

PREREQUISITI

• Equazioni

- 1) $x + 13 - 2(x - 1) = 3(x + 2) - 4$
- 2) $12x + 7(3x + 2) = 0$
- 3) $\frac{2}{3}(x - 6) + 4x = -4$
- 4) $2x^2 + 3x - 2 = x^2 + 5(x + 3) - 18$
- 5) $(x - 3)(x - 6) = 3 - x(x + 1) + x^2$
- 6) $x(x + 1) + 2 - 3x + x(x + 2) = 0$
- 7) $x(x^2 + 2x) - 3x^3 + x^4 = 1 - 2x^3$ $(x = \pm\sqrt{\sqrt{2} - 1})$
- 8) $2x^4 + 4x^2 + 2 = 0$ (impossibile)
- 9) $-18x^2 + 18x^3 = -18 - 9x(x - 3x^2)$ $(x = 1)$
- 10) $2x^3 + 10x = 8x^2 + 4$ $(x = 1; 2)$
- 11) $3x^3 - x^2 - 1 = 2 - 3x + 2x^2$ $(x = 1)$
- 12) $x^4 + x^3 - 6x^2 - 4x + 8 = 0$ $(x = 1; \pm 2)$

• Sistemi

- 1) $\begin{cases} 3x + 2y - 5 = 0 \\ x + y = 7 \end{cases}$
- 2) $\begin{cases} 4x + 2(x - y) = 5(y + 3) \\ x = 3(x + 2y) \end{cases}$
- 3) $\begin{cases} x^2 + 2 = 5x - 4 \\ y + 3x^2 = 0 \end{cases}$
- 4) $\begin{cases} 2y^2 = 3x^2 + 5x \\ 2x^2 + 2y^2 = 6x - 2 \end{cases}$ (impossibile)
- 5) $\begin{cases} x(x - 1) = y(y + 1) \\ 2y - x^2 = 0 \end{cases}$
- 6) $\begin{cases} 2x^2 + 1 = y(y - 2) \\ y = 2x^2 \end{cases}$ $(\pm \frac{\sqrt{3+\sqrt{13}}}{2}, \frac{3+\sqrt{13}}{2})$

• Disequazioni

- 1) $3(x - 2) < 5(x + 7/2)$
- 2) $2x(x + 4) < 4x^2 + 1$
- 3) $10x^2 - 2x + 4 < 0$
- 4) $x^4 - \frac{3}{2}x^2 - \frac{1}{4} > 0$ $(x < -\frac{\sqrt{3+\sqrt{13}}}{2}, x > \frac{\sqrt{3+\sqrt{13}}}{2})$
- 5) $\frac{1}{2}x^3 - x^2 + 2x \leq 0$
- 6) $x(5x + 2) - 3(x + 1) > -5$

• Divisione di polinomi (riscrivere le seguenti espressioni razionali fratte come somma di un polinomio e di una razionale fratta, in cui il grado del polinomio a numeratore sia minore del grado del polinomio a denominatore)

$$\begin{aligned}
 1) \quad & \frac{3x^2 - 2y^2}{\sqrt{3}x + \sqrt{2}y} && (\sqrt{3}x - \sqrt{2}y) \\
 2) \quad & \frac{x^3 + y^3}{x^2 - xy + y^2} && (x + y) \\
 3) \quad & \frac{x^4 + 2x^2 + x - 4}{\sqrt{2}x^3 + 3x} && \left(\frac{1}{\sqrt{2}}x + \frac{(2-3/\sqrt{2})x^2 + x - 4}{\sqrt{2}x^3 + 3x} \right) \\
 4) \quad & \frac{x^4 + 1}{x^2 + 1} && \left(x^2 - 1 + \frac{2}{x^2 + 1} \right)
 \end{aligned}$$

• Equazioni e disequazioni razionali fratte

$$\begin{aligned}
 1) \quad & \frac{6(x-2)}{x+1} = 5 \\
 2) \quad & \frac{4x}{x-1} + 10 = \frac{4}{x-1} \\
 3) \quad & \frac{4}{x+2} = \frac{3}{x-2} \\
 4) \quad & \frac{x+2}{x-3} = x \\
 5) \quad & \frac{x^2 + 1 - 2x}{4x + 8} = \frac{1-x}{x-2+x^2} && \text{(impossibile)} \\
 6) \quad & \frac{3x^2}{x^2 - x - 1} = 2 \\
 7) \quad & \frac{3(x-2)}{x+1} \geq 2 \\
 8) \quad & \frac{x-1}{2+x} \leq 2 \\
 9) \quad & \frac{5}{x-4} > \frac{10}{x+4} \\
 10) \quad & \frac{1}{x^2 + 1} < -(x^2 + 2) && \text{(impossibile)} \\
 11) \quad & \frac{2x+1}{(x-1)^2} > \frac{2}{x-1} && (x \neq 1) \\
 12) \quad & \frac{x^3 - 2x^2 + x - 2}{x^2 - x - 2} > \frac{1}{x+1} && (x > -1; x \neq 0, 2)
 \end{aligned}$$

• Proprietà delle potenze ($x, y, z > 0$)

$$\begin{aligned}
 1) \quad & \frac{x^3 \cdot x^7}{x^{15}} = ? \\
 2) \quad & [(x^4)^7 \cdot x^2]^{1/2} = ? \\
 3) \quad & \frac{(x^3 \cdot y^3)^2}{z^6} = ? \\
 4) \quad & \frac{x^2 + x^3}{x^{1/2}} = ? \\
 5) \quad & \frac{x^4 + y^4}{(xy)^3} = ? \\
 6) \quad & \frac{\sqrt{\sqrt{x}} \cdot \sqrt[3]{x}}{\sqrt[7]{x^4}} = ?
 \end{aligned}$$

