ESERCIZI LEZIONE 17

1. $\vec{u} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

(I) $\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y = -5 + 6 = 1$

(II) $\|\vec{u}\| = \sqrt{u_x^2 + u_y^2} = \sqrt{1+9} = \sqrt{10}$

$\|\vec{v}\| = \sqrt{25+4} = \sqrt{29}$

$\cos\vartheta = \dfrac{u_x v_x + u_y v_y}{\|\vec{u}\| \, \|\vec{v}\|} = \dfrac{-5+6}{\sqrt{10}\sqrt{29}} = \dfrac{1}{\sqrt{290}}$

(III) Versore parallelo e concorde con $\vec{v}$

$$\vec{r} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \frac{1}{+\sqrt{29}} = \begin{pmatrix} \frac{5}{\sqrt{29}} \\ \frac{2}{\sqrt{29}} \end{pmatrix}$$

$$\boxed{\vec{r} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \frac{1}{\pm\sqrt{v_x^2 + v_y^2}}}$$

poiché sia concorde con $\vec{v}$ scelgo il versore positivo.

(IV) retta r parallela e concorde con $\vec{v}$

$r: \; 2x - 5y = 0$

componente di $\|\vec{u}\|$ sulla retta r

$$u_r = \vec{u} \cdot \vec{r} = \|\vec{u}\| \, \underset{1}{\underset{\shortparallel}{\|\vec{r}\|}} \cos\vartheta = \cancel{\sqrt{10}} \cdot \frac{1}{\cancel{\sqrt{10}}\sqrt{29}} = \frac{1}{\sqrt{29}}$$

2. $P(1, -2)$ $\vec{u} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$$\begin{vmatrix} x-1 & y+2 \\ -1 & 3 \end{vmatrix} = 3x - 3 + y + 2 = \boxed{3x + y - 1 = 0}$$

$$\boxed{\begin{cases} x = -t + 1 \\ y = 3t - 2 \end{cases}} \quad \begin{cases} t = 1 - x \\ y = 3 - 3x - 2 \end{cases} \quad \begin{cases} \longrightarrow \\ \boxed{3x + y - 1 = 0} \end{cases}$$

3. $r: \; 3x + y - 1 = 0$ $O(0,0)$ $\vec{r} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

retta parallela ad r passante per l'origine

$$\begin{vmatrix} x & y \\ -1 & 3 \end{vmatrix} = \boxed{3x + y = 0}$$

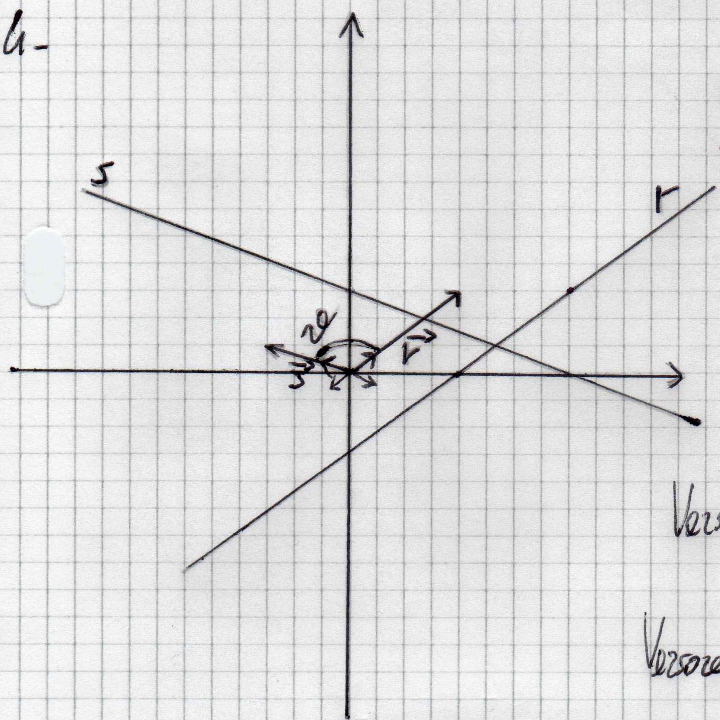
retta perpendicolare ad r passante per l'origine
- condizione di perpendicolarità

$$aa'+bb'=0 \qquad \begin{pmatrix}-1\\3\end{pmatrix}\cdot\begin{pmatrix}a'\\b'\end{pmatrix}=0 \qquad \begin{pmatrix}-1\\3\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix}=0$$

$\vec{s}=\begin{pmatrix}3\\1\end{pmatrix}$ vettore perpendicolare alla retta r

$$\boxed{x-3y=0}$$

4-



$r:\ 3x-4y=12 \rightarrow y=\frac{3}{4}x-3$

$s:\ x+3y=9 \rightarrow y=-\frac{x}{3}+3$

$\vec{r}=\begin{pmatrix}4\\3\end{pmatrix} \qquad \vec{s}=\begin{pmatrix}-3\\1\end{pmatrix}$ parametri direttori

