Global existence for semilinear integrodifferential equations of hyperbolic type.

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This talk is concerned with an abstract semilinear Volterra equation of the form

$$\ddot{u}(t) + Au(t) + \int_0^t B(t-s)u(s)ds = F(u(t)) + f(t), \qquad t \geq 0\,, \tag{1}$$

where A and $B(t)$ are linear unbounded self-adjoint operators on a Hilbert space X with domains $D(A)$ and $D(B(t))$ respectively, such that $D(A) \subset D(B(t))$ and $D(A)$ is dense in X. We assume that A is positive and $B(t)x$ is absolutely continuous in $[0, \infty)$ for any $x \in D(A)$.

First, under suitable estimates on $\langle B(t)x, y\rangle$ and $\langle \dot{B}(t)x, y\rangle$, $x \in D(A)$, $y \in D(\sqrt{A})$ the linear version of the above equation is studied and a maximal regularity result is obtained.

Then, the linear theory is applied to prove a global existence and uniqueness result for the nonlinear problem. Regarding the nonlinear term, we assume that

$$F = \nabla\phi\,,$$

where $\phi : D(\sqrt{A}) \to \mathbb{R}$ is Gâteaux differentiable and satisfies a suitable condition, allowing superlinear growth at ∞.

It is worthy of mention that (1) can be regarded as a model problem for some elastic systems with memory. Therefore, our abstract results can be applied to study, for example, a partial integro-differential equation arising in the theory of viscoelasticity, in case of materials for which memory effects cannot be neglected. For this reason, the analysis of a concrete example concludes the talk.

These results are contained in a joint paper with P. Cannarsa.