- Identità fondamentali della trigonometria

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$$

$$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$$

$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\sin \theta = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\sin \theta \pm \sin \phi = 2 \sin \frac{\theta \pm \phi}{2} \cos \frac{\theta \mp \phi}{2}$$

$$\cos \theta + \cos \phi = 2 \cos \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2}$$

$$\cos \theta - \cos \phi = -2 \sin \frac{\theta + \phi}{2} \sin \frac{\theta - \phi}{2} .$$

- Equazioni e disequazioni trigonometriche

- 1) $\sin^2 \theta + 3 \sin \theta = -2$

- 2) $\cos \theta + 3 \sin^2 \theta = 0$

- 3) $2 \cos 2\theta + 2 \sin \theta = 2$

- 4) $\cos \theta + \sin \theta \tan \theta > 1$

- 5) $\frac{\sin \theta}{\cos \theta - \sin \theta} + \frac{\sin \theta}{\cos \theta + \sin \theta} \leq 1$

- 6) $\sin^2 \theta \geq 1 + \cos^2 \theta .$

- Geometria analitica (disegnare i seguenti luoghi geometrici)

- 1) $3y + 5x + 2 = 0$

- 2) $6x^2 + 3(y - 1)^2 - 1 = 0$

- 3) $x^2 + y^2 + 2x - 2 = 0$

- 4) $36x^2 + 12x - y^2 + 4 = 0$

- 5) $3x^2 + 2y - 1 = 0$

- 6) $y^2 + 3x = 2$

- 7) $4y + 12x = 1$

- 8) $4x^2 + 2x + 4y^2 - 2y = 3/2$

- 9) $4x^2 - y^2 + 2y = 9 .$

- Logaritmi ed esponenziali

- 1) $\frac{e^{x^2}}{e^{2e^x}} = e^3$ $(x = \frac{1 \pm \sqrt{21}}{2})$
- 2) $\frac{e^{3x^3} e^3}{e^{2x}} = e^{2x^2 + 1 + x}$ $(x = 2/3; \pm 1)$
- 3) $(10)^{2x} + 2 \cdot (10)^x = 3$ $(x = 0)$
- 4) $e^{3x} + 3e^{2x} > 3e^x + 1$ $(x > 0)$
- 5) $\frac{e^{3x^2} + e^{x^2}}{(e^x)^x} \leq 3e^{x^2}$ $(-\sqrt{\log \frac{3+\sqrt{5}}{2}} \leq x \leq \sqrt{\log \frac{3+\sqrt{5}}{2}})$
- 6) $e^{4x} + e^{2x} = e^{2x+2x^2} + e^{2x^2}$ $(x = 0; 1)$
- 7) $\log 10 + \log 2 - \log 5 = ?$ $(2 \log 2 = \log 4)$
- 8) $\frac{3 \log 2 - \log 10}{1/2(\log 4 - \log 5)} = ?$ (2)
- 9) $\log_2(6x) + 2 \log_2 x - \log_2(3x) = 4$ $(x = \sqrt{8})$
- 10) $\log(3x^2 + 2x - 1) - \log(2x + 1) > 0$ $(x > \sqrt{2/3})$
- 11) $\frac{\log(3x - 2)}{\log(4x - 1)} > 1$ (impossibile)
- 12) $\log^3 x - 2 \log x \geq 0$ $(e^{-\sqrt{2}} \leq x \leq 1; x \geq e^{\sqrt{2}})$

- Valore assoluto

- 1) $|x + 10| = -3$ (impossibile)
- 2) $|x^2 + 2x| = 4$ $(x = -1 \pm \sqrt{5})$
- 3) $|x^2 + 4x| = 4x - 1$ (impossibile)
- 4) $|x^2 + 2x + 1| < 0$ (impossibile)
- 5) $|x^2 + 1| > 1/2$ $(\forall x)$
- 6) $|x + 2| \leq |x + 3|$ $(x \geq -5/2)$
- 7) $|x + 4| \leq x^2 + 2$ $(x \leq -1; x \geq 2)$
- 8) $|x^2 - 2| > x - 1$ $(x < \frac{-1+\sqrt{13}}{2}; x > \frac{1+\sqrt{5}}{2})$
- 9) $|x + 12| < 1$ $(-13 < x < -11)$

- Equazioni e disequazioni irrazionali

- 1) $\sqrt{x^2 + 2x} = \sqrt{x + 1}$ $(x = \frac{-1+\sqrt{5}}{2})$
- 2) $\sqrt[3]{x - 4} = 4 - x$ $(x = 4)$
- 3) $\sqrt{x^2 + 1} = 2x + 1$ $(x = 0)$
- 4) $\sqrt{x^4 + 1} \geq -7,5$ $(\forall x)$
- 5) $\sqrt{x^2 - 3x + 5} \leq x + 3$ $(x \geq -4/9)$
- 6) $\sqrt[3]{x^3 - 8} \geq x - 2$ $(x \leq 0; x \geq 2)$
- 7) $\sqrt{x^3 + 3x^2 + 3x + 1} \geq x + 1$ $(x \geq 0; x = -1)$

prerequisiti

- 8) $\sqrt[3]{x+125} < x+5$ ($x > 0$)
- 9) $\sqrt[4]{x+\frac{1}{2}} > \sqrt{5-x}$ ($\frac{11-\sqrt{23}}{2} < x \leq 5$)
- 10) $\sqrt{x-1} + \sqrt{-x+3} \leq 4$ ($1 \leq x \leq 3$)
- 11) $\sqrt{x^2-3x+1} + \sqrt{x^2-3} < -2$ (impossibile)
- 12) $\sqrt{\frac{18-x}{x+2}} > 2x-3$ ($-2 < x < \frac{1+\sqrt{15}}{2}$)