Versori paralleli ad esse

$$\text{Versore}(\vec{r})=\begin{pmatrix}4\\3\end{pmatrix}\frac{1}{\pm\sqrt{16+9}}=\begin{pmatrix}\pm\frac{4}{5}\\ \pm\frac{3}{5}\end{pmatrix}$$

$$\text{Versore}(\vec{s})=\begin{pmatrix}-3\\1\end{pmatrix}\frac{1}{\pm\sqrt{9+1}}=\begin{pmatrix}\mp\frac{3}{\sqrt{10}}\\ \pm\frac{1}{\sqrt{10}}\end{pmatrix}$$

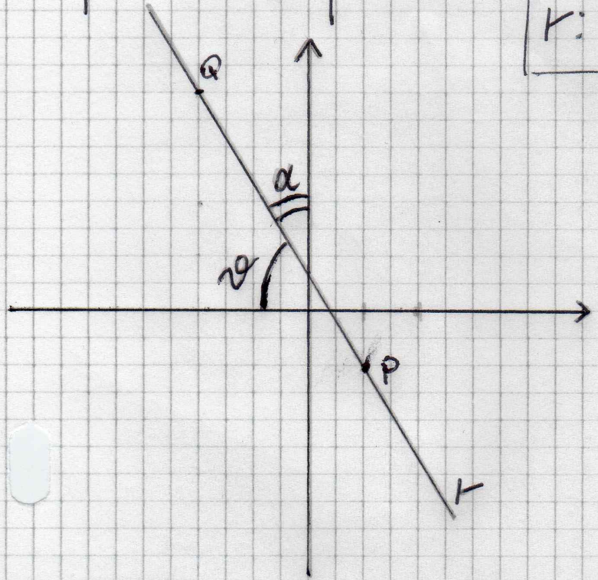
$$\cos\vartheta=\frac{r_x s_x+r_y s_y}{\|\vec{r}\|\ \|\vec{s}\|}=\frac{-12+3}{5\sqrt{10}}=-\frac{9}{5\sqrt{10}}$$

$$\vartheta=\arccos\left(-\frac{9}{5\sqrt{10}}\right)\simeq 2,17634099$$

5- $P(1,-1) \qquad Q(-2,4)$

$$\begin{vmatrix} x-1 & y+1 \\ -2-1 & 4+1 \end{vmatrix} = 0 \Rightarrow (x-1)5 + 3(y+1) = 5x + 3y - 2 = 0$$

$$\boxed{r:\ 5x+3y-2=0}$$



$\vec{r} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$ parametri direttori

$\vec{i} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \qquad \vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

versori paralleli agli assi cartesiani

$$\cos\vartheta = \frac{r_x i_x + r_y i_y}{\|\vec{r}\| \ \|\vec{i}\|} = \frac{3}{\sqrt{34}}$$

$$\vartheta = \arccos\left(\frac{3}{\sqrt{34}}\right) \simeq 1,03037$$

$$\cos\alpha = \frac{r_x j_x + r_y j_y}{\|\vec{r}\| \ \|\vec{j}\|} = \frac{5}{\sqrt{34}}$$

$$\alpha = \arccos\left(\frac{5}{\sqrt{34}}\right) \simeq 0,540419$$

6. $ax+by+c=0$ con $a\neq 0$, $b\neq 0$

$y=mx+q$ (FORMA RIDOTTA DELLA RETTA)

m = coefficiente angolare q = intercetta

parametri direttori $\begin{pmatrix} -b \\ a \end{pmatrix} = \begin{pmatrix} l \\ m \end{pmatrix}$ dove $l=-b$
$m=a$

m (coeff. angolare) $= \dfrac{m}{l} = \dfrac{a}{-b}$

CONDIZIONE DI PERPENDICOLARITA'

$r: y=mx+q$ $m=\dfrac{a}{-b}$ $\vec{r}=\begin{pmatrix} -b \\ a \end{pmatrix}$

$r': y=m'x+q'$ $m'=\dfrac{a'}{-b'}$ $\vec{r'}=\begin{pmatrix} -b' \\ a' \end{pmatrix}$

se $\boxed{\vec{r}\cdot\vec{r'}=0}$ $bb'+aa'=0 \rightarrow \dfrac{\cancel{bb'}}{\cancel{bb'}}+\dfrac{aa'}{bb'}=0 \rightarrow \dfrac{a}{b}\dfrac{a'}{b'}+1=0$

$mm'=-1 \rightarrow \boxed{m'=-\dfrac{1}{m}}$

viceversa se $m'=-\dfrac{1}{m}$

$-\dfrac{a'}{b'}=\dfrac{b}{a} \rightarrow \dfrac{aa'+bb'}{\cancel{ab'}}=0 \rightarrow aa'+bb'=\boxed{\vec{r}\cdot\vec{r'}=0}$