1

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 8 & 8 & 8 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -8 & -16 & -25 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & -8 & -16 & -25 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = U$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 4 & -3 & 0 \\ 8 & -8 & 1 \end{pmatrix} \qquad A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 4 & -3 & 0 \\ 8 & -8 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 8 & 8 & 8 & 8 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & -3 & -3 \\ 0 & 6 & 8 \end{pmatrix} \to$$

$$\to \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & 6 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \boxed{2} \end{pmatrix} = U$$

$$L = \begin{pmatrix} 2 & 0 & 0 \\ 4 & -3 & 0 \\ -2 & 6 & 2 \end{pmatrix} \qquad A = LU = \begin{pmatrix} 2 & 0 & 0 \\ 4 & -3 & 0 \\ -2 & 6 & 2 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{pmatrix}$$

$$A = \begin{pmatrix} \boxed{1} & 2 & 3 \\ \boxed{4} & 5 & 6 \\ \boxed{7} & 8 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & \boxed{-3} & -6 \\ 0 & \boxed{-6} & -4 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & \boxed{8} \end{pmatrix} = U$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 4 & -3 & 0 \\ 7 & -6 & 8 \end{pmatrix} \qquad A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 4 & -3 & 0 \\ 7 & -6 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{pmatrix}$$

② SARRUS

$$(1\cdot 5\cdot 8) + (2\cdot 6\cdot 7) + (3\cdot 4\cdot 0) - (7\cdot 5\cdot 3) - (0\cdot 6\cdot 1) - (8\cdot 2\cdot 4) = -45$$

LAPLACE (SECONDA COLONNA)

$$-2 \begin{vmatrix} 4 & 6 \\ 7 & 8 \end{vmatrix} + 5 \begin{vmatrix} 1 & 3 \\ 7 & 8 \end{vmatrix} = -2(32-42) + 5(8-21) = 20 - 65 = -45$$

③ $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 8 \end{pmatrix}$

$$C_{11} = \begin{vmatrix} 5 & 6 \\ 0 & 8 \end{vmatrix} = 40 \qquad C_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = -3$$

$$C_{12} = -\begin{vmatrix} 4 & 6 \\ 7 & 8 \end{vmatrix} = 10 \qquad C_{32} = -\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = 6$$

$$C_{13} = \begin{vmatrix} 4 & 5 \\ 7 & 0 \end{vmatrix} = -35 \qquad C_{33} = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = -3$$

$$C_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 8 \end{vmatrix} = -16$$

$$C_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 8 \end{vmatrix} = -13$$

$$C_{23} = -\begin{vmatrix} 1 & 2 \\ 7 & 0 \end{vmatrix} = 14$$

④

$$A = \begin{pmatrix} 1 & 2 & 0 & 4 \\ 4 & 5 & 0 & 7 \\ 8 & 8 & 8 & 8 \\ 1 & 2 & 0 & -4 \end{pmatrix}$$

Per calcolare il determinante uso il Teorema di Laplace lungo la terza colonna.

$$\det A = 8 \begin{vmatrix} 1 & 2 & 4 \\ 4 & 5 & 7 \\ 1 & 2 & -4 \end{vmatrix} = 8 \left[1 \begin{vmatrix} 5 & 7 \\ 2 & -4 \end{vmatrix} - 2 \begin{vmatrix} 4 & 7 \\ 1 & -4 \end{vmatrix} + 4 \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix} \right] =$$

$$= 8 \left(-34 + 46 + 12 \right) = 192$$

⑤

$$\begin{cases} 2x + y + 3z = 1 \\ 4x - y + 3z = -4 \\ -2x + 5y + 5z = 9 \end{cases} \qquad A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{pmatrix}$$

è la stessa dell'esercizio ①

$$LY = B \Rightarrow \begin{pmatrix} 2 & 0 & 0 \\ 4 & -3 & 0 \\ -2 & 6 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix} \Rightarrow \begin{matrix} y_1 = \frac{1}{2} \\ y_2 = 2 \\ y_3 = -1 \end{matrix}$$

$$UX = Y \Rightarrow \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 2 \\ -1 \end{pmatrix} \Rightarrow \begin{matrix} x_1 = 0 \\ x_2 = \frac{5}{2} \\ x_3 = -\frac{1}{2} \end{matrix}$$