processing Conference (EUSIPCO), IEEE , pp. 722-726, **2018**

[2] Bruni, V., Tartaglione, M., Vitulano, D., *An iterative spectrogram reassignment of frequency modulated multicomponent signals*. In 15th Meeting On Applied Scientific Computing And Tools (MASCOT), **2018**

[3] Bruni, V., Tartaglione, M., Vitulano, D., *A Fast and Robust Spectrogram Reassignment Method*. Mathematics, 7(4), 358, **2019**

[4] Bruni, V., Tartaglione, M., Vitulano, D., *An iterative approach for spectrogram reassignment of frequency modulated multicomponent signals*. Mathematics and Computers in Simulation, **2019**

[5] Bruni, V., Tartaglione, M., Vitulano, D., *Instantaneous frequency modes separation via a Spectrogram-Radon based approach*. In 2019 11th International Symposium on Image and Signal Processing and Analysis (ISPA), pp. 347-351, IEEE, **2019**

[6] Bruni, V., Tartaglione, M., Vitulano, D., *Radon spectrogram-based approach for automatic IFs separation.* EURASIP Journal on Advances in Signal Processing, **2020**

[7] Bruni, V., Tartaglione, M., Vitulano, D., *A Signal Complexity-Based Approach for AM–FM Signal Modes Counting*. Mathematics, 8, MDPI, 2170, **2020**

[8] Bruni, V., Tartaglione, M., Vitulano D., *Coherence of PRNU weighted estimations for improved source camera identification*, Multimedia Tools and Applications, **2021**

[9] Bruni, V., Tartaglione, M., Vitulano, D., *A pde-Based Analysis of the Spectrogram Image for Instantaneous Frequency Estimation*. Mathematics, 9(3):247, **2021**