SUCCESSIONI E SERIE NUMERICHE

Successioni numeriche

- Calcolare i seguenti limiti di successioni:

1) $\displaystyle\lim_{n\to+\infty}\frac{1+\sqrt{n}}{(1+n^2)n^\alpha}$, $(\alpha\in I\!R)$

2) $\displaystyle\lim_{n\to+\infty}\frac{\left(\frac{x+10}{5}\right)^n}{n^2}$, $(x\in I\!R)$

3) $\displaystyle\lim_{n\to+\infty}\frac{\sqrt[3]{1+\frac{1}{n^2}}-1}{\mathrm{e}^{n/2}\log\left(1+\frac{1}{n^3}\right)}$

4) $\displaystyle\lim_{n\to+\infty}\left(1+\tan\frac{1}{n}\right)^{\log(n^n)}$

5) $\displaystyle\lim_{n\to+\infty}\frac{\log\left|2-\left[1+\frac{1}{(n+5)^\alpha}\right]^{n^2}\right|}{\frac{1}{n}}$, $(\alpha\in I\!R)$

6) $\displaystyle\lim_{n\to+\infty}\frac{\sin\left[\log\left(1+\frac{3}{n}\right)\right]-\log\left(1+\frac{3}{\sqrt{n}}\right)}{\left(\frac{1}{n}\right)^\alpha}$, $(\alpha\in I\!R)$

7) $\displaystyle\lim_{n\to+\infty}\frac{\log(1+\mathrm{e}^n)}{\sqrt{1+n^2}}$

8) $\displaystyle\lim_{n\to+\infty}\frac{2^{\frac{\log n}{n}}-1}{i^{2n}+\log n^2}\,n$

9) $\displaystyle\lim_{n\to+\infty}3^n n!\log(1+2^{-n})$.

Serie Numeriche

- Studiare il carattere delle seguenti serie:

1) $\displaystyle\sum_{n=1}^{+\infty} \frac{\log\left|2 - \left[1 + \frac{1}{(n+5)^\alpha}\right]^{n^2}\right|}{\frac{1}{n}}$, $\qquad (\alpha \in I\!\!R)$

2) $\displaystyle\sum_{n=1}^{+\infty} \frac{\sin\left[\log\left(1 + \frac{3}{n}\right)\right] - \log\left(1 + \frac{3}{\sqrt{n}}\right)}{\left(\frac{1}{n}\right)^\alpha}$, $\qquad (\alpha \in I\!\!R)$

3) $\displaystyle\sum_{n=1}^{+\infty} \frac{\sqrt{n+15}}{\log(n^2+3)}$

4) $\displaystyle\sum_{k=1}^{+\infty} \frac{(4^x + 1 - |4^x - 1|)^k}{\sqrt{k}}$, $\qquad (x \in I\!\!R)$

5) $\displaystyle\sum_{n=1}^{+\infty} \frac{(2^x - 3)^n}{n^{1/20} + 3}$

6) $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{1}{x}\right)^n \frac{n!}{n^n}$, $\qquad (x \in I\!\!R \setminus \{0\})$

7) $\displaystyle\sum_{n=1}^{+\infty} \left(1 - \frac{\cos x}{n}\right)^{n^2}$, $\qquad (x \in I\!\!R)$

8) $\displaystyle\sum_{n=2}^{+\infty} (-1)^n \frac{n\log n + \sin n}{n^2 + 5}$

9) $\displaystyle\sum_{n=1}^{+\infty} \frac{\left(1 + \frac{1}{n}\right)^{\sqrt{n}} - 1 - \frac{1}{\sqrt{n}}}{n}$

10) $\displaystyle\sum_{n=1}^{+\infty} \frac{[(\log_2 x)^2 - 3]^n}{n^{\sqrt{2}} + 3}$, $\qquad (x \in I\!\!R^+)$

11) $\displaystyle\sum_{n=1}^{+\infty} \left[\left(\frac{\sqrt{x^2+2}}{x^2}\right)^n + \left(\frac{\sqrt{2n^2+1}}{n^2}\right)^{x+\frac{1}{n}}\right]$, $\qquad (x \in I\!\!R)$

12) $\displaystyle\sum_{n=1}^{+\infty} \frac{2}{n(n+2)}$, $\qquad$ (calcolarne la somma)

- Disegnare nel piano complesso l'insieme degli $z = x + iy$ tali che convergano le serie

$$\sum_{n=57}^{+\infty} \frac{n^{|x-4|-2}}{\log^8 n} \qquad \text{e} \qquad \sum_{n=57}^{+\infty} \frac{n^2 + 2}{n - 3} \left(\frac{y-3}{2}\right)^n \ .$$