# Min-max representation in ergodic type Bellman equation of first order under general stability conditions

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**Abstract**: It is known that there exist principal eigenvalues and eigenfunctions in eigenvalue problems of Schrödinger operators for suitable potential functions. It can be formulated as a generalized eigenvalue problem for linear differential operators of second order. In the generalized problem, positive harmonic functions for a given constant are considered and the critical value among the constants corresponds to the principal eigenvalue. As for the representation of the critical value, a min-max type formula was proved by Donsker and Varadhan and it is considered as a generalization of the variational formula for the principal eigenvalue. On the other hand, the generalized eigenvalue problems for the linear differential operators can be reduced to the ergodic type Bellman equations of second order through logarithmic transformation. Thus, the Donsker-Varadhan min-max formula gives the representation in the ergodic type equation of second order.

In this talk, by taking the second order case into consideration, we shall study a min-max type formula for the critical value in the ergodic type Bellman equation of first order. This would be expected because we can formally deduce a formula from the min-max representation in second order case by small noise limit argument. We first discuss the min-max formula under some stability assumption, which is crucial to control the large time behavior of the dynamical system at infinity. We shall next try to extend the representation result under general situations. Due to the general stability assumption, we need to restrict the class of running costs. More precisely, the min-max formula can be obtained for bounded running cost functions. The min-max formula can be written in term of a sort of $I$-function which is a counterpart of $I$-function in Donsker-Varadhan formula. We also discuss some properties on the $I$-function in first order case.