Analisi Matematica I (A.A. 2010/2011)

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<u>Esercizi</u> su: calcolo integrale

Esercizio 1. Calcolare i seguenti integrali:

$$\int_0^{\ln(2)} \sqrt{\frac{e^x-1}{4}}\,dx, \qquad \int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx, \qquad \int \sin(x)\cdot e^x\,dx,$$

$$\int_0^1 \frac{x\cdot e^{\arctan(x)}}{\sqrt{1+x^2}^{\,3}}\,dx, \qquad \int_0^1 \frac{x}{1+\sqrt{x}}\,dx, \qquad \int_0^{\pi^2-1} \frac{x\cdot\sin\sqrt{x+1}}{\sqrt{x+1}}\,dx,$$

$$\int_0^1 \sqrt{1+\sqrt{x}}\,dx, \qquad \int_1^3 e^{\sqrt[3]{x}}\,dx, \qquad \int_0^3 \cos\big(\sqrt{x+1}-1\big)\,dx,$$

$$\int_0^{\frac{\pi}{4}} \frac{\arctan(x)}{1+x^2}\,dx, \qquad \int_0^{\infty} \frac{1}{\sqrt{e^x-1}}\,dx, \qquad \int \frac{3x}{x^2-2x+1}\,dx,$$

$$\int_0^{\frac{\pi}{4}} x^2\cdot\cos(x)\,dx, \qquad \int \ln^2(x)\,dx, \qquad \int_{\ln(2)}^{\ln(3)} e^{2x}\cdot\ln\big(e^x-1\big)\,dx,$$

$$\int_1^{+\infty} \frac{\ln^2(x^2)}{x^2}\,dx, \qquad \int x\cdot\arctan(x)\,dx, \qquad \int x\cdot\sqrt{x-1}\,dx,$$

$$\int_0^4 \frac{x}{x^2-4x+8}\,dx, \qquad \int_0^{\pi/6} \frac{\cos(x)}{1-2\sin(x)^2}\,dx, \qquad \int_1^{\frac{e}{2}} \frac{\ln(x^2)}{x\cdot\ln(2x)}\,dx,$$

$$\int \frac{\cos^3(x)}{\sin(x)\big(1+\sin(x)\big)}\,dx, \qquad \int \frac{x^4-2x^2+10}{x^2-3x+2}\,dx \qquad \int \frac{e^x}{\sqrt{1-\big(e^x-1\big)^2}}\,dx,$$

$$\int_0^{\infty} e^{-x}\sqrt{e^x-1}\,dx, \qquad \int_1^{+\infty} \frac{\ln(x)}{(x+1)^2}\,dx, \qquad \int_0^{\infty} \frac{1}{1+e^x}\,dx,$$

$$\int_e^{+\infty} \frac{1}{x\cdot\ln^2(x)}\,dx, \qquad \int \frac{2x-1}{x^2+4x+4}, \qquad \int \operatorname{arcsinh}(x)\,dx.$$

Esercizio 2. Calcolare, al variare del parametro α, i seguenti integrali impropri:

$$\int_0^{\frac{1}{2}} \frac{1}{x\cdot|\ln x|^{\alpha}}\,dx, \qquad \int_2^{\infty} \frac{1}{x\cdot(\ln x)^{\alpha}}\,dx.$$

Esercizio 3. Studiare, al variare del parametro α, la convergenza della serie

$$\sum_{n=2}^{\infty} \frac{1}{n\cdot(\ln n)^{\alpha}}.$$

Esercizio 4. Studiare la convergenza dei seguenti integrali impropri:

$$\int_1^{\infty} \frac{\sin^6(\frac{1}{x})}{\ln(x^5+1)-5\ln(x)}\,dx, \qquad \int_1^{\infty} \frac{4e^{x^4}}{1+e^{4x^4}}\,dx, \qquad \int_0^1 \frac{\sqrt{x}\,(1-\sqrt{x})}{1-\cos\sqrt{x}}\,dx.$$

Esercizio 5. La funzione

$$f(x) = \frac{x^2 - x}{\sqrt[7]{(x-2)^8} \cdot \sqrt[5]{(x-1)^8}}$$

è (in senso improprio)

$\boxed{\text{A}}$ integrabile su $(0, 1)$

$\boxed{\text{B}}$ non integrabile su $(-1, 0)$

$\boxed{\text{C}}$ integrabile su $(3, +\infty)$

$\boxed{\text{D}}$ integrabile su $(2, 3